

ANSWERS

1967 PAPER C

1. (i) See sketch below (ii) $\frac{2}{(\cos^2 x - \sin^2 x)}$ (iii) P(0, -3); Q(4, -11) (iv) (a) $x^{-4/5}$ (b) $\cos x - x \sin x$

(v) (a) $\frac{-1}{2(2x+1)} + C$ (b) $\frac{1}{2} \log_e (2x+1)$ (vi) $k = 2$ (vii) $\frac{59}{299}$ (viii) $\frac{10}{49}$ (ix) 2 units³

(x) (a) positive (b) negative (c) negative

2. (i) R(3, 4) (ii) $x = \frac{2\pi}{3}, -\frac{2\pi}{3}$

3. (i) 20 (ii) $-2n + 22$ (iii) -2 (iv) 12

4. (i) -2 (ii) (a) $\frac{1}{\sqrt{x^2 - 1}}$ (b) $e^{x - \frac{1}{x}} \cdot (1 + \frac{1}{x^2})$

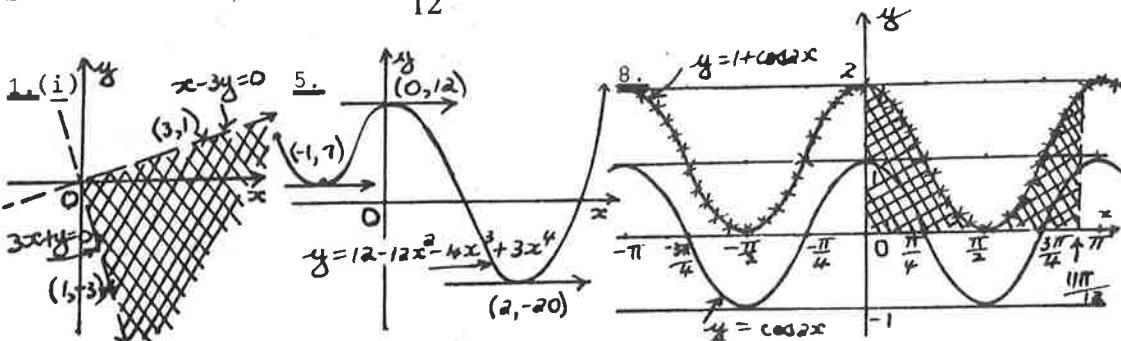
5. Stationary points: (-1, 7) minimum; (0, 12) maximum; (2, -20) minimum. (i) $-1 < x < 0$ or $x > 2$ (ii) $x < -1$ or $0 < x < 2$. Curve sketched below.

6. (3 Unit) $x^2 = 4ay$; $(\pm 2\sqrt{2}a, 2a)$ {Hint. Use chord of contact result}.

Alternative Question (2 Unit) $x^2 = 4ay$; $(\pm 4, 2)$; C(0, -2)

7. 98 seconds approximately. (Note $v = 30e^{-kt}$ where $k = \frac{1}{10} \log 1.2 \approx 0.01823$)

8. See sketch below; area = $\frac{(11\pi - 3)}{12}$ units².


1968 PAPER C

1. (i) (a) 1 (b) $-\frac{1}{2}$ (c) $\frac{-1}{\sqrt{2}}$ (ii) $5x - 4y = 10$ (iii) (4, 4); line is tangent to parabola. (iv) $x > 0, y > 0$,

$x + y < 1$

2. (i) Neither (a) nor (b) (ii) $\frac{7\pi}{6}, \frac{11\pi}{6}$ (iii) (a) $\frac{-1}{x^5}$ (b) $\frac{-2}{(2x+1)^2}$ (c) $\frac{3\sqrt{x}}{2}$

3. (i) (a) $-\frac{1}{x} + C$ (b) $\log_e x + C$ (c) $-\cos x + C$ (ii) 2 units² (iii) $a = 2$

4. Probability both white, with replacement $\frac{4}{25}$, without replacement $\frac{2}{20} = \frac{1}{10}$. Probability one white and one red, with replacement $2 \times \frac{6}{25} = \frac{12}{25}$, without replacement $2 \times \frac{6}{20} = \frac{3}{5}$.

5. Sketched below - should be drawn on graph paper.

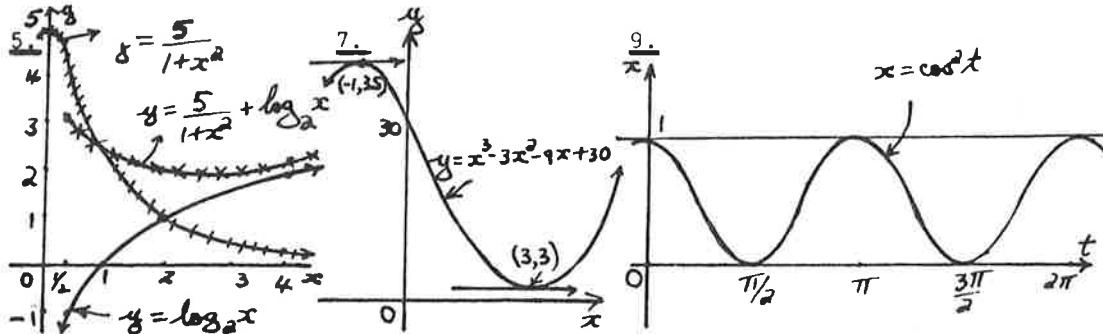
6. $\frac{5\sqrt{2}}{4}$ units; {Note join of (-1, 2) and $(\frac{1}{4}, \frac{3}{4})$ is hypotenuse.}

7. (i) Stationary points: (-1, 35) maximum; (3, 3) minimum; sketch below (ii) $9x + y = 26$, $x - 9y + 70 = 0$ (iii) line touches curves at (0, 30) and intersects it at (3, 3)

8. (i) -1 (ii) (a) $\frac{-9x^2}{(1+x^3)^4}$ (b) $\frac{1}{2} \left\{ \frac{1}{x+1} - \frac{1}{x-1} \right\} = \frac{-1}{x^2-1}$

9. (i) $x = 4t^2 - t^4$ (ii) sketch below; $v = -2\sin t \cos t$, $f = 2\sin^2 t - 2\cos^2 t$.

10. (i) (a) $\cot x$ (b) $-\sin x \cdot e^{\cos x}$ (ii) (a) $\log_e 30 \approx 3.4012$ (b) $\frac{1}{2} \log_e 2 \approx 0.3466$



1969 PAPER C

1. (i) $x = 1$; (1, -2) (ii) sketch below (iii) 14.07 (iv) $\frac{1}{7}$ (Note Sunday must be first day of year)

2. (i) (a) π (b) $\frac{2\pi}{3}$ (ii) (a) $-\sin \frac{1}{2}x$ (b) $-\sin x - \cos x$ (iii) 90°

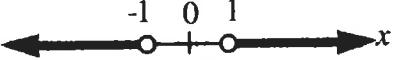
3. (i) $-xe^{-\frac{x^2}{2}}$; $(x^2 - 1) \cdot e^{-\frac{x^2}{2}}$; concave down when $x^2 - 1 < 0$, i.e. $-1 < x < 1$ (ii) (a) $\frac{1}{2} \log(2x+3) + C$

(b) $\frac{\cos \pi x}{\pi} + C$ (c) $\frac{x^5}{25} + C$ (iii) $\frac{1}{5} + \frac{2}{11} = \frac{21}{55}$

4. (i) locus of P is a circle centre O, radius 2 units {Note: If A is (-2, 0) and $\angle PAO = \alpha$, $\angle APO = \theta$,

$\angle XOP = \beta$, then $m = \tan \alpha$ and $\frac{m' - m}{1 + m'm} = \tan \theta$; thus $\alpha = \theta$, i.e. $OP = OA = 2$ units, i.e. P is always 2 units from O} (ii) {Note $\beta = 2\theta$; result is "angle at centre of a circle is twice the angle at the circumference on the same arc."}

5. (i) $\frac{1}{x \log_e x}$; $\log_e 2$ (ii) (2, 1)

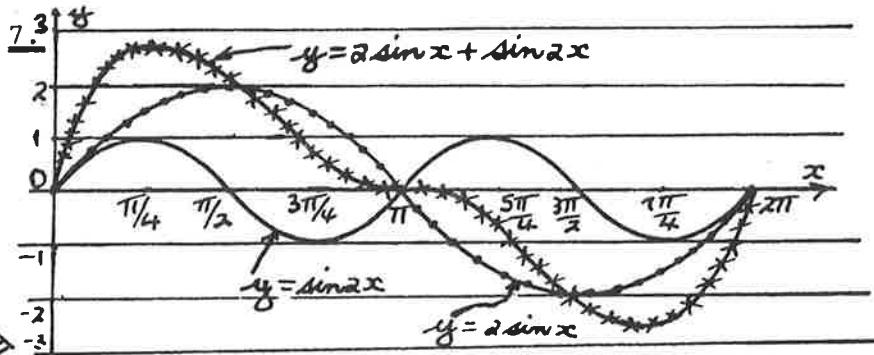
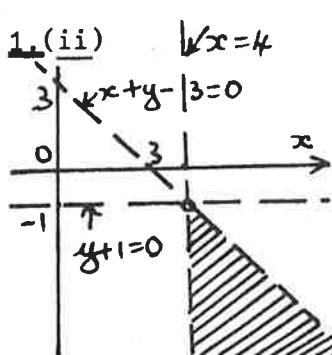
6. (i) $\frac{1}{4}(3^n + 3^{3n+1})$ {Note G.P., $r = -3$, $n = 2n + 1$ } (ii)  {Note $|x - 1|$, $|x + 1|$ represent the distances of point representing x from 1, -1 respectively.}

7. Sketches given below; (a) 3 units² (b) $2\sqrt{2}$ units²

8. (i) $-1 \leq x \leq 1$ {Note $\sqrt{1+x}$ exists if $x \geq -1$ and $\sqrt{1-x}$ exists if $x \leq 1$ } (ii) {Note $y' > 0$ } (iii) least value $-\sqrt{2}$ (when $x = -1$) and greatest value $\sqrt{2}$ (when $x = 1$)

9. (i) 6190 micrograms (Note $m = 1000e^{kt}$ where $k \approx 0.1823$) (ii) $k = 2$

10. (i) when $t = 1$, $\frac{5}{3}$ (ii) $x = -\cos t + \frac{1}{2}\cos 2t + \frac{1}{2}$; maximum 2 (when $t = \pi$)



1970 PAPER C

1. (i) (a) 0.037 (b) 3.0 (c) 4.0 (ii) $x - 3y + 2 = 0$ (iii) $3x - 2y + 1 = 0$ (iv) (a) 16 (b) -2

2. (i) -1 (ii) $-\cos(\frac{\pi}{4} - x)$ (iii) $\frac{\sqrt{6} + \sqrt{2}}{4}$

3. (i) $\frac{4}{3}$ units² (ii) (a) $-\frac{1}{x^2} + C$ (b) $\log_e(x+2) + C$ (iii) $\frac{5}{4}$ cm

4. (i) sketch below (ii) $-2 \leq f(x) \leq 2$ (iii) 2 (iv) 0

5. (i) $x - y = 1$ (ii) 90° (iii) (0, -1)

6. (i) limiting position is tangent at point (2, 4) whose equation is $y = 4x - 4$ (ii) (a) $x \cos x + \sin x$ (b) $-\frac{1}{x}$

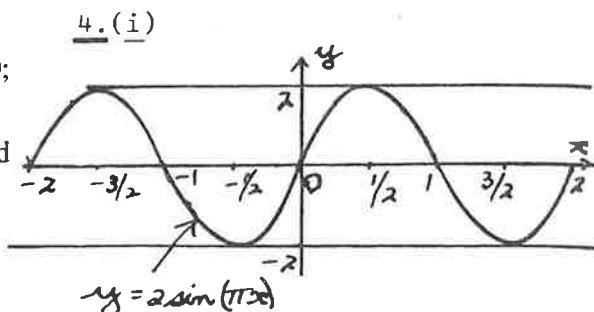
7. (ii) 14 cm/s (iii) 16 cm/s

8. (i) -3, 1 (ii) $x > 0$; $-\frac{\pi}{2} < x < \frac{\pi}{2}$ (Hint: use a sketch of $y = \cos x$; when is $\cos x > 0$?)

9. (i) (a) $\frac{\sin x - \cos x}{(\sin x + \cos x)^2}$ (b) $\frac{x+1}{(x^3 + 3x)^{2/3}}$ (ii) 130;

distance travelled by a particle in the time period $t=0, t=4$ is 130 cm.

10. (i) (a) $\frac{5}{36}$ (b) $\frac{6}{36} = \frac{1}{6}$ (ii) $1 - \frac{11}{36} = \frac{25}{36}$



1971 PAPER C

1. (i) $-\frac{4}{5}$ (ii) sketch below (iii) 1 (iv) (a) $-e^{-x}$ (b) $2 \sec^2 2x$ (c) $\frac{-2}{x^3}$

2. (i) $-2e^x \sin x$ (ii) $x = 1, 3$ (iii) not positive definite

3. (i) 6 units² (ii) $\frac{8}{45}$ (iii) $\frac{7}{50}$

4. (i) 0.3894; by integration 0.3894 (ii) (a) $\log_e(1 + 2x^2) + C$ (b) $\frac{-2}{\sqrt{x}} + C$

5. (i) (a) not A.P. or G.P. (b) G.P. (c) A.P. (d) not A.P. or G.P. (ii) A.P. $a = -3$, $d = 2$; $u_{10} = 15$, $S_{10} = 60$

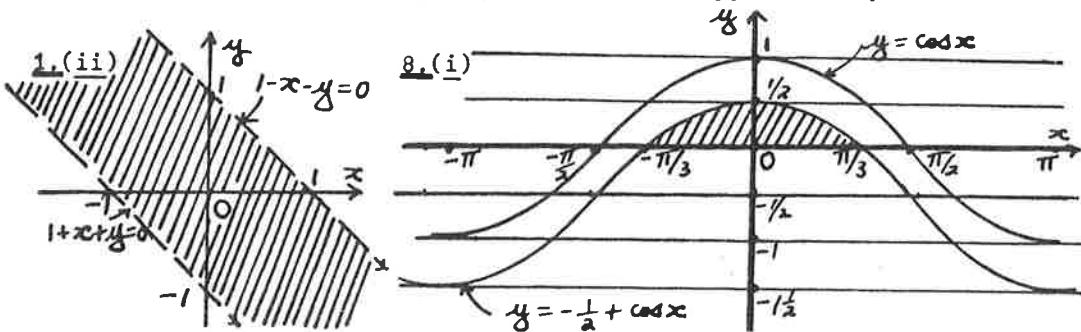
6. (i) (1, 3) (ii) $2x + y = 5$, $2x - y = -1$ (iii) $\frac{4}{3}$

7. (i) $x - y = 1$ (ii) $2x + y = 2$ (iii) $(\frac{4}{3}, \frac{2}{3})$

8. (i) sketch below (ii) $(\pm \frac{\pi}{3}, 0)$ (iii) $(\sqrt{3} - \frac{\pi}{3})$ units²

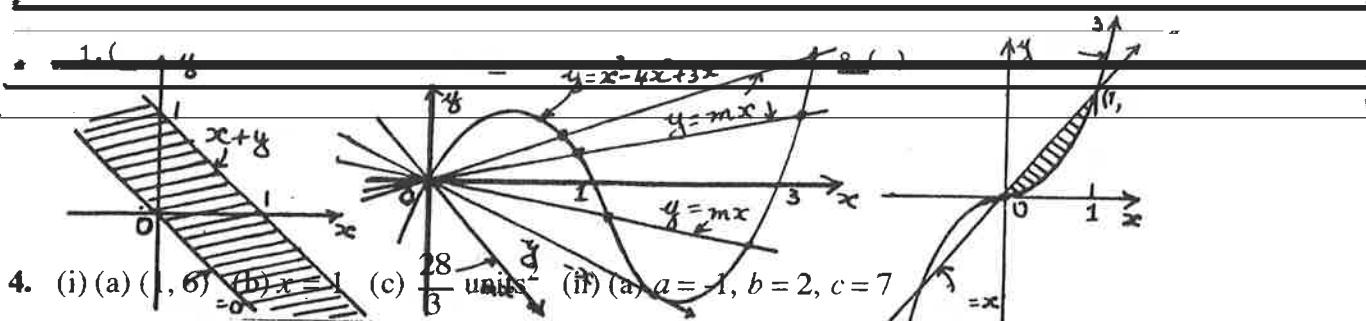
9. (i) -9 (ii) $t = 1, 3$ (iii) 0 (when $t = 0, 3$)

10. (i) $y = \frac{1}{4}(4 - x^2)$; {Use definition: distance from P(x, y) to focus (0, 0) is equal to distance from P to directrix, i.e. $\sqrt{x^2 + y^2} = |2 - y|$ and square} (ii) 8282 approximately



1972 PAPER C

1. (i) sketch below (ii) $y = 4x + 1$ (iii) (a) $\frac{-2x}{(1+x^2)^2}$ (b) $-2(\cos x - \sin x)(\sin x + \cos x)$ (iv) $\frac{1}{12}$

 -2 $\sqrt{2} \quad \sqrt{2}$ 5 

5. (i) (a) $n(55 - 3n)$ (b) least n is 19 (ii) (a) 83 (b) $\frac{3}{2}(3^n - 1) + 2n$ {Hint: sum of G.P. and A.P.}

6. (i) (b) $t = 1 + \frac{\pi}{2}x, x = 2$ (ii) (a) $\frac{7\pi}{6}, \frac{11\pi}{6}$; (b) no solutions (c) no solutions

7. (i) sketch below (ii) {Hint: line $y = mx$ cuts curve at least once where $x = 0$; it will not cut the curve

3. (i) 31.415 cm^2 (ii) 8 (iii) 7 cm

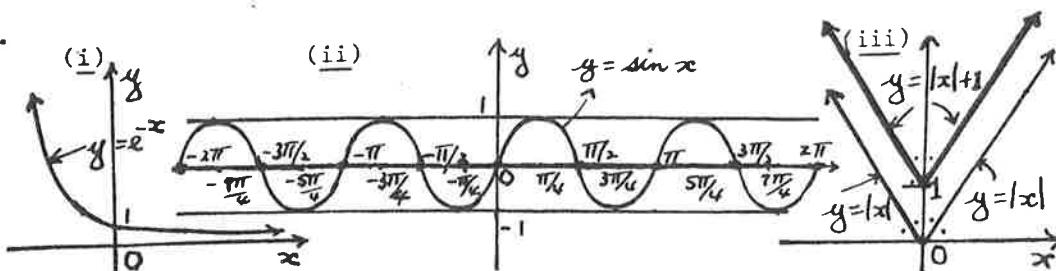
4. (i) (a) $\frac{2}{9}$ (b) $2 \times \frac{1}{6} = \frac{1}{3}$ (c) $\frac{1}{45}$ (ii) A.P. common difference $\log 2$ (b) $1325 \log 2$

5. (i) (a) $\frac{2}{3}$ (b) $\log 2 - \frac{1}{2}$ (ii) $0 < f(x) \leq 1$ (iii) $\frac{1}{5}$

6. (i) $5x - y = 1$ (ii) (4, -8) & (2, 4) (iii) Note $S = \pi \{ c^2 - 2cr + 2r^2 \}$; $R + r = c$

7. (i) (a) $t = 0, 4$ (b) 6 m/s^2 (c) $5 \text{ m} = \int_3^4 (12t - 3t^2) dt$ (ii) $\frac{9\pi}{2} \text{ units}^3$

8.



9. (i) $k \leq 1$ or $k \geq 9$ (ii) $1 < k < 9$ (iii) $b = -24$

10. (3 Unit) (i) $y - px + ap^2 = 0$ (ii) $\{a(p+q), apq\}$ (iii) $\{a(p+q), \frac{1}{2}a(p^2+q^2)\}$. Alternative Question (2 Unit) (i) $3x - y = 9$ (ii) (2, -3) (iii) (2, 5)

1974 PAPER C

1. (i) $\frac{7}{5}, \sqrt{2}, \frac{10}{7}, 1.4$ (ii) (0, 8) & (0, 0) (iii) $(4 + 2\sqrt{2}) \text{ units}^2$ (iv) The 26 letters do not occur with equal frequency; 'e' occurs far more often on any page than any other letter; probability 'e' being chosen is greater than $\frac{1}{26}$.

2. (i) (a) $3x^2 - 5$ (b) $10(2x - 3)^4$ (c) $x(2\cos x - x \sin x)$ (ii) (a) $\frac{2}{3}x^{3/2} + C$ (b) $-\frac{x^{-0.3}}{0.3} + C$ (iii) $\frac{33}{2}$

3. (i) (a) 4 (b) $9\left(\frac{2}{3}\right)^{n-1}$ (ii) (a) 148 (b) 3725 (iii) $50 + 50 = 100 \text{ cm}$

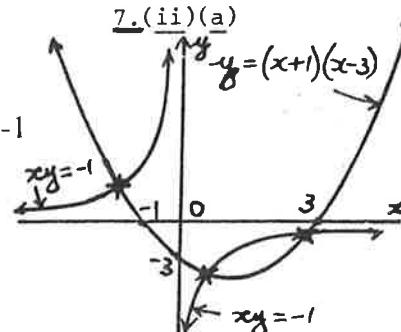
4. (i) (a) 0.596 (b) $6\cos 2t$ (ii) $\frac{121}{9} \text{ m/s}$ ($v = 4t + 1 + \frac{4}{t}$)

5. (i) (a) stationary points (-3, 27) max. and (1, -5) min. (b) $x > -1$

(ii) $a = \frac{1}{4}, c = 4$

6. (i) (a) 1226 m^2 (b) 110 m (ii) 200 540 km approximately

7. (i) $x = -\frac{8}{3}$ ($x \neq -6$) (ii) (a) sketch at side (b) $x^3 - 2x^2 - 3x + 1 = 0$



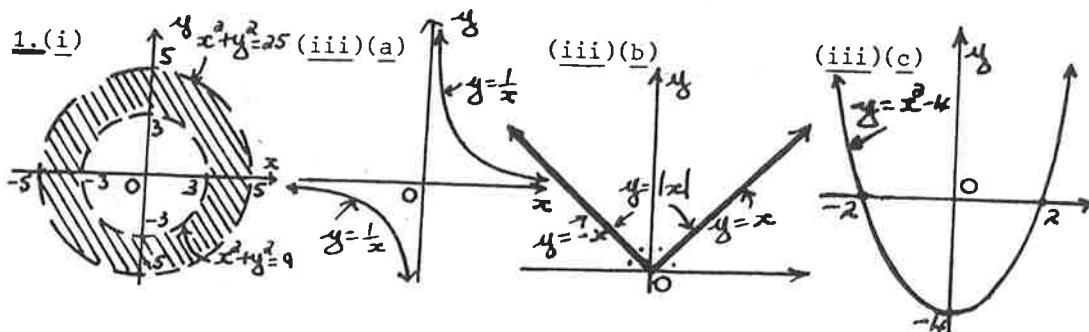
8. (i) (a) 72.9% (b) 27.1% (ii) $6y = x^2 - 2x + 10$; parabola

9. (i) $x - y = 1$ (ii) $b = e$ (iii) 0.2

10. (i) 19.9 units³ (ii) (a) approximately 1073 (b) approximately 7.5 bacteria/min.

1975 PAPER C

1. (i) sketch below (ii) (a) -0.3640 (b) -1.0642 (iii) sketches below (iv) (1, -3)



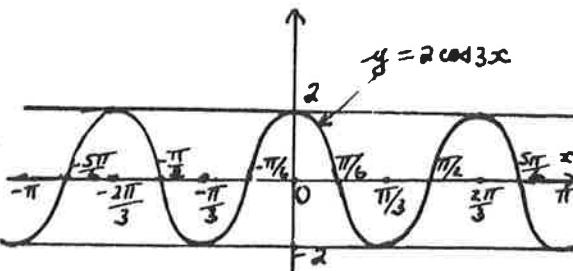
2. (iii) $\frac{3\sqrt{3}a^2}{4}$ units²

3. (i) (a) $3x^2 + 4$ (b) $1 + \log_e x$ (ii) (a) $\frac{1}{4}x^4 + \frac{2}{3}x^{3/2} + C$ (b) $\log_e(x+3) + C$ (c) $2\sqrt{x} + C$

4. (i) first term 5, common difference 4 (ii) (a) $\frac{1}{xy}$ (b) $\frac{2}{x}$ (iii) $\frac{e^x(x^2 - 2x + 2)}{x^3}$

5. (i) $\frac{4}{5}$ (b) $\frac{12}{13}$ (c) $\frac{63}{65}$ (ii) 5.71 km approximately

6. (i) $x = \pm 2, \pm 3$ (ii) $k \leq -5$ or $k \geq 3$ (iii) $\frac{1}{7}$



7. (i) $\frac{64\pi}{3}$ units³ (ii) sketch at side; period $\frac{2\pi}{3}$

8. (i) stationary points $(-\frac{1}{3}, \frac{23}{27})$ maximum and $(5, -75)$ minimum (ii) $a = -3, b = -5$

9. (i) (a) $x = 4, v = 2$ (b) $a = 4e^{2t}$ (c) $a = 4(x - 3)$ (ii) Note $v = 3(t - 1)^2 + 1 \geq 1$

10. (3 Unit) (i) $y - \frac{1}{2}(p+q)x + apq = 0$ (ii) $pq = 1$ (iii) $\{a(p+q), \frac{1}{2}a(p^2 + q^2)\}$.

Alternative Question (2 Unit) (i) $5x - 4y = 8$ (iii) $(5, \frac{17}{4})$