v. $\left(\operatorname{cosec}^2\frac{\pi}{2m+1}-1\right)+\left(\operatorname{cosec}^2\frac{2\pi}{2m+1}-1\right)+\cdots+\left(\operatorname{cosec}^2\frac{m\pi}{2m+1}-1\right)$

$$=\left(\operatorname{cosec}^2\frac{\pi}{2m+1}+\operatorname{cosec}^2\frac{2\pi}{2m+1}+\cdots+\operatorname{cosec}^2\frac{m\pi}{2m+1}\right)-m=\frac{m(2m-1)}{3}$$

$$\therefore \operatorname{cosec}^2\frac{\pi}{2m+1}+\operatorname{cosec}^2\frac{2\pi}{2m+1}+\cdots+\operatorname{cosec}^2\frac{m\pi}{2m+1}=m+\frac{m(2m-1)}{3}$$

$$=\frac{3m+2m^2-m}{3}$$

$$=\frac{2m(m+1)}{3}$$

vi. $\operatorname{cosec}^2\theta>\frac{1}{\theta^2}$ $\therefore \sum_{k=1}^{m}\operatorname{cosec}^2\frac{k\pi}{2m+1}=\frac{2m(m+1)}{3}>\sum_{k=1}^{m}\frac{(2m+1)^2}{k^2\pi^2}$

$$=\frac{(2m+1)^2}{\pi^2}\sum_{k=1}^{m}\frac{1}{k^2}$$

$$\therefore \left(\sum_{k=1}^{m}\frac{1}{k^2}\right)\cdot\frac{(2m+1)^2}{4m(m+1)}<\frac{\pi^2}{6}.$$

vii. From 8a iv & vi

$$1-\frac{4m(m+1)}{(2m+1)^2}=\frac{1}{(2m+1)^2}<1-\frac{6}{\pi^2}\sum_{k=1}^{m}\frac{1}{k^2}<1-\frac{2m(2m-1)}{(2m+1)^2}=\frac{6m+1}{(2m+1)^2}$$

viii. $\lim_{m\to\infty}\frac{1}{(m+1)^2}=0$ & $\lim_{m\to\infty}\frac{6m+1}{(2m+1)^2}=\lim_{m\to\infty}\frac{\frac{6}{m}+\frac{1}{m^2}}{\left(2+\frac{1}{m}\right)^2}=0$

$$\therefore 0\le 1-\frac{6}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{k^2}\le 0 \quad \therefore \frac{6}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{k^2}=1$$

$$\therefore \sum_{k=1}^{\infty}\frac{1}{k^2}=\frac{\pi^2}{6}.$$