# 1998 HIGHER SCHOOL CERTIFICATE SOLUTIONS

# 3 UNIT (ADDITIONAL) AND 3/4 UNIT (COMMON) MATHEMATICS

## QUESTION 1

(a) $$\frac{d}{dx}\left(2x\tan^{-1}x\right) = \tan^{-1}x \times 2 + \frac{2x}{1+x^2}$$
$$= 2\tan^{-1}x + \frac{2x}{1+x^2}.$$

(b) Gradient of $3y = 2x + 8$ is $\frac{2}{3}$.
Gradient of $y = 5x - 9$ is 5.
Let $\theta$ be the acute angle between the lines.
$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$
$$= \left|\frac{\frac{2}{3} - 5}{1 + \frac{2}{3} \times 5}\right|$$
$$= 1$$
$$\therefore \quad \theta = 45°.$$

(c) $$\lim_{x\to 0}\frac{\sin 3x}{5x} = \frac{3}{5} \times \lim_{3x \to 0}\frac{\sin 3x}{3x}$$
$$= \frac{3}{5} \times 1 \qquad \left(\lim_{x\to 0}\frac{\sin x}{x} = 1\right)$$
$$= \frac{3}{5}.$$

(d) $$\log_2 14 = \log_2(7 \times 2)$$
$$= \log_2 7 + \log_2 2$$
$$\doteqdot 2{\cdot}807 + 1$$
$$= 3{\cdot}807 \text{ (to 3 decimal places).}$$

(e) If $ax^3 + bx^2 + cx + d = 0$, $\alpha\beta\gamma = -\frac{d}{a}$,
$\therefore$ for $2x^3 - 14x - 1 = 0$, $\alpha\beta\gamma = -\frac{(-1)}{2} = \frac{1}{2}$.

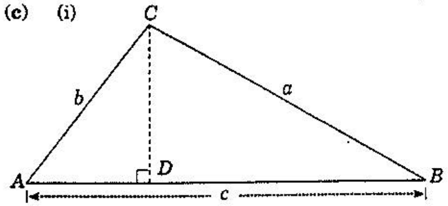
(f) $$\int_0^{\frac{\pi}{3}} \sin^2 x \, dx$$
$$= \int_0^{\frac{\pi}{3}} \frac{1 - \cos 2x}{2}\, dx \quad \left(\text{using } \cos 2x = 1 - 2\sin^2 x\right)$$
$$= \left[\frac{x}{2} - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{3}}$$
$$= \left(\frac{\pi}{6} - \frac{\sin\frac{2\pi}{3}}{4}\right) - (0 - \sin 0)$$
$$= \frac{\pi}{6} - \frac{\sqrt{3}}{8} - 0$$
$$= \frac{4\pi - 3\sqrt{3}}{24}.$$

## QUESTION 2

(a)
$$\begin{array}{r} x^2 - 2 \\ x^2+1 \overline{\smash{)}\, x^4 - x^2 + 1} \\ \underline{x^4 + x^2 \phantom{{}+1}} \\ -2x^2 + 1 \\ \underline{-2x^2 - 2} \\ 3 \end{array}$$
$\therefore$ $Q(x) = x^2 - 2$ and $R(x) = 3$.

(b) $\$P$, invested for $n$ years at $r\%$ per annum, compounded annually, grows to $\$P(1 + r\%)^n$.
$\therefore$ 1st contribution grows to
$$\$500(1 + 8\%)^{40} = \$500(1{\cdot}08)^{40}$$
2nd contribution grows to $\$500(1{\cdot}08)^{39}$
$\vdots$
40th contribution grows to $\$500(1{\cdot}08)^1$.
$\therefore$ Amount in fund on 65th birthday
$$= \$500\left[(1{\cdot}08)^{40} + (1{\cdot}08)^{39} + (1{\cdot}08)^{38} + \cdots + (1{\cdot}08)\right]$$
$$= \$500 \times a\left(\frac{r^n - 1}{r - 1}\right) \quad \text{where } a = 1{\cdot}08,\ r = 1{\cdot}08,\ n = 40$$
$$= \$500 \times 1{\cdot}08\left[\frac{(1{\cdot}08)^{40} - 1}{1{\cdot}08 - 1}\right]$$
$$= \$139\,890{\cdot}52.$$

(c) (i)




In $\Delta CBD$, $\sin B = \dfrac{CD}{a}$

$\therefore$ $CD = a\sin B$. —①

In $\Delta CAD$, $\sin A = \dfrac{CD}{b}$

$\therefore$ $CD = b\sin A$. —②

$\therefore$ From ① and ②,
$a\sin B = b\sin A$. —③

(ii) In $\Delta CBD$, $\cos B = \dfrac{DB}{a}$

$\therefore$ $DB = a\cos B$.

In $\Delta ACD$, $\cos A = \dfrac{AD}{b}$

$\therefore$ $AD = b\cos A$.

$\therefore$ $AD + DB = b\cos A + a\cos B$.

That is, $c = a\cos B + b\cos A$. —④

(iii) $c^2 = (a\cos B + b\cos A)^2$ from ④

$= a^2\cos^2 B + 2ab\cos B\cos A + b^2\cos^2 A$

and $c^2 = 4ab\cos A\cos B$ (given)

$\therefore$ $a^2\cos^2 B + 2ab\cos B\cos A + b^2\cos^2 A$
$= 4ab\cos A\cos B$

$\therefore$ $a^2\cos^2 B - 2ab\cos A\cos B + b^2\cos^2 A = 0$

$\therefore$ $(a\cos B - b\cos A)^2 = 0$

$\therefore$ $a\cos B = b\cos A$. —⑤

Also, $a\sin B = b\sin A$ from ③ —⑥

⑥ + ⑤ $\rightarrow$ $\tan B = \tan A$

$\therefore$ $B = A$ (since $A, B < 180°$)

$\therefore$ $b = a$ (sides opposite equal $\angle$s in $\Delta ACB$).

## QUESTION 3

(a) When $n = 1$, $4^n + 14 = 4^1 + 14$
$= 18$
$= 6\times 3$.

$\therefore$ Result is true for $n = 1$.

Let $k$ be a value of $n$ for which the result is true.

Then $4^k + 14 = 6M$, where $M$ is an integer. —①

Consider when $n = k+1$,

then $4^{k+1} + 14 = 4^k\times 4 + 14$
$= (6M - 14)\times 4 + 14$ from ①
$= 6\times 4M - 4\times 14 + 14$
$= 6\times 4M - 42$
$= 6(4M - 7)$.

That is, it is a multiple of 6 and so the result is true for $n = k+1$.

Hence, since the result is true when $n = k = 1$, it is also true for $n = k = 1+1 = 2$ and by mathematical induction the result will be true for all positive integral values of $n \geq 1$.

(b) (i)

$\sin 4t + \sqrt{3}\cos 4t \equiv R\sin(4t + \alpha)$
$\equiv R(\sin 4t\cos\alpha + \cos 4t\sin\alpha)$
$\equiv R\sin 4t\cos\alpha + R\cos 4t\sin\alpha$

$\therefore$ $R\cos\alpha = 1$, —①

and $R\sin\alpha = \sqrt{3}$. —②

② + ① $\rightarrow$ $\tan\alpha = \sqrt{3}$

$\therefore$ $\alpha = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$, etc.

Choose $\alpha = \dfrac{\pi}{3}$:

①$^2$ + ②$^2$ $\rightarrow$ $R^2(\cos^2\alpha + \sin^2\alpha) = 4$

$\therefore$ $R^2 = 4$

$\therefore$ $R = \pm 2$.

N.B. $R = 2$ since $\alpha = \dfrac{\pi}{3}$ implies $R > 0$.

Hence $\sin 4t + \sqrt{3}\cos 4t \equiv 2\sin\left(4t + \dfrac{\pi}{3}\right)$. —③

$\left\{\text{Other answers are possible, for example: } -2\sin\left(4t + \dfrac{4\pi}{3}\right).\right\}$

(ii) $\sin 4t + \sqrt{3}\cos 4t = 0$

$\therefore$ $2\sin\left(4t + \frac{\pi}{3}\right) = 0$

$\therefore$ $4t + \frac{\pi}{3} = \ldots, -2\pi, -\pi, 0, \pi, 2\pi, \ldots$

$4t = \ldots, -2\pi - \frac{\pi}{3}, -\pi - \frac{\pi}{3},$
$0 - \frac{\pi}{3}, \pi - \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, \ldots$
$= n\pi - \frac{\pi}{3}$ for $n$ any integer.

$\therefore$ $t = \frac{1}{4}\left(n\pi - \frac{\pi}{3}\right)$
$= \frac{\pi}{12}(3n - 1)$ for $n$ any integer.

(c) (i) $x = 1 + \sin 4t + \sqrt{3}\cos 4t$
$= 1 + 2\sin\left(4t + \frac{\pi}{3}\right)$ from ③. —④

This is of the form $x = b + a\sin(nt + \alpha)$ and hence represents SHM about $x = b = 1$.





Also, $x = 1 + 2\sin\left(4t + \frac{\pi}{3}\right)$
$= 1 + 2\cos\left[\frac{\pi}{2} - \left(4t + \frac{\pi}{3}\right)\right]$
$= 1 + 2\cos\left(\frac{\pi}{6} - 4t\right)$
$= 1 + 2\cos\left(4t - \frac{\pi}{6}\right)$ (since cos is an even function).

This is of the form $x = b + a\cos(nt + \alpha)$ which is the form stated in the syllabus to represent SHM about $x = b = 1$.

OR $x = 1 + 2\sin\left(4t + \frac{\pi}{3}\right)$ —⑤

$\therefore$ $\dot{x} = 8\cos\left(4t + \frac{\pi}{3}\right)$

$\therefore$ $\ddot{x} = -32\sin\left(4t + \frac{\pi}{3}\right)$
$= -16\times 2\sin\left(4t + \frac{\pi}{3}\right)$ —⑥
$= -4^2(x - 1)$ from ⑤.

This is of the form $\ddot{x} = -n^2(x - b)$, and hence represents SHM about the point $x = b = 1$.

(ii) Amplitude = 2.

(iii) Particle is at maximum speed when $\ddot{x} = 0$.

$\ddot{x} = -32\sin\left(4t + \frac{\pi}{3}\right)$ from ⑥
$= 0$ when $\sin\left(4t + \frac{\pi}{3}\right) = 0$.

That is, when $4t + \frac{\pi}{3} = 0, \pi, \ldots$

$4t = \frac{-\pi}{3}, \frac{2\pi}{3}, \ldots$

$t = \frac{-\pi}{12}, \frac{\pi}{6}, \ldots$

$\therefore$ First time particle reaches maximum speed is at $t = \frac{\pi}{6}$.

## QUESTION 4

(a) (i) $V = \pi\displaystyle\int_0^h x^2\,dy$ where $x^2 + (y - 15)^2 = 15^2$

$= \pi\displaystyle\int_0^h \left[15^2 - (y - 15)^2\right] dy$

$= \pi\displaystyle\int_0^h (30y - y^2)\,dy$

$= \pi\left[15y^2 - \dfrac{y^3}{3}\right]_0^h$

$= \pi\left(15h^2 - \dfrac{h^3}{3}\right) - 0$

$= 15\pi h^2 - \dfrac{\pi h^3}{3}$ as required.

(ii) $V = 15\pi h^2 - \dfrac{\pi h^3}{3}$

$\dfrac{dV}{dh} = 30\pi h - \pi h^2$

$\dfrac{dV}{dt} = 3$.

Find $\dfrac{dh}{dt}$ when $h = 6$.

$\dfrac{dV}{dt} = \dfrac{dV}{dh}\times\dfrac{dh}{dt}$

$\therefore$ $3 = (30\pi h - \pi h^2)\dfrac{dh}{dt}$

$\therefore$ $\dfrac{dh}{dt} = \dfrac{3}{30\pi h - \pi h^2}$.

When $h = 6$, $\dfrac{dh}{dt} = \dfrac{3}{30\pi\times 6 - \pi\times 6^2}$
$= \dfrac{1}{48\pi}$.

$\therefore$ Water is rising at a rate of $\dfrac{1}{48\pi}$ cm s$^{-1}$ when the water level is 6 cm.

(b) (i) $f(x) = 1 + \dfrac{3}{x-2}$, $x > 2$.

Let $y = f(x)$.

Then $y = 1 + \dfrac{3}{x-2}$, $x > 2$.

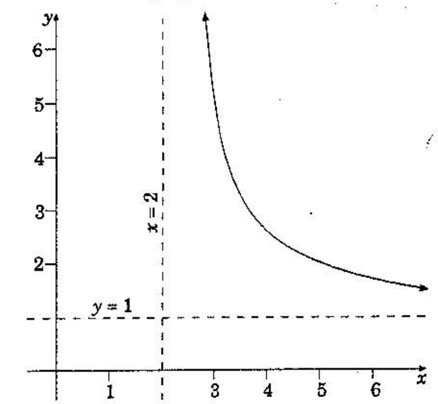
$\therefore$ $y - 1 = \dfrac{3}{x-2}$, $x > 2$.

This is one branch of a rectangular hyperbola of the form

$Y = \dfrac{3}{X}$

translated 2 units to the right and 1 unit up. Hence the horizontal asymptote is $y = 1$ and the vertical asymptote is $x = 2$.

**Sketch of $y = f(x)$ for $x > 2$.**




(ii) The inverse function is obtained from

$x = 1 + \dfrac{3}{y-2}$, where $y > 2$, $x > 1$.

$$x-1=\frac{3}{y-2}$$
$$y-2=\frac{3}{x-1}$$
$$y=2+\frac{3}{x-1}$$
$$\therefore\quad f^{-1}(x)=2+\frac{3}{x-1},\ x>1.$$

(iii) The range of $f(x)$ is $y>1$ (see sketch) so the domain of $f^{-1}(x)$ is $x>1$.

**(c)** (i) If an interval $(BC)$ subtends two equal angles at two points $(D, E)$ on the same side of it, then the endpoints of the interval and the two points are concyclic.

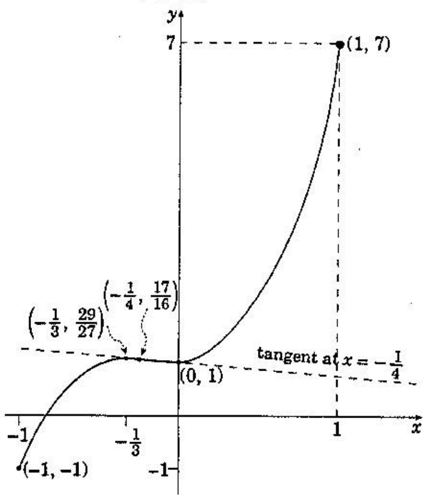
(ii) Exterior $\angle AED$ of cyclic quadrilateral $EDCB$ equals the interior opposite $\angle DCB$.

**QUESTION 5**

**(a)** $8+27+\cdots+n^3=2^3+3^3+\cdots+n^3$
$$=\sum_{r=2}^{n}r^3.$$

**(b)** (i) $P(x)=4x^3+2x^2+1$
$$P(0)=1$$
$$P(1)=7$$
$$P(-1)=-1$$
$$P'(x)=12x^2+4x$$
$$=4x(3x+1)$$
$$=0\text{ when }x=0\text{ or }-\tfrac{1}{3}.$$

$\therefore$ $(0,1)$ and $\left(-\frac{1}{3},\frac{29}{27}\right)$ are stationary points.




The continuous nature of polynomial functions and the positions of the stationary points determine their nature as turning points.

**OR** $P''(x)=24x+4$
$$P''(0)=4>0$$
$\therefore$ $(0,1)$ is a minimum turning point.
$$P''\left(-\tfrac{1}{3}\right)=-4<0$$
$\therefore$ $\left(-\frac{1}{3},\frac{29}{27}\right)$ is a maximum turning point.

(ii)
$$P(x)=4x^3+2x^2+1$$
$$P'(x)=12x^2+4x.$$
Using Newton's method with $x_1=-\frac{1}{4}$, another approximation is
$$x_2=x_1-\frac{P(x_1)}{P'(x_1)}$$
$$=-\frac{1}{4}-\frac{4\times\left(-\frac{1}{4}\right)^3+2\left(-\frac{1}{4}\right)^2+1}{12\left(-\frac{1}{4}\right)^2+4\left(-\frac{1}{4}\right)}$$
$$=4.$$

(iii) Newton's method is calculating the $x$ intercept of the tangent to this curve at $x=-\frac{1}{4}$ and using this value as the next approximation to the root. Because there is a turning point between $x=-\frac{1}{4}$ and the root, the tangent to the curve at $x=-\frac{1}{4}$ will not cut the $x$ axis closer to the root. (See graph.)

**(c)** (i) $P(4\text{ untagged fish})$
$$=\frac{50}{60}\times\frac{49}{59}\times\frac{48}{58}\times\frac{47}{57}$$
$=0{\cdot}4723$ to 4 significant figures.

(ii) $P(\text{at least one tagged fish})$
$=1-P(4\text{ untagged fish})$
$=1-0{\cdot}4723$
$=0{\cdot}5277$ to 4 significant figures.

(iii) $P(\text{no tagged fish for 7 days})$
$=(0{\cdot}4723)^7$
$=0{\cdot}005\,24$ to 3 significant figures.

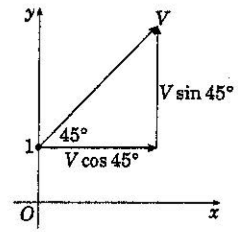
(iv) $P(\text{no tagged fish for exactly 2 days})$
$={}^7C_2\times(0{\cdot}4723)^2\times(0{\cdot}5277)^5$
$=0{\cdot}192$ to 3 significant figures.





**QUESTION 6**

**(a)** (i)




$$\ddot{x}=0$$
$$\dot{x}=\int 0\,dt=C_1.$$
When $t=0$, $\dot{x}=V\cos45°=\frac{V}{\sqrt2}$
$$\therefore\quad C_1=\frac{V}{\sqrt2}$$
$$\therefore\quad \dot{x}=\frac{V}{\sqrt2}$$
$$x=\int\frac{V}{\sqrt2}\,dt$$
$$=\frac{Vt}{\sqrt2}+C_2.$$
When $t=0$, $x=0$
$$\therefore\quad C_2=0$$
$$\therefore\quad x=\frac{Vt}{\sqrt2}\qquad —①$$
$$\ddot{y}=-g$$
$$\dot{y}=\int -g\,dt=-gt+C_3.$$
When $t=0$, $\dot{y}=V\sin45°=\frac{V}{\sqrt2}$
$$\therefore\quad C_3=\frac{V}{\sqrt2}$$
$$\therefore\quad \dot{y}=-gt+\frac{V}{\sqrt2}$$
$$y=\int\left(-gt+\frac{V}{\sqrt2}\right)dt$$
$$=\frac{-gt^2}{2}+\frac{Vt}{\sqrt2}+C_4.$$
When $t=0$, $y=1$
$$\therefore\quad C_4=1$$
$$\therefore\quad y=\frac{-gt^2}{2}+\frac{Vt}{\sqrt2}+1.\qquad —②$$
From ①, $t=\frac{\sqrt2x}{V}$.

Substitute in ②:
$$y=\frac{-g\cancel{2}x^2}{\cancel{2}V^2}+\frac{V}{\cancel{\sqrt2}}\times\frac{\cancel{\sqrt2}x}{V}+1$$
$$=1+x-g\frac{x^2}{V^2}.\qquad —③$$

(ii) When $x=9{\cdot}3$, $y=2{\cdot}3$.

Substitute in ③: $2{\cdot}3=1+9{\cdot}3-9{\cdot}8\times\frac{(9{\cdot}3)^2}{V^2}$
$$\therefore\quad \frac{9{\cdot}8\times(9{\cdot}3)^2}{V^2}=8$$
$$V^2=\frac{9{\cdot}8\times(9{\cdot}3)^2}{8}$$
$$V=9{\cdot}3\sqrt{\frac{9{\cdot}8}{8}}$$
$$=10{\cdot}2932\ldots$$
$$\doteqdot 10{\cdot}3$$
$\therefore$ Initial speed $\doteqdot 10{\cdot}3\text{ ms}^{-1}$.

(iii) The ball is at ground level when $y=0$.

Substitute $y=0$, $V=10{\cdot}3$, $g=9{\cdot}8$ in ③:
$$0=1+x-\frac{9{\cdot}8x^2}{(10{\cdot}3)^2}$$
$$9{\cdot}8x^2-(10{\cdot}3)^2x-(10{\cdot}3)^2=0$$
$$x=\frac{(10{\cdot}3)^2\pm\sqrt{(10{\cdot}3)^4+4\times9{\cdot}8\times(10{\cdot}3)^2}}{2\times9{\cdot}8}$$
$=11{\cdot}747\,06\ldots$ (other answer not relevant as it is less than 9·3, by inspection)
$=11{\cdot}7$ m.

The ball lands approximately $(11{\cdot}7-9{\cdot}3)$ metres from the net, that is, approximately 2·4 m from the net.

**(b)** (i)
$$\frac{d}{dx}\left(\tfrac{1}{2}v^2\right)=3\left(1-x^2\right)$$
$$\tfrac{1}{2}v^2=\int 3\left(1-x^2\right)dx$$
$$=3x-x^3+C.$$
When $x=0$, $v=4$
$$\therefore\quad C=8$$
$$\therefore\quad \tfrac{1}{2}v^2=3x-x^3+8$$
$$\therefore\quad v^2=6x-2x^3+16$$
$$=16+6x-2x^3.$$

(ii) Yes.

Consider the graph of $v^2=16+6x-2x^3$.

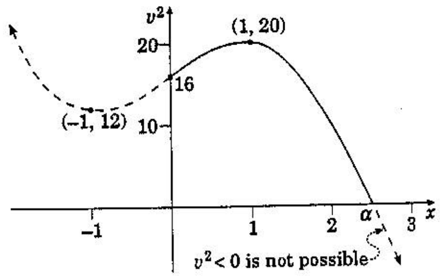
When $x=0$, $v^2=16$.

When $x=2$, $v^2=12$.

When $x=3$, $v^2=-20$.

$\therefore$ Curve cuts $x$ axis between $x=2$ and $x=3$, (at $\alpha$ say).
$$\frac{d}{dx}\left(v^2\right)=-6x^2+6$$
$$=6\left(1-x^2\right)$$
$$=0\text{ when }x=\pm1.$$
When $x=1$, $v^2=20$.

When $x=-1$, $v^2=12$.




From the graph, it can be seen that the particle stops at $x = \alpha$ (where $v^2 = 0$).

Acceleration $a = 3(1-x^2) < 0$ for $x > 1$, and since $\alpha > 1$, the particle begins to move towards the left. It continues without stopping and will reach the origin since $v^2 \neq 0$ for any other value of $x$ (see graph).

## QUESTION 7

(a) (i) $(1+x)^{2n} + (1-x)^{2n}$

$= 1 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + {}^{2n}C_3 x^3 + \cdots + {}^{2n}C_{2n} x^{2n}$
$+ 1 - {}^{2n}C_1 x + {}^{2n}C_2 x^2 - {}^{2n}C_3 x^3 + \cdots + {}^{2n}C_{2n} x^{2n}$

$= 2\left(1 + {}^{2n}C_2 x^2 + {}^{2n}C_4 x^4 + \cdots + {}^{2n}C_{2n} x^{2n}\right)$.

(ii) If $x = 1$ and $n = 10$, (i) becomes

$2^{20} + 0 = 2\left(1 + {}^{20}C_2 + {}^{20}C_4 + \cdots + {}^{20}C_{20}\right)$

$\therefore\ 1 + {}^{20}C_2 + {}^{20}C_4 + \cdots + {}^{20}C_{20} = 2^{19}$.

(b) (i) $y = \sqrt{x}$

$\therefore\ x = y^2$

$\therefore\ \dfrac{dx}{dy} = 2y$

$$\therefore \int \frac{dx}{\sqrt{x(1-x)}} = \int \frac{2y\,dy}{y\sqrt{1-y^2}} = 2\int \frac{dy}{\sqrt{1-y^2}} = 2\sin^{-1} y + C, = 2\sin^{-1}\sqrt{x} + C, \quad 0 < x < 1.$$

(ii) $z = x - \frac{1}{2}$

$\therefore\ x = z + \frac{1}{2}$

$\therefore\ \dfrac{dx}{dz} = 1$

$$\therefore \int \frac{dx}{\sqrt{x(1-x)}} = \int \frac{dz}{\sqrt{\left(z+\frac{1}{2}\right)\left(\frac{1}{2}-z\right)}} = \int \frac{dz}{\sqrt{\left(\frac{1}{2}\right)^2 - z^2}}$$

$$= \sin^{-1}\left(\frac{z}{\frac{1}{2}}\right) + C, = \sin^{-1} 2z + C, = \sin^{-1}(2x-1) + C, \quad 0 < x < 1.$$

(iii) Hence $\sin^{-1}(2x-1)$

$= 2\sin^{-1}\sqrt{x} + C, \quad 0 < x < 1.$

To evaluate constant $C$, put, say, $x = \frac{1}{2}$,

then $\sin^{-1} 0 = 2\sin^{-1}\frac{1}{\sqrt{2}} + C$

$0 = 2 \times \frac{\pi}{4} + C$

$\therefore\ C = -\frac{\pi}{2}$.

Hence $\sin^{-1}(2x-1)$

$= 2\sin^{-1}\sqrt{x} - \frac{\pi}{2}, \quad 0 < x < 1.$

**Note**

The domain $0 < x < 1$ is the domain of the function

$$\frac{1}{\sqrt{x(1-x)}}$$

from which this equation was obtained. However this equation is also true for $x = 0$ and $x = 1$.

(c) $|4x-1| > 2\sqrt{x(1-x)}$.

Using a definition of $|x|$ as $\sqrt{x^2}$, the inequality becomes

$\sqrt{(4x-1)^2} > 2\sqrt{x(1-x)}$. —①

$\sqrt{x}$ is defined only for $x \geq 0$

$\therefore\ x(1-x) \geq 0$

$\therefore\ 0 \leq x \leq 1$. —②

Squaring both sides of ①, we get

$(4x-1)^2 > 4x(1-x)$

$\therefore\ 16x^2 - 8x + 1 > 4x - 4x^2$

$\therefore\ 20x^2 - 12x + 1 > 0$

$\therefore\ (10x-1)(2x-1) > 0$

$\therefore\ x < \frac{1}{10}$ or $x > \frac{1}{2}$. —③

From ② and ③, the solution is

$0 \leq x < \frac{1}{10}$ or $\frac{1}{2} < x \leq 1$.





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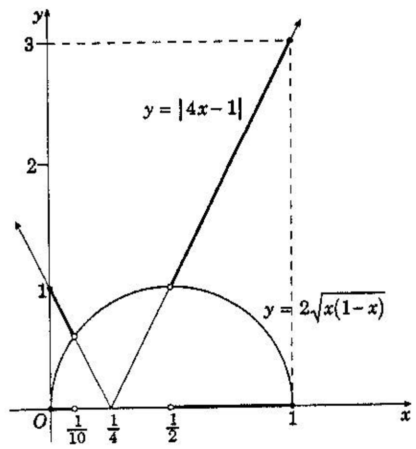


The graph confirms the restriction on the domain.

**END OF 3 UNIT (ADDITIONAL) AND 3/4 UNIT (COMMON) MATHEMATICS SOLUTIONS**