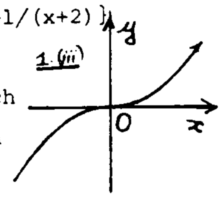
1967 PAPER B

1. (i) $2\pi/5$ units^3 (ii) $x-\log_e(x+2)+C$ {Hint: $(x+1)/(x+2)=1-1/(x+2)$}

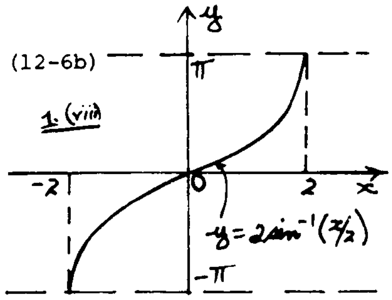


(iii) $3x\{2\cos(x^3)-3x^3\sin(x^3)\}$; horiz. inflexion on rising curve at $x=0$. {Hint: check sign of y' through $x=0$}; sketch at side. (iv) {Note $p-q=1$}; locus $x^2=4a(y-a/4)$; parabola vertex $(0, a/4)$, axis the y- axis, focus at $(0, 5a/4)$

(v) 972/3125 (vi) $X^2+Y^2=4$; circle centre (1, 2) radius 2 units

(vii) $10\sin(\theta+\pi/6)$ (viii) $-2\le x\le 2$; $-\pi\le y\le\pi$; sketch at side

(ix) $\pi/2$ (x) (a) $2^{n+1}-2^{-n}$ (b) no limit.



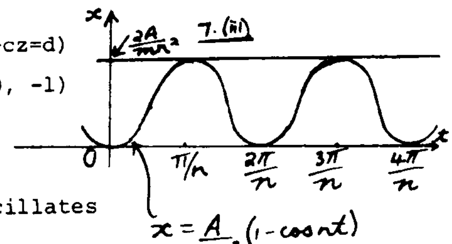
2. (i) quotient $x^2+(2-b)x+(4-3b)$; remainder $(12-6b)$ (ii) $b=2$ (iii) 2, $\pm\sqrt{2}$.

3. (i) $(2n+3)(n+2)$ (ii) 1/3

4. (ii) (a) $2x\{1+\log(x^2+1)\}$ (b) $(x^2+1)^{x^2+1}.2x\{1+\log(x^2+1)\}$

5. (i) (a) 420 (b) 456 (c) 468 (ii) 468 exact value; %age error in (a) is approx. 10.3%, in (b) is 2.5%, in (c) no error.

6. (i) $x+y+z=1$ (Hint: Let plane be $ax+by+cz=d$) (ii) $x+y=2$ (iii) (a) (0, 2, -1) (b) (2, 0, -1)



7. (i) $a=\frac{A}{m}\cos nt$ (ii) $v=\frac{A}{mn}\sin nt$

(iii) $x=\frac{A}{mn^2}(1-\cos nt)$; the particle oscillates between $x=0$, $x=2A/(mn^2)$; we can show from (i) and (iii) that $\ddot{x}=-n^2(x-A/mn^2)$; i.e. the motion is S.H.M. with centre at $x=A/mn^2$ and period $2\pi/n$.

8. {Hint: Write out in full the sample spaces; in (i) there are 7 events in the sample space whilst in (ii) there are 22 events; taking into account in each part, the announcements made by the man.} (i) (a) 3/7 (b) 4/7 (ii) 6/22=3/11.

1968 PAPER B

1. (i) 1/4 units^2 (ii) $\frac{1}{2}\tan^{-1}\frac{1}{2}x+C$ (iii) $2\sec^2(x^2)\{1+4x^2\tan(x^2)\}$ (iv) locus is $(x+2)^2=36ay$. {If P^1 has coords (x, y) then $x=6ap-2$, $y=ap^2$.}

2. (i) $a=4$, $b=-3$ (ii) 18/343 (iii) 2/3; true value $2/\pi$

3. (i) $Y=X+3$ (ii) (a) $\{3^{-n+2}-3^{-3n}\}/8$ (b) limit 0 (iii) $b=-15$, $c=14$

4. (i) $-1\le x\le 1$; $-\pi/2\le\sin^{-1}(x^2)\le\pi/2$ (ii) $\sin^{-1}(x^2)+2x^2/\sqrt{1-x^4}$; (a) horiz. inflexion on rising curve at $x=0$ (b) tangent to curve at $x=1$ is vertical (iii) greatest value $\pi/2$ (at $x=1$) and least value $-\pi/2$ (at $x=-1$); there are no local max. or min. (note $y^1>0$ for $-1<x<1$).

5. (ii) Use sum of midordinate rectangles and result in (i); limit = 4.

6. (i) $(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+{}^nC_rx^r+\ldots+{}^nC_nx^n$; relation is ${}^{n+1}C_r={}^nC_{r-1}+{}^nC_r$

(ii) Hint: by direct integration result is 0; other method is to expand by binomial theorem and integrate term by term; then substitute limits.

7. (i) Successive approximations $x_0, x_1, x_2 \ldots$ to a root of an equation $f(x)=0$ are related in Newton's Method by $x_{n+1}=x_n-f(x_n)/f^1(x_n)$ (ii) $x_{n+1}=\frac{2}{3}x_n+a/(3x_n{}^2)$ (iii) By Newton's Method $x_2=25/12 \doteqdot 2.083$; from tables, $\sqrt[3]{9}=2.080$; %age error $\doteqdot$ 0.2%.

8. (i) Surface of circular cylinder with generators parallel to z- axis, cutting xy- plane in circle centre (2, 0, 0) radius 1 unit (ii) interior of circular cylinder with generators parallel to x- axis, cutting yz- plane in circle centre (0, 2, 0) radius 1 unit. (iii) empty set (iv) (a) circumference of circle centre (2, 0, 0) in xy- plane, with radius 1 unit (b) circumf. of circle in plane z=2, with centre (2, 0, 2) and radius 1 unit (c) infinite rectangular strip in xy- plane, of width 2 units, bounded by (but not including) planes y=1, y=3 (d) empty set.

9. (i) Eqns. of motion $m\ddot{x}=0$, $m\ddot{y}=-mg$; with initial conditions t=0, x=0, y=0, $\dot{x}=V_1$, $\dot{y}=V_2$ (ii) $T=2V_2/g$; time of flight does not depend on the initial horiz. vel. of the projectile (iii) Here we are given $\ddot{y}=-f(y)$, where initially t=0, $\dot{y}=V_2$, y=0 (a) If we were to integrate this equation twice, then since the constants of integration could not involve V_1, so y does not involve V_1, and hence T (obtained by setting y=0) is still indep. of V_1 (b) As the projectile rises, the acceleration is numerically less than g, and hence the time taken to reach the highest point (i.e. reduce the velocity to zero) is greater. Since the time taken on the downward flight is equal to that of the upward flight, the total time T is increased.

10. (i) 5; not a possible result since can only draw 2, 4, 6 or 8. The expected value simply means that over a large number of drawings, the average value of the scores on the balls drawn would be 5. (ii) (a) 675/2048 (b) {Hint: proby. 621/4096}

1969 PAPER B

1. (i) $\frac{1}{2}\log_e(1+x^2)+C$ (ii) $-6x/(9+x^2)^2$ (iii) $y=-4x^2-4$ (iv) 0.7081 units^2

2. (i) 1/13 {Hint: list all 13 possible outcomes} (ii) 1 (iii) 61/31

3. (i) $-1\leq x\leq 1$; $0\leq f(x)\leq \pi/4$ (ii) 0, 0; conclusion: there is stationary pt. at $x=\frac{1}{2}$, since $f'(\frac{1}{2})=0$; however $f''(\frac{1}{2})=0$ does not give any further information about this stat. pt. (it need not be a horizontal pt. of inflexion). {Note $f''(x)=6(2x-1)^2>0$ for all x except $x=\frac{1}{2}$; i.e. curve concave up and there is a min. at $x=\frac{1}{2}$} (iii) $-1/3 < x < 3$; note $\{(2x-1)/(2+x)\}^2<1$.

4. (ii) 3.133 approx. (iii) 0.3%

5. (i) all points in space outside the sphere centre (2, 0, 1) radius 1 unit (ii) pair of planes parallel to xy- plane through (0, 0, ±1) (iii) set of

points inside and on the circumference of the circle centre (2, 0, 1) radius 1 unit and on plane z=1.

6. (i) ${}^{40}C_5=658008$ (ii) $10.36/{}^{40}C_5 = 5/9139$ (iii) $4.{}^{10}C_5/{}^{40}C_5=14/9139$

7. (i) $k(x-2)^2(x-3)$ where k is constant (ii) $1(x-2)^2(x-3)$; unique (iii) $1(x-2)^2(x-3)(x-a)$ where $a\neq 2, 3$

8. $y-\frac{1}{2}(p+q)x+apq=0$; $pq=-1$; locus $x^2=a(y-3a)$

9. (i) Hint: Equate two results for the derivative of e^x (ii) find inner rectangle sum, and use sum of G.P. and the result in (i)

10. (iii) constant $= \frac{1}{2}Aa^4$; Note $v^2=\frac{1}{2}A(a^4-x^4)\geq 0$ if $-a\leq x\leq a$; particle oscillates between $x=\pm a$, but not in S.H.M. since $\ddot{x}$ is not of form $-n^2x$.

1970 PAPER B

1. (i) $6(1+6x^2)e^{3x^2}$ (ii) 1 units^2 (iii) $\log_e(1+\sin x)+C$

2. (i) (a) 32 (b) 16 (ii) $XY+2X+Y=0$ (iii) $\{e^x-e^{-(2k+3)x}\}/(1-e^{-2x})$

3. (i) (0, 1) min. (ii) 2/10=1/5 (iii) −0.03

4. (i) a=12 (ii) 2, −2 (others unreal)

5. (i) $4\pi/15$ units^3 (ii) Hint: Use reflection property of parabola.

6. (i) $\pi/2$ (ii) $y+z=0$ {Hint: Let plane be $ax+by+cz=d$ and find a, b in terms of c} (iii) (1/3, 1/3, −1/3)

7. (ii) $1/\sqrt{1-x^2}$

8. (ii) limit 1/5

9. (i) 1/6 (ii) 5/6. 4/5. 3/4. 2/3. 1/2. 1 =1/6 (iii) 2·5 (iv) 125/3888

10. (i) 32/9 (ii) t=85 (note x=16) (iii) $14/\frac{1}{2} = 28$ (iv) $a=-3/\{8(1+3t)^{5/4}\}$; $a=-12v^5$.

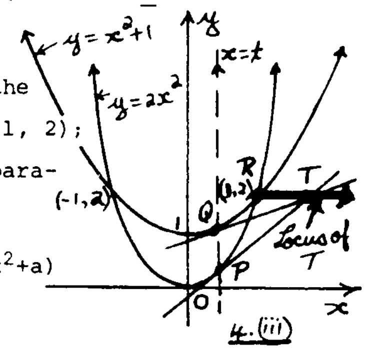
1971 PAPER B

1. (i) $9e^{3x}$ (ii) $(1-e^{-1})$units^2 (iii) $\frac{1}{3}\log x + C$ (iv) $3/(1+9x^2)$

2. (i) −0.05 (ii) (0, 1) max. (check sign of y^1 through x=0) (iii) $\cos^2 x$

3. (i) $3x^2/2\sqrt{1+x^3}$ (ii) $Y=-6/(X^2+2)$ (iii) $x\geq 1$ {Note $x^2-1\geq 0$ as well as $x+\sqrt{x^2-1}>0$ } derivative $1/\sqrt{x^2-1}$

4. (i) $y=4tx-2t^2$ (iii) precise locus of T is the ray y=2 for $x\geq 1$ {Note the curves intersect at (±1, 2); also the tangents cannot intersect 'inside' the parabolas.}



5. (i) $1x(x-2)(x+2)$; unique (ii) $1(x-2)(x+2)(x^2+a)$ where a>0; not unique; many such polynomials.

6. (i) 0.1296 (ii) 0.4752 (iii) 2.4

7. (i) $-m(\cos t)^{m-1}\sin t$; $1/(2k+1)$ (ii) {Hint: Expand by Binomial Theorem, integrate term by term between the limits $\pi/2$, 0 using the result in (1)} (iii) 8/15

8. (i) For each number y in the range of a function f there exists an x (perhaps more than one) in the domain of f such that $y=f(x)$, i.e. $f(x)=y$. Now if $f(x)$ is strictly increasing, then for each y in the range, there is one and only one value of x; this value of x is a function of y, called the inverse function, written f^{-1}; i.e. $x=f^{-1}(y)$ (ii) If P (a, b) lies on $y=f(x)$, then $b=f(a)$; its reflection Q in $y=x$ has coordinates (b, a) and thus Q lies on curve $y=f^{-1}(x)$ since $a=f^{-1}(b)$ i.e. $b=f(a)$ which is true. The locus of Q (i.e. the reflected curve) is the graph of $y=f^{-1}(x)$.
(iii) $y=\log_e x-1$.

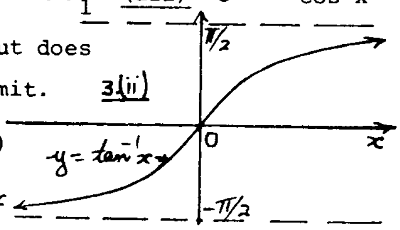
9. (i) $x-y=0$; plane perp. to xy- plane containing the z- axis and bisecting the angle between the positive x, y axes. (ii) $4/\sqrt{18}$ {Hint: we can find points on L by taking values of x and finding y, z; then obtain the eqns. of L} (iii) line touches sphere at (-2/3, -2/3, 1/3) {Use parameters}

10. (ii) $\frac{1}{2}\omega^2-\frac{g}{L}$ (iii) {at top $E=\frac{1}{2}(\frac{d\theta}{dt})^2+\frac{g}{L}>\frac{g}{L}$} (iv) {Hint $\frac{1}{2}\omega^2-\frac{g}{L}>\frac{g}{L}$ }

1972 PAPER B

1. (i) $3/(4x^{5/2})$ (ii) $\pi/6$ (iii) $(x-3)^2+(y-4)^2=4$ (iv) $y=\frac{1}{2}x^{1/3}$

2. (i) 1 units^2 (ii) since $f^1(0)=0$, tangent $x=0$ is parallel to x- axis and thus does not cut it; hence we cannot obtain x_1 (iii) $e^{\sin x}\cos x$



3. (i) 3.5 (ii) no max; curve approaches but does not reach $\pi/2$. (iii) $\{n^{k+1}-n\}/(n-1)$; no limit.

4. (i) $y=2tx-t^2$ (ii) locus $y=0$ (the x- axis)

5. (i) $x^2+y^2+z^2=25$ (ii) surface of circular cylinder generators parallel to y- axis, cutting xz plane in circle centre O radius 4 units (iii) two parallel circles, in planes $y=\pm3$, with centres $(0, \pm3, 0)$ and radius 4 units.

6. {Hints (i) $f(-0)=-f(0)$, i.e. $2f(0)=0$ (ii) use Remainder Theorem (iii) P(x) has at least 3 roots, $x=0, \pm5$} (iv) $Q(x)=1x(x-5)(x+5)$; unique (v) degree must be 7; form $x(x-5)(x+5)(x^2+a)$ where $a\neq-25$.

7. (ii) 31/16 (iii) not S.H.M. since $\ddot{x}$ not of form $-n^2x$ {Hint: $v^2=\frac{1}{8}(4x^2-31)(4x^2-1)$; factorise and sketch} (iv) 18; particle always moves to right {Hint: $v^2=2\{(x^2-4)^2+2\}>0$}

8. (i) ${}^{28}C_5=28.27.26.5$ (ii) $7.24/{}^{28}C_5=1/585$ (iii) $4.6/{}^{28}C_5=1/4095$

10. (i) 2 (iii) 2.2; 10% (iv) exact value 2

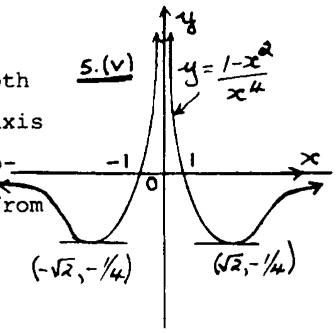
1973 PAPER B

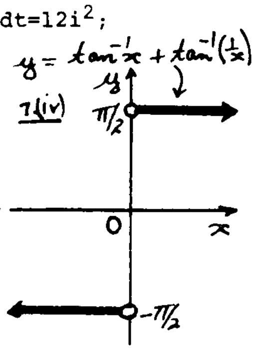
1. (i) $-2\{\sin(x^2)+2x^2\cos(x^2)\}$ (ii) $-e^{-3x}/3+C$ (iii) $2\log_e 3$ units^2 (iv) $\sqrt{3}$ units

2. (ii) (4, 3) (iii) $22/6=3.\dot{6}$

3. (i) 2.000125 (ii) 55+1023/1024; (sum of A.P. and G.P.) (iii) 1

4. (i) (a) $\pi/8$ (b) 3 (iii) 384 calories {Hint: $dh/dt=12i^2$; $h=384\int_0^1(1-\cos 200\pi t)dt$}





5. (i) ±1 (ii) $(\pm\sqrt{2}, -\frac{1}{4})$; both min. (iii) curve approaches y- axis (x=0) vertically (iv) curve approaches x- axis horizontally (from below)

6. (i) u^3-v^3 (iii) $\frac{1}{3}x^{-2/3}$

7. (i) 0 (ii) $\pi/2$ (iv) no; we cannot find $f(0)=\pm\pi/2$ simultaneously.

8. (i) (a) $-p$ (b) q (c) p^2-2q (ii) (a) 2 (b) -44 (c) 92

9. (i) 3 sec (ii) 90 m (iii) $\pi/4$ (iv) The arrow is shot off horizontally with vel. 30m/s; if air friction is very large, the horiz. vel. will become zero, and hence the only motion is vertically downwards. That is, the arrow will hit the ground at an angle nearly $\pi/2$ and thus θ is larger than $\pi/4$.

10. (i) $q=0.99998$ (iii) $p<0.000001$ {Hint: $1-10000p\geq0.99$} (iv) 0.164 approx.

1974 PAPER B

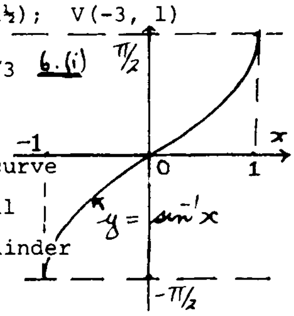
1. (i) $\frac{1}{2}e^{2x}+C$ (ii) $-\sec^2 x$ (iii) 62/5 (iv) 0.9372

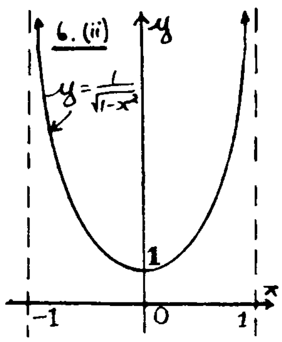
2. (i) $y=4x-2$ (ii) $\pi/4$ (iii) (a) $a=3$, $b=-2$ (b) any pair of values so $3a-2b+1=0$, e.g. $a=1$, $b=2$ or $a=0$, $b=\frac{1}{2}$ etc.

3. (i) $\frac{1}{2}\log_e 2$ units^2 (ii) $x=0, \pm\sqrt{\pi}, \pm\sqrt{2\pi}, \pm\sqrt{3\pi}, \ldots$ (iii) $-3<x<4$

4. (i) $Y=X^2/(X^2+1)$ (ii) $S(-3, 1\frac{1}{2})$; $V(-3, 1)$

5. (i) (b) $C=2$, $\alpha=\pi/6$ (ii) $x=\pm\pi/3$





6. (i) $-1\leq x\leq 1$; $-\pi/2\leq\sin^{-1}x\leq\pi/2$ (ii) as $x\to\pm1$, $f^1(x)\to\infty$, i.e. the curve is vertical (iii) If a polynomial $f(x)$ is divided by $(x-a)$, the remainder is $f(a)$; factors are $(x-1)^2(x+5)$.

7. (i) (a) $\frac{1}{2}$ (b) 17/70 (ii) (a) $n/3$ (b) (1) 1/27 (2) 2/9 (3) 4/9; expected number =1

8. (ii) interval is $0\leq x\leq 6$ {Hint: $v^2=n^2x(6-x)\geq 0$ for $0\leq x\leq 6$ from a sketch}

(iii) change origin to $x=3$, i.e. $X=x-3$, then $\ddot{X}=-n^2X$; period $2\pi/n$

9. (i) (18, 11, 13) (ii) (a) $x^2+y^2+z^2=25$; $(x-8)^2+y^2+z^2=25$ (b) plane $x=4$, radius 3 units (Hint: solve eqns. simult, then use Pythagoras' Thm)

10. {Hint: $A=2\pi r^2+2V/r$; $dA/dr=0$ when $r^3=V/2\pi$; note $r:h=r:V/\pi r^2$ etc.}

1975 PAPER B

1. (i) (-1, 1); $x=-1$ (ii) $e^x(\cos x-\sin x)$ (iii) $x+2y=2$ (iv) sketch below; 9/2 units^2

2. (i) $\frac{1}{2}n(3n-37)$; least n is 13 (ii) 1 (iii) 35

3. (i) $x-2y=2$ (ii) 1.260 (iii) $t=1$, $x=5$

4. (i) 3.73205 (ii) Hint: there are only 2 terms on R.H.S. when $n=1$

5. (ii) A $(at, 0)$; B $(0, -at^2)$ (iii) 1:-2 (iv) $(t^2-1)/2t$

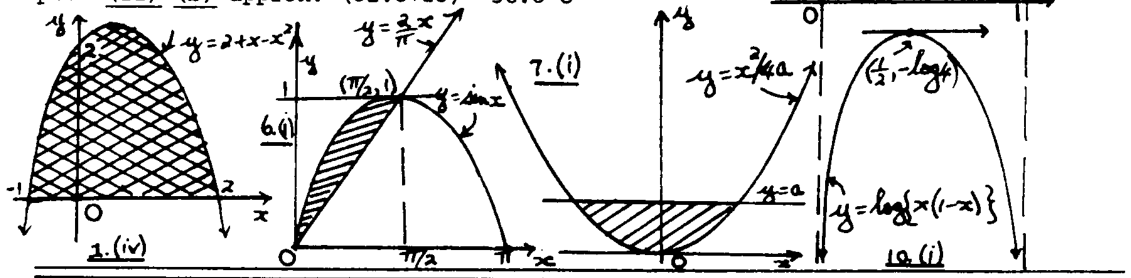
6. (i) sketch below; $(1-\pi/4)$ units^2 (ii) $1/(x\sqrt{x^2-1})$

7. (i) sketch below; $2\pi a^3$ units^3 (ii) (a) 1/36 (b) $(35/36)^n$ (c) $n\geq 25$ {Hint: $1-(35/36)^n>\frac{1}{2}$}

8. (i) $(x-a)$, note $f(a)=0$; other factor $(x+a+1)$ (ii) (a) 3 units (b) (2, 1, 2) (c) sphere on OA as diamater; $(x-1)^2+(y-\frac{1}{2})^2+(z-1)^2=9/4$.

9. (ii) $v^2=(9-x^2)(x^2-1)$ (iii) particle oscillates in interval $1\leq x\leq 3$ (Hint: factorise v^2 and draw a sketch to find where $v^2\geq 0$); not S.H.M. since $\ddot{x}$ is not of form $-n^2x$.

10. (i) Sketch below; note domain $0<x<1$ and $(\frac{1}{2}, -\log 4)$ is a max. stat. pt. (ii) (b) approx. $(31.8+25)^\circ=56.8^\circ$C



1976 3 UNIT PAPER

1. (i) (-3/5, -6/5) (ii) (a) $-2/(1+x)^3$ (b) $x\cos x+\sin x$ (iii) (a) $7n-55$ (b) $\frac{1}{2}n(7n-103)$ (iv) (a) 1/6 (b) 3/8 (c) 13/24

2. (i) (a) $x+\frac{1}{2}x^2+C$ (b) $-2/\sqrt{x}+C$ (c) $\sin x-\cos x+C$ (ii) (a) $\frac{1}{2}$ (b) 1 (iii) $t=1$, $s=0$

3. (i) sketch below (ii) $X^2+Y^2=16$ (iii) $(x+1)^2=1(y+1)$ i.e. $y=x^2+2x$

4. (i) $2\cos 65^\circ\cos 15^\circ$ (ii) sketch below (iii) $2\pi/3$

5. (i) stat. pts. $(\pm 3, -49)$ both min. and $(0, 32)$ max.; sketch below (ii) (a) $x=500$ (when $t=5$) (b) $t=7$

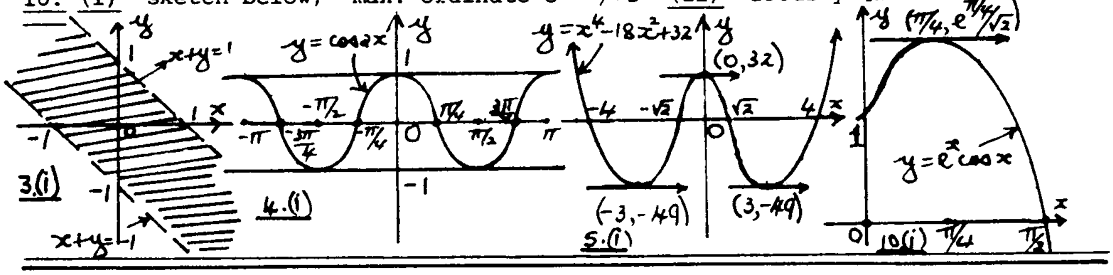
6. (i) $-1\le x\le 1$ (ii) ½ (iii) $0\le f(x)\le \pi/6$ (iv) $-\tan x$

7. (i) 14.45m^2

8. (i) 12 units (ii) 2/3, 1/3, 2/3 (iii) $\pi/2$ (iv) $2x+y+2z=0$
(v) $(x-1)^2+(y+1)^2+(z-1)^2=36$

9. (i) $(4+4+12)/6^4=5/324$ {Hint: list possible scores of 7 carefully}
(ii) $12.3^{14}/10^9$

10. (i) sketch below; max. ordinate $e^{\pi/4}/\sqrt{2}$ (ii) locus $y=\frac{1}{2}x^2-x+1$



1977 3 UNIT PAPER

1. (i) (a) $-2x/(1+x^2)^2$ (b) $x\sec^2 x+\tan x$ (ii) (a) $\frac{1}{2}e^{2x}+C$
(b) $\log_e(1+x^2)+C$ (c) $-\cos x+\tan x+C$ (iii) $\frac{1}{2}(\cos 35^\circ-\cos 65^\circ)$
(iv) $x^2+4y^2=4$

2. (i) (a) 9 (b) -4 (ii) sketch below (iii) (a) 1 (b) log 2

3. (i) sketch below (ii) (b) $-1<x<3$

4. (ii) form $(x-2)^2=4.\frac{1}{4}(y+1)$; focal length ¼ unit (iii) sketch below;
$y=2x-6$; 1/3 units^2

5. (i) 0.7472 (ii) approx $79^\circ 1'$ (iii) $f(x)=\frac{1}{2}x-\frac{1}{4}\sin 2x+\frac{1}{4}$; note
$\sin^2 x=\frac{1}{2}(1-\cos 2x)$

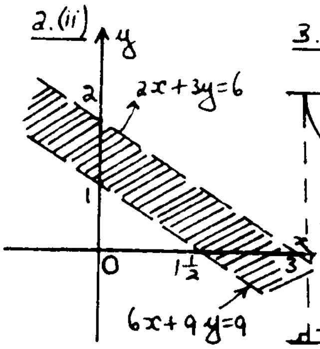
6. (i) (a) $|x|=x$ if $x\ge 0$ and $|x|=-x$ if $x<0$; or $|x|=+\sqrt{x^2}$ (b) sketch below
(ii) (a) A function $y=f(x)$ is a set of ordered pairs (x, y) so that for each first element there is one and only one second element. The set of first elements is called the domain and the set of second elements is called the range. (b) not the graph of a function; note $y=\pm\sqrt{a^2-x^2}$ and for $x=k$ in the domain, there are 2 values of y; (or the eqn. represents a circle centre 0 radius a units; we can draw vertical lines $x=k$ to cut the graph in two places). (iii) 8/3

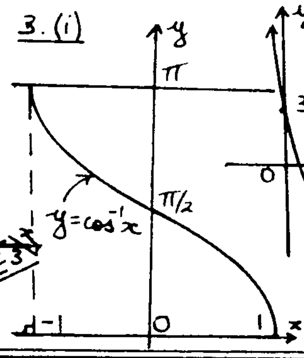
7. (i) max. value $1/e$ (when $x=1/e$) and min. value 0 (when $x=1$) {Hint: sketch curve} (ii) $\sqrt{2}\cos(\theta+\pi/4)$; $\theta=0$ only

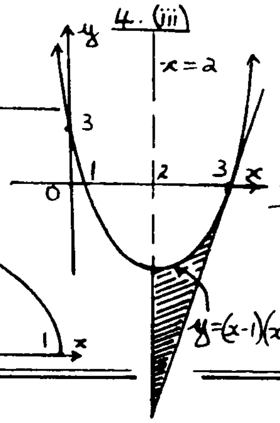
8. (i) Hint: put $x=1$ in expansion (ii) (b) Hint: show $v^2=2gR^2(x^{-1}-10^{-9})$

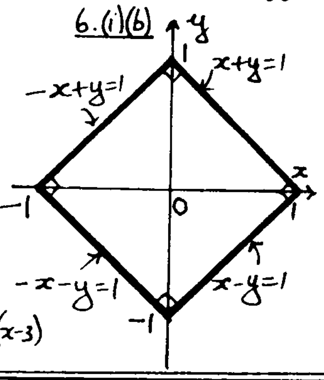
9. (i) (a) $1/2^{2n-1}$ (b) $\frac{2}{3}\{1-2^{-2n}\}$ (c) $\frac{2}{3}$ (ii) (a) $1/2^9=1/512$
(b) $\frac{2}{3}\{1-2^{-2n}\}$ (c) 2/3

10. (i) (a) $8\pi\ \text{units}^3$ (b) $(32\pi-32\pi/5)=128\pi/5\ \text{units}^3$ (ii) approx. 73.5%







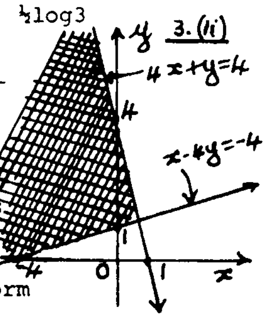


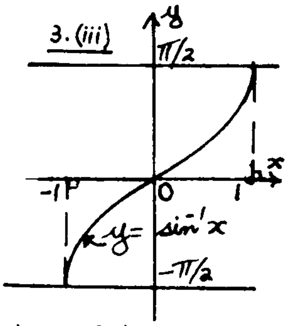
1978 3 UNIT PAPER

1. (i) (a) 5/21 (b) 3/8 (ii) 0.209 (iii) (a) 4·91 cm (b) 4.27 cm

2. (i) (a) $1/(1+x)^2$ (b) $x(x \cos x+2 \sin x)$ (ii) (a) $-\frac{1}{2} \cos 2x+C$ (b) $\log_e(1+e^x)+C$ (c) $\sin^{-1}(x/2)+C_1$ or $-\cos^{-1}(x/2)+C_2$ (iii) $\frac{1}{2}(\sin 65^\circ+\sin 15^\circ)$ (iv) (a) 2 (b) $\frac{1}{2}\log 3$

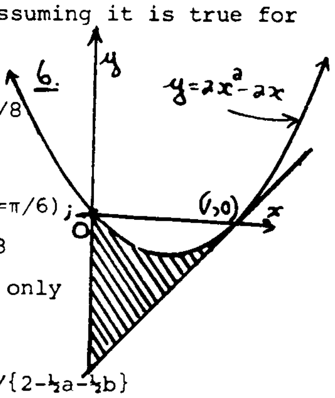
3. (i) (a) -8 (b) $\frac{1}{2}$ (ii), (iii) sketches at side





4. (i) $10\pi/3$ cm; $50\pi/3$ cm^2 (ii) (a) $125\sqrt{35}\pi/81$ cm^3(show radius of cone 5/3 cm) (b) $5\sqrt{3}$ cm (Hint: when cone is straightened out to form a flat sector, string becomes a straight line; use cosine rule)

5. (i) (a) 5.5-3.5n; $\frac{1}{2}n(7.5-3.5n)$ (b) $2(-3/4)^{n-1}$; $\frac{8}{7}\{1-(-3/4)^n\}$; from (a) answers are -8.5; -13.0; no limits as $n\to\infty$; from (b) answers are -27/32; 25/32; limits are respectively 0, 8/7. (ii) sum = n^2. Method - if a proposition is true for n=1, and it is true for n=k+1 assuming it is true for n=k, then it is true for all positive integers n.



6. (i) $y=2x^2-2x$ (ii) 1/8 unit; (1/2, -3/8), y=-5/8 (iii) y=2x-2; 2/3 units ^{2}

7. (i) max. value 2 (at θ=2) and min. value $\sqrt{3}$ (at $\theta=\pi/6$); Hint draw a sketch; test endpoints (ii) (a) x+y+z=3 (b) $x^2+y^2+z^2\leq 3$ (c) $|x|\leq 1$, $|y|\leq 1$, $|z|\leq 1$; $\sqrt{3}$ units; only point common to sets is vertex (1, 1, 1)

8. (i) (a) 1/2 (b) $\{1-\frac{1}{2}a-\frac{1}{2}b\}/\{2+\frac{1}{2}a+\frac{1}{2}b\}\leq V\leq\{1+\frac{1}{2}a+\frac{1}{2}b\}/\{2-\frac{1}{2}a-\frac{1}{2}b\}$

9. (i) (a) 0 (ii) ${}^{16}C_8(1/2)^{16}$ (iii) (a), (b) both ${}^{16}C_8(1/2)^{16}$, using results in (i) (iv) Expt. 1 toss coin 2n times, the proby. of n heads is ${}^{2n}C_n(1/2)^{2n}$; Expt. 2 toss a coin n times then toss it again n times, the proby. of the sum of heads being n (in the two sets of n tosses) is $\sum_{j=0}^{n} ({}^nC_j)^2(1/2)^{2n}$; then equate

10. (i) $3\pi/2$ units^3 (ii) (b) x=1 (c) 1.048 approx

1979 3 UNIT PAPER

1\. (i) $-17/120$ (ii) $4/\{(x-1)(x+1)\}$ (iii) (a) $\sqrt{7}$ cm (b) $\sqrt{13}$ cm (c) $-8/\sqrt{91}$

2\. (i) (a) $x+y=1$ (b) 3 (ii) (a) $1+\log x+2\sin x\cos x$ (b) $\frac{2}{3}x^{3/2}+e^{-x}+C$ (iii) $x>0$

3\. (i) (a) 4 (b) $\pi/8$ (ii) (a) 4 units^2 (b) $\pi(16+\log 9)$ units^3

4\. (ii) $x=2+\sqrt{3}\doteqdot 3.73$

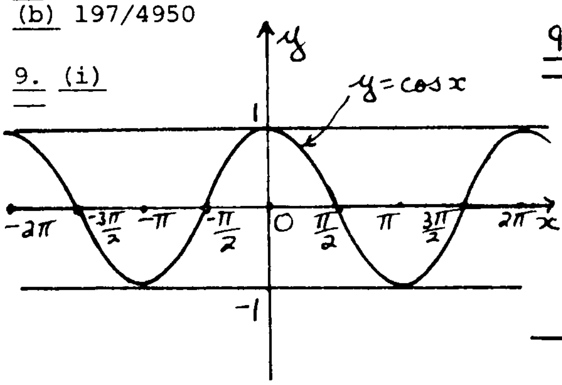
5\. (i) 60 (when $x=30$) (ii) 30 km/h

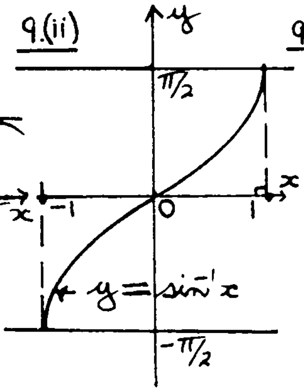
6\. (i) $(x+1)^2<-4(y+2)$ (ii) Hint: $\sin t_1+\sqrt{3}\cos t_1$ can be written $2\sin(t_1+\pi/3)$

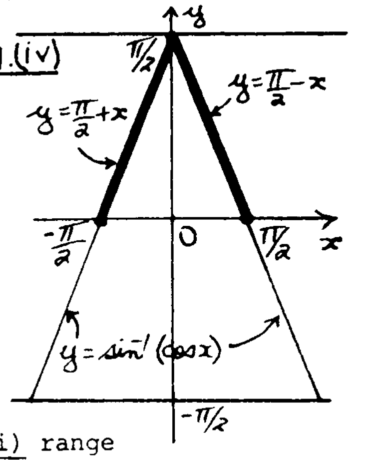
7\. (0, 0, 1); 3 units; line is $x/1=y/2=(z-1)/2$; $c=11$, $P(1, 2, 3)$; $c=2$ (noting plane passes through centre of sphere)

8\. {Hint: Use tree diagram} (i) (a) 1/100 (b) 1/50 (c) 3/100 (ii) (a) 1/50 (b) 197/4950

9\. (i)







(ii) for $\sin^{-1}x$, domain $-1\le x\le 1$, range $-\pi/2\le y\le \pi/2$ (iii) range $\cos 1\le \cos(\sin x)\le 1$ (note $-1\le \sin x\le 1$) (iv) {range of $y=\sin^{-1}(\cos x)$ is $-\pi/2\le y\le \pi/2$; the graph is shown over this whole range; the section requested in the question is the heavy part.}

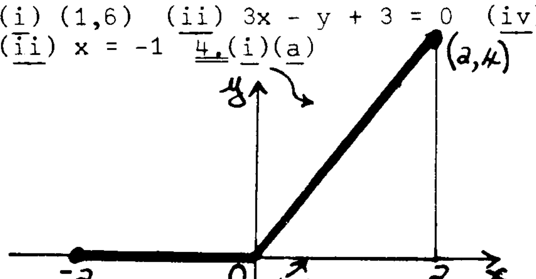
10\. (i) (a) $57-7n$ (b) 12 (c) 138

ANSWERS - 1980 3 UNIT PAPER

1\.(i)(a) $\dfrac{1-x^2}{(1+x^2)^2}$ (b) $\cos(\cos^2 x+e^x).\{-2\sin x\cos x+e^x\}$ (c) $1-e^{-1}+\log 2+\frac{\pi}{2}$ (iii)(a) 0 (b) $\pi/2$, note $f(x)=$ constant $\pi/2$

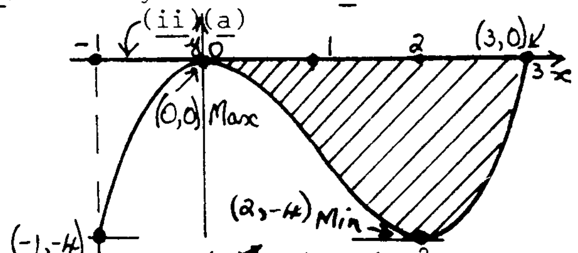
2\.(i) (1,6) (ii) $3x-y+3=0$ (iv) $17x-5y+25=0$ (v) 1/28

3\.(ii) $x=-1$ 4.(i)(a)



(b) 4

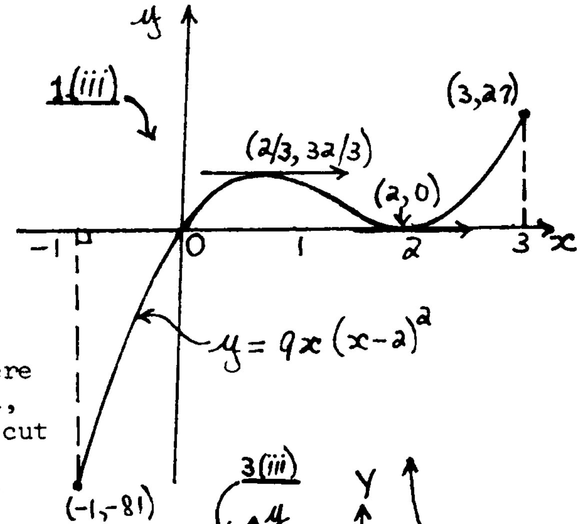
(ii)(a)



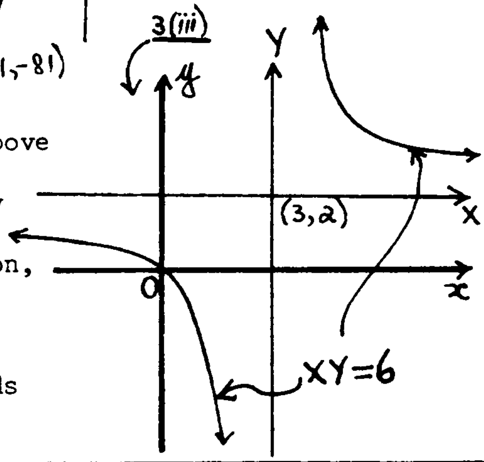
(b) 27/4 units^2

5.(i) $\frac{1}{2}\log_e(x^2+4)+\frac{1}{2}\tan^{-1}\frac{x}{2}+C$ {*Hint break into 2 integrals*} (ii) $\sqrt{108}=6\sqrt{3}$ (note $v^2=144-9x^2$) (iii) $x=\pi/2,\ 3\pi/2;\ \pi/6,\ 5\pi/6$
6. eqn. tangent at P is $y-tx+4t^2=0$; line ℓ is $2tx+(t^2-1)y=4t^2-4$; locus is directrix $y=-4$, i.e. $c=-4$
7.(i) $\frac{x}{3}=\frac{y}{2}=\frac{z}{6}$ (ii) $\frac{x-12}{-9}=\frac{y}{2}=\frac{z}{6}$ (iii) $\frac{13}{77}$ (iv) $3y-z=0$ (*Let plane be $Ax+By+Cz=D$*) Sphere S has radius 6 units, noting if 2 spheres touch externally distance between centres equals sum of radii. *For locus, note centre (X,Y,Z) is distant 7 units from Q and 11 units from R.*
8.(i)(a) $h=\dfrac{100\sin 7^{0}12'\sin 9^{0}42'}{\sin 2^{0}30'}$ (b) 48.4 m (ii)(a) $x(t)=3t^2+e^{-t}-1$
(b) particle Q is always to the right of P on the given line ℓ.
9.(i) $P(x)=3x^2-12x+15$ (ii) $P(x)=2x^2-20x+32$ {*Hint for (i),(ii) let $P(x)=ax^2+bx+c$*} (iii) $P(x)=2x^4-16x^2+32$ {*Hint $P(x)=a(x+2)^2(x-2)^2$ where $P(3)=50$*} (iv) $P(x)=-1\frac{1}{2}x^3+3x^2+1\frac{1}{2}x$ {*Hint $P(x)=ax(x-\alpha)(x-\beta)=ax[x^2-(\alpha+\beta)x+\alpha\beta]$ where $\alpha=1+\sqrt{2}$, $\beta=1-\sqrt{2}$*}
10.(i)(a) $1\,000\,000\times\frac{299}{300}\doteqdot 996\,667$ seeds (b)(i) If $q=1/300$, $p=299/300$ answers are (1) p^{100} (2) ${}^{100}C_{99}q^1p^{99}$ (3) $1-\{p^{100}+100qp^{99}\}$ (ii) *Hint Consider the order of size of coeffts. in the expansion of $(1+x)^{2n}$, especially the middle term.*

ANSWERS: 1981 - 3 UNIT PAPER



1.(i)(a) $-30(5-3x)^9$ (b) $x^2\{1+3\log_e 2x\}$
(ii) 1/6 (iii) range is $-81\le f(x)\le 27$
2.(i) $\frac{13}{4}-\frac{3\sqrt{21}}{4}$ (ii) $x\le -1$ or $x\ge 3$
(iii)(a) $\frac{x-5}{-1}=\frac{y+6}{2}=\frac{z-5}{1}$ (b) (3, −2, 7)
3.(i) $-1/(x^2+1)$ (ii)(a) $2x\sec^2(x^2)$ (b) 0
(iii) XY = 6
4.(i) 240 (ii) −1 or 0 (iii) 35
5.(i)(b) 13/7 (c) finding x co-ord. of pt. where tangent at x = 2 on curve cuts x-axis; at x = 1, tang. is parl. to x-axis and thus tang. cannot cut x-axis. (ii) a = 2, b = 1 or a = −1, b = −8
6.(i)(a) $(-1/2,\ 1/2)$ and (1, 2) (b) 9/8 units^2
(ii) $-8<m<0$ (iii) $y=0$, $y=-8(x+1)$
7.(i) $m\ddot{x}=0$, $m\ddot{y}=-mg$; $x=8t$, $y=-\frac{1}{2}gt^2+6t+27$
(ii) 3 sec, 24 m (iii) 28.8 m above O, i.e. 1.8 m above top of cliff (iv) $y=-5x^2/64+3x/4+27$
8.(i) invalid, since all persons in State not equally likely to be killed on the roads; risk varies from person to person depending on use of roads, occupation, chance, etc. (ii) 0.2 (b) 0.15 (b) 0.69
9.(i) Area = $25\sin\theta(1+\cos\theta)$; $\theta=\pi/3$
(ii)(b) $\Delta BPA=\frac{1}{2}PA.PB\sin\alpha$
10.(i)(a) $P(t)=P(0)e^{kt}$ where $k=\frac{1}{2}\ell n\,3$ (b) 52 birds
(ii)(b) P approaches L (c) dP/dt approaches 0



1. (i) $\dfrac{23+17\sqrt{3}}{13}$ (ii) 125 (iii) sketch below (iv)(a) log 2 (b) $\frac{1}{2}(\sqrt{2}-\sqrt{3})$

2. (i) $-3x^2\sin(1+x^3)$ (ii) 31/5 units^2 (iii)(a) values in order are −5.4, 0, 0.7, 0.5, 0.3, 0.1 (b) $(\frac{1}{2}, 2e^{-1})$; maximum (c) sketch below

3. (i) $\pi^2/2$ units^3 (ii) AC=3.46m, AD=3.61m

4. (i)(a) $S(n)=\dfrac{a(1-r^n)}{1-r}$ (b) 100 000 (ii)(a) sketches below (b) $\pi/4$ (c) value π

5. (i)(a) 3/5 (b) 7/25 (ii)(b) $x+y=3$; Q(−6,9)

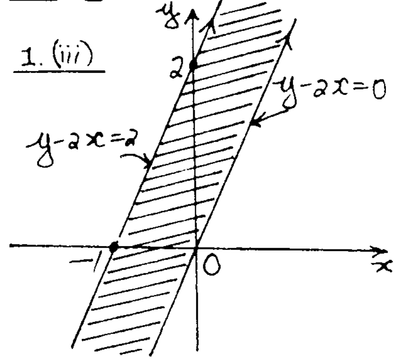
6. (i)(a) 0.02835 (b) 4 heads and 1 tail; 0.36015 {*Note* $np=5\times0.7=3.5$, *but 3.5 heads and thus 1.5 tails are not possible with 5 tosses; by calculation 4 heads and 1 tail are more likely than 3 heads and 2 tails.*}
(ii) at least 34 times

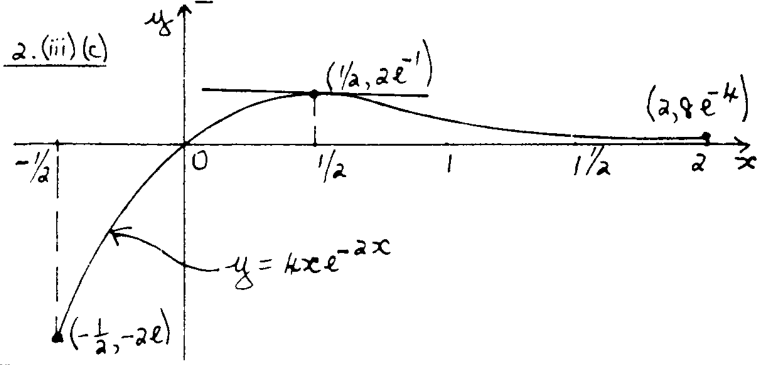
7. (i)(a) sketch below (c) locus of P is circle centre origin with radius 2 units (ii)(a) 2/5 (b) 1/60

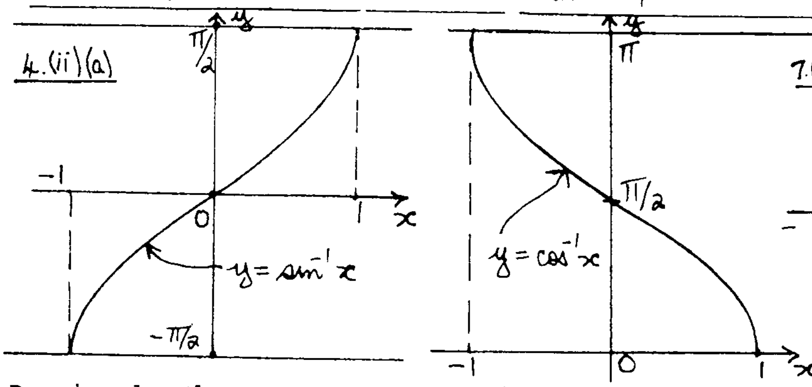
8. (i)(a) 9 units (b) 10/27 (c) 12.5 units^2 (ii) $x-12y+22z=0$; $x/1=y/-12=z/22$

9. (ii) $y=6\left(x+\dfrac{900}{x}\right)$; \$360

10. (i) $x=Vt\cos\alpha$, $y=\frac{1}{2}gt^2+Vt\sin\alpha$ (ii) max. height $V^2\sin^2\alpha/2g$
(iii)(a) $5\sqrt{29} \doteqdot 26.9$ m/s; $\tan^{-1}(2/5) \doteqdot 21.8°$ (b) (5+1)=6m

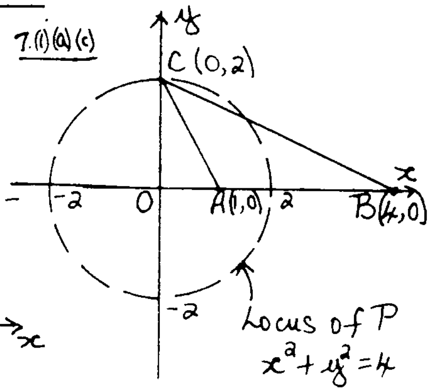






Domain $-1\le x\le 1$ Domain $-1\le x\le 1$

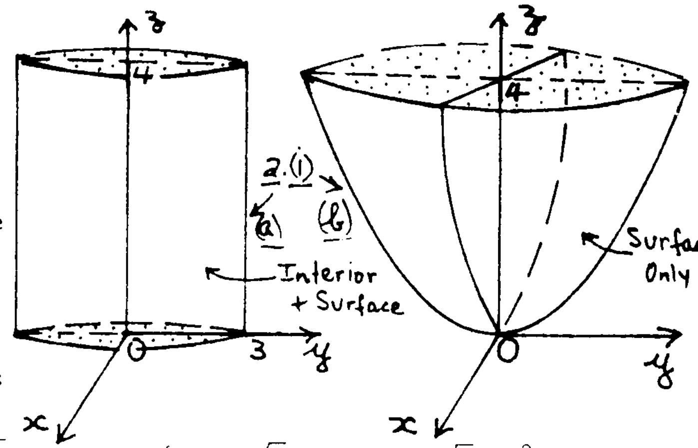
Range $-\pi/2\le\sin^{-1}x\le\pi/2$ Range $0\le\cos^{-1}x\le\pi$



ANSWERS 1983 H.S.C.: 3UNIT - 4UNIT COMMON PAPER

1. (i)(a) $(2-x^2)/(1-x^2)^{3/2}$ (b) $a = 1/6\sqrt{3}$ (ii) $\sin\theta = 2t/(1+t^2)$, $\cos\theta = (1-t^2)/(1+t^2)$

2. (i) (a) Interior and surface of a closed right circular cylinder, of radius 3 units, generators parallel to the z axis, centred at the origin, and closed off by the planes $z = 0$, $z = 4$. (b) Surface of a closed paraboloid of revolution, closed by the plane $z = 4$.



(ii) (a) $\dfrac{x-5}{-1} = \dfrac{y+6}{2} = \dfrac{z-5}{1}$ (b) $(3,-2,7)$

3. (i) 56^0, 124^0, 270^0 (ii) 382.5 m

4. (i) $a = -1$, $b = 0$, $c = 3$, $d = -2$
(ii) (a) 1/99; $1/\{(2k+1)(2k+3)\}$ (c) Use Induction

5. (a) $2\sqrt{10}$s (b) $10\sqrt{41} \doteqdot 64.0$m/s, $\doteqdot 98^0 59'$ (c) $\doteqdot 107^0 33'$

6. (i) 13440 (ii) (a) 2.8+2 = 4.8 hits (b) 3+2 = 5 hits (iii) $\doteqdot$ \$37194.30

7. (i) (a) $y - t_1 x + t_1{}^2 = 0$ (ii) $t_2 = (-3\pm2\sqrt{2})t_1$; M is $\{(-2+2\sqrt{2})t_1,\ (-3+2\sqrt{2})t_1{}^2\}$ or $\{(-2-2\sqrt{2})t_1,\ (-3-2\sqrt{2})t_1{}^2\}$