DRAFT SENIOR SECONDARY CURRICULUM – ESSENTIAL MATHEMATICS

Organisation

1. Overview of senior secondary Australian Curriculum

ACARA has developed draft senior secondary Australian Curriculum for English, Mathematics, Science and History according to a set of design specifications (see http://www.acara.edu.au/curriculum/development_of_the_australian_curriculum.html). The ACARA Board approved these specifications following consultation with state and territory curriculum, assessment and certification authorities.

Senior secondary Australian Curriculum will specify content and achievement standards for each senior secondary subject. Content refers to the knowledge, understanding and skills to be taught and learned within a given subject. Achievement standards refer to descriptions of the quality of learning (the depth of understanding, extent of knowledge and sophistication of skill) demonstrated by students who have studied the content for the subject.

The senior secondary Australian Curriculum for each subject has been organised into four units. The last two units are cognitively more challenging than the first two units. Each unit is designed to be taught in about half a 'school year' of senior secondary studies (approximately 50–60 hours duration including assessment). However, the senior secondary units have also been designed so that they may be studied singly, in pairs (that is, year-long), or as four units over two years. State and territory curriculum, assessment and certification authorities are responsible for the structure and organisation of their senior secondary courses and will determine how they will integrate the Australian Curriculum content and achievement standards into courses. They will also provide any advice on entry and exit points, in line with their curriculum, assessment and certification requirements.

States and territories, through their respective curriculum, assessment and certification authorities, will continue to be responsible for implementation of the senior secondary curriculum, including assessment, certification and the attendant quality assurance mechanisms. Each of these authorities acts in accordance with its respective legislation and the policy framework of its state government and Board. They will determine the assessment and certification specifications for their courses that use the Australian Curriculum content and achievement standards and any additional information, guidelines and rules to satisfy local requirements.

These draft documents should not, therefore, be read as proposed courses of study. Rather, they are presented as draft content and achievement standards that will provide the basis for senior secondary curriculum in each state and territory in the future. Once approved, the content and achievement standards would subsequently be integrated by states and territories into their courses.

2. Senior Secondary Mathematics subjects

The Senior Secondary Australian Curriculum: Mathematics consists of four subjects in mathematics. The subjects are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students. Each subject is organised into four units.

Essential Mathematics focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This subject provides the opportunity for students to prepare for post-school options of employment and further training.

General Mathematics focuses on using the techniques of discrete mathematics to solve problems in contexts that include financial modelling, network analysis, route and project planning, decision making, and discrete growth and decay. It provides an opportunity to analyse and solve a wide range of geometrical problems in areas such as measurement, scaling, triangulation and navigation. It also provides opportunities to develop systematic strategies based on the statistical nvestigation process for answering statistical questions that involve comparing groups, investigating associations and analysing time series.

Mathematical Methods focuses on the development of the use of calculus and statistical analysis. The study of calculus in Mathematical Methods provides a basis for an understanding of the physical world involving rates of change, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics in Mathematical Methods develops the ability to describe and analyse phenomena involving uncertainty and variation.

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers, matrices and recursive methods. Specialist Mathematics is the only mathematics subject that has been designed to not be taken as a stand-alone subject.

3. Structure of Essential Mathematics

Essential Mathematics has four units which are structured to contain separate topics. It is intended that the topics be taught in a context relevant to students' needs and interests. In Essential Mathematics, students use their knowledge and skills to investigate realistic problems of interest which involve the application of mathematical relationships and concepts.

	Unit 1	Unit 2	Unit 3	Unit 4
Essential Mathematics	Calculations, percentages and rates Measurement Algebra Graphs	Representing and comparing data Percentages Rates and ratios Time and motion	Measurement Scales, plans and models Graphs Data collection	Probability and relative frequencies Earth geometry Loans and compound interest

Units

Unit 1 provides students with the mathematical skills and understanding to solve problems relating to calculations, applications of measurement, the use of formulas to find an unknown quantity, and the interpretation of graphs. Teachers are encouraged to apply the content of Topics 1, 2, 3 and 4 in a context which is meaningful and of interest to their students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit to achieve this goal are *Mathematics and foods* and *Mathematics and independent living*. However, these contexts may not be relevant for all students, and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

Unit 2 provides students with the mathematical skills and understanding to solve problems related to representing and comparing data, percentages, rates and ratios, the mathematics of finance, and time and motion. Teachers are encouraged to apply the content of Topics 1, 2, 3 and 4 in a context which is meaningful and of interest to the students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit to achieve this goal are *Mathematics and cars* and *Mathematics and independent living*. However these contexts may not be relevant for all students, and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

Unit 3 provides students with the mathematical skills and understanding to solve problems related to measurement, scales, plans and models, drawing and interpreting graphs and data collection. Teachers are encouraged to apply the content of Topics 1, 2, 3 and 4, in a context which is meaningful and of interest to the students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit to achieve this goal are *Mathematics and design* and *Mathematics and medicine*. However these contexts may not be relevant for all students, and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

Unit 4 provides students with the mathematical skills and understanding to solve problems related to probability, earth geometry and time zones, and loans and compound interest. Teachers are encouraged to apply the content of Topics 1, 2, and 3 in a context which is meaningful and of interest to the students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit to achieve this goal are *Earning and managing money and Mathematics of travelling*. However these contexts may not be relevant for all students, and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

Organisation of achievement standards

The achievement standards have been organised into two dimensions: 'Concepts and Techniques' and 'Reasoning and Communication'. These two dimensions reflect students' understanding and skills in the study of mathematics.

Role of technology

It is assumed that students wll be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in these subjects.

4. Links to F-10

For all content areas of Essential Mathematics, the proficiency strands understanding, fluency, problem solving and reasoning from the F-10 curriculum are still very much applicable and should be inherent in students' learning of the subject. Each of these is essential and mutually reinforcing. For all content areas, practice allows students to develop fluency in their skills. They will encounter opportunities for problem solving, such as finding the volume of a solid to enable the amount of liquid that is held in the container to be compared with what is written on the label, or finding the interest on an amount in order to be able to compare different types of loans. In Essential Mathematics, reasoning includes critically interpreting and analysing information represented through graphs, tables and other statistical representations to make informed decisions. The ability to transfer mathematical skills between contexts is a vital part of learning in this subject. For example, familiarity with the concept of a rate enables students to solve a wide range of practical problems, such as fuel consumption, travel times, interest payments, taxation, and population growth.

5. Representation of General Capabilities

The seven general capabilities of *Literacy*, *Numeracy*, *Information and Communication Technology* (ICT) *capability*, *Critical and creative thinking*, *Personal and social capability*, *Ethical behaviour, and Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

General capabilities that are specifically covered in Essential Mathematics include *Literacy*, *Numeracy*, *Information and communication technology (ICT) capability*, *Critical and creative thinking* and *Ethical behaviour*.

Literacy is of fundamental importance in students' development of Essential Mathematics as they develop the knowledge, skills and dispositions to interpret and use language confidently for learning. Students will be taught to read, understand and gather information presented in a wide range of genres, modes and representations (including text, symbols, graphs and tables). They are taught to communicate ideas logically and fluently and to structure arguments.

Numeracy involves students recognising and understanding the role of mathematics in the world and to use mathematical knowledge and skills purposefully. Essential Mathematics has a central role in the development of numeracy in a manner that is more explicit and foregrounded than is the case in other learning areas. Essential Mathematics provides the opportunity to apply mathematical understanding and skills in a real world context.

Critical and creative thinking is inherent in Essential Mathematic. Students develop their critical and creative thinking as they learn to generate and evaluate knowledge, clarify concepts and ideas, seek possibilities, consider alternatives and solve problems. Critical and creative thinking is integral to activities that require students to think broadly and deeply using skills, behaviours and dispositions such as reason, logic, resourcefulness, imagination and innovation in all learning areas at school and their lives beyond school.

Ethical behaviour involves students exploring the ethics of their own and other others' actions. Students develop the capability to behave ethically as they identify and investigate the nature of ethical concepts, values, character traits and principles, and understand how reasoning can assist ethical judgement. There are opportunities in Essential Mathematics to explore, develop and apply ethical behaviour in a range of contexts.

Information and Communication Technology (ICT) is a key part of Essential Mathematics. Students develop ICT capability as they learn to use ICT effectively and appropriately to access, create and communicate information and ideas, solve problems, perform calculations, draw graphs, collect, analyse and interpret data. Digital technologies can engage students and promote the understanding of key concepts.

There are also opportunities within Essential Mathematics to develop the general capabilities of *Intercultural understanding* and *Personal and social capability*, with an appropriate choice of activities and contexts provided by the teacher.

6. Representation of Cross-curriculum priorities

The Cross-curriculum priorities of Aboriginal and Torres Strait Islander histories and cultures, Asia and Australia's engagement with Asia, and Sustainability, are not overtly evident in the content descriptions of the Essential Mathematics subject. However, opportunities exist for teachers to reference them in the context of their teaching of relevant topics, especially those topics which use real data to develop mathematical and statistical concepts.

Aboriginal and Torres Strait Islander histories and cultures

Students will deepen their understanding of the lives of Aboriginal and Torres Strait Islander Peoples through the application of mathematical concepts in appropriate contexts. Teachers could develop statistical and mathematical learning opportunities based on information and data pertinent to Aboriginal and Torres Strait Islander histories and cultures.

Asia and Australia's engagement with Asia

In Essential Mathematics, the priority of Asia and Australia's engagement with Asia provides rich and engaging contexts for developing students' mathematical knowledge, skills and understanding.

In this learning area, students develop mathematical understanding in fields such as measurement, symmetry and statistics by drawing on knowledge of and examples from the Asia region. These could include calculation, money, art, architecture, design and travel. Investigations involving data collection, representation and analysis can be used to examine issues pertinent to the Asia region.

Sustainability

In Essential Mathematics, the priority of sustainability provides rich, engaging and authentic contexts for developing students' abilities in number, measurement and statistics.

Essential Mathematics provides opportunities for students to develop problem solving and reasoning essential for the exploration of sustainability issues and their solutions. Mathematical understandings and skills are necessary to measure, monitor and quantify change in social, economic and ecological systems over time. Statistical analysis enables prediction of probable futures based on findings and helps inform decision making and actions that will lead to preferred futures.

In this learning area, students can observe, record and organise data collected from primary sources over time and analyse data relating to issues of sustainability from secondary sources. They can apply spatial reasoning, measurement, estimation, calculation and comparison to gauge local ecosystem health and can cost proposed actions for sustainability.

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Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring, it has evolved in highly sophisticated and elegant ways to become the language used to describe much of the physical world. Statistics is the study of ways of collecting and extracting information from data and of using that information to describe and make predictions about the behaviour of aspects of the real world in the face of uncertainty. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

Essential Mathematics focuses on enabling students to use mathematics effectively, efficiently and critically to make informed decisions in their daily lives. The emphasis of Essential Mathematics is to provide students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This subject provides the opportunity for students to prepare for post-school options of employment and further training.

For all content areas of Essential Mathematics, the proficiency strands of understanding, fluency, problem solving and reasoning from the F-10 curriculum are still very much applicable and should be inherent in students' learning of the subject. Each of these is essential and mutually reinforcing. For all content areas, practice allows students to develop fluency in their skills. They will encounter opportunities for problem solving, such as finding the volume of a solid to enable the amount of liquid that is held in the container to be compared with what is written on the label, or finding the interest on an amount to be able to compare different types of loans. In Essential Mathematics, reasoning includes critically interpreting and analysing information represented through graphs, tables and other statistical representations to make informed decisions. The ability to transfer mathematical skills between contexts is a vital part of learning in this subject. For example, familiarity with the concept of a rate enables students to solve a wide range of practical problems, such as fuel consumption, travel times, interest payments, taxation, and population growth.

It is intended that the content of the Essential Mathematics subject is taught within a context that is relevant to the needs of the particular student cohort. The skills and understandings developed throughout the subject will be further enhanced and reinforced through presentation in an area of interest to the students.

Aims

Essential Mathematics aims to develop students':

- understanding of concepts and techniques drawn from mathematics and statistics
- ability to solve applied problems using concepts and techniques drawn from mathematics and statistics
- reasoning and interpretive skills in mathematical and statistical contexts
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately.

Unit 1

Unit Description

This unit provides students with the mathematical skills and understanding to solve problems relating to calculations, applications of measurement, the use of formulas to find an unknown quantity, and the interpretation of graphs. Teachers are encouraged to apply the content of Topics 1, 2, 3 and 4 in a context which is meaningful and of interest to their students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit to achieve this goal are *Mathematics and foods* and *Mathematics and independent living*. However, these contexts may not be relevant for all students, and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

It is assumed that students will be taught this subject with an extensive range of technological applications and techniques. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills.

Learning outcomes

By the end of this unit students:

- understand the concepts and techniques in calculations, measurement, algebra and graphs
- apply reasoning skills and solve practical problems in calculations, measurement, algebra and graphs
- communicate their arguments and strategies when solving problems using appropriate mathematical language
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Calculations, percentages and rates

Calculations:

- solve practical problems requiring basic number operations
- apply arithmetic operations according to their correct order
- ascertain the reasonableness of answers to arithmetic calculations
- use leading digit approximation to obtain estimates of calculations
- use a calculator for multi-step calculations
- check results of calculations for accuracy
- recognise the significance of place value after the decimal point
- evaluate decimal fractions to the required number of decimal places
- round up or round down numbers to the required number of decimal places
- apply approximation strategies for calculations

Percentages:

- calculate a percentage of a given amount
- determine one amount expressed as a percentage of another

Rates

- identify common usage of rates, such as: km/h as a rate to describe speed or beats/minute as a rate describing pulse rate
- convert units of rates occurring in practical situations to solve problems
- use rates to make comparisons. For example, using unit prices to compare best buys; comparing heart rates after exercise

Topic 2: Measurement

Linear measure:

- use of metric units of length, their abbreviations, conversions between them, and appropriate levels of accuracy and choice of units.
- estimation of lengths
- conversions between metric units of length and other length units

Area measure:

- use metric units of area, their abbreviations, conversions between them and appropriate choices of units
- estimate the areas of different shapes
- convert between metric units of area and other area units
- calculate areas of rectangles and triangles

Mass:

- use metric units of mass, their abbreviations, conversions between them and appropriate choices of units
- estimate the mass of different objects

Volume and capacity:

- use metric units of volume, their abbreviations, conversions between them and appropriate choices of units
- understand the relationship between volume and capacity
- estimate volume and capacity of various objects

Units of energy:

- use units of energy to describe electricity used such as kilowatt hours
- use units of energy used for foods including calories
- convert from one unit of energy to another

Topic 3: Algebra

Single substitution:

- substitute numerical values into algebraic expressions. For example, substitute *different values of x to evaluate the expressions* $\frac{3x}{5}$, $5\ (2x - 4)$

General Substitution:

- substitute given values for the other pronumerals in a mathematical formula to find the value of the subject of the formula

Topic 4: Graphs

Reading and Interpreting graphs:

- interpret information presented in graphs, such as: conversion graphs, line graphs, step graphs, column graphs and picture graphs
- interpret information presented in two-way tables
- discuss and interpret graphs found in the media and in factual texts

Drawing graphs:

- determine which type of graph is the best one to display a data set
- use spreadsheets to tabulate and graph data
- draw a line graph to represent any data that demonstrates a continuous change. For example, *hourly temperature*

Unit 1 – Examples in Context

Topic 1: Calculations, percentages and rates

Calculation – for example

- *creating a budget for living at home and for living independently*
- *calculating food, clothing, transport costs per day, week or month using tables, spreadsheets, and estimation*
- *calculating clothing costs per week or month using tables, spreadsheets, and estimation*
- *creating and evaluating daily menus to meet the minimum daily nutritional and energy needs*

Percentages – for example

- *calculating and comparing monthly and weekly amounts available for accommodation with varying income levels using percentages*
- *using percentages to compare the different components of personal expenditure*
- *expressing ingredients of packaged food as percentages of the total quantity, or per serving size, or per 100 grams*
- *comparing the quantities, both numerically and in percentage terms, of additives within a product or between similar products. For example, flavours*

Rates – for example

- *calculating cost of maintaining a residence using rates per hour, call-out fees and penalty rates for cost of repairs for various tradesman*
- *using rates to compare and evaluate nutritional information. For example, quantity per serve and quantity per 100g*
- *using unit prices to determine 'best' buys;*
- *calculating the costs of various levels of insurance cover using tables and rates*

Topic 2: Measurement.

Length – for example

- *determining the dimensions/measurements of food packaging*
- *determining the dimensions of a room for the purpose of painting the room*

Mass – for example

- *comparing and discussing the components of different food types for the components of packaged food expressed as grams.*

Area and volume – for example

- *finding the volume of water collected from a roof under different conditions*
- *finding the volume of water that can be stored in tanks and the time taken to fill tanks of different sizes and shapes and with different rainfall rates*
- *finding the surface area and volume of various packaging*

Topic 3: Algebra

- *using formulas to calculate the surface areas and volumes of various packaging*
- *using formulas to calculate total electricity, water and gas use*
- *using formulas to convert from one unit to another*

Topic 4: Graphs

Reading and interpreting graphs – for example

- *analysing and interpreting a range of graphical information of global weather patterns that affect food growth*
- *analysing and interpreting a range of graphical information given on gas and electricity bills*

Drawing graphs – for example

- *expressing ingredients of particular food types as percentages of the total quantity, or per serving size, or per 100 grams, presenting the information in different formats. For example, column graphs, and pie graph*
- *interpreting water, electricity, and gas bills and using graphs to describe the variation between months and comparing year to year variation*

Unit 2

Unit Description

This unit provides students with the mathematical skills and understanding to solve problems related to representing and comparing data, percentages, rates and ratios, the mathematics of finance and time and motion. Teachers are encouraged to apply the content of Topics 1, 2, 3 and 4 in a context which is meaningful and of interest to the students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit to achieve this goal are *Mathematics and cars* and *Mathematics and independent living*. However these contexts may not be relevant for all students and teachers are encouraged to find a suitable context that will make the topics of this unit relevant for their particular student cohort.

It is assumed that students will be taught this subject with an extensive range of technological applications and techniques. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques used in representing and comparing data, percentages, rates and ratios and time and motion
- apply reasoning skills and solve practical problems in representing and comparing data, percentages, rates and ratios and time and motion
- communicate their arguments and strategies when solving mathematical and statistical problems using appropriate mathematical or statistical language
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Representing and comparing data

Classifying data:

- identify examples of categorical data
- identify examples of numerical data

Data presentation and interpretation:

- display categorical data in tables and column graphs
- display numerical data as frequency distributions, dot plots, stem and leaf plots and histograms
- recognise and identify outliers
- compare the suitability of different methods of data presentation in real-world contexts

Summarising and interpreting data:

- identify the mode
- calculate measures of central tendency, the arithmetic mean and the median
- for ungrouped and grouped data
- investigate the suitability of measures of central tendency in various real-world contexts
- investigate the effect of outliers on the mean and the median
- calculate and interpret quartiles, deciles and percentiles
- use informal ways of describing spread, such as: spread out/dispersed, tightly packed, clusters, gaps, more/less dense regions, outliers
- calculate and interpret statistical measures of spread, such as: the range, interquartile range and standard deviation
- investigate real-world examples from the media illustrating inappropriate uses, or misuses, of measures of central tendency and spread

Comparing data sets:

- compare back to back stem plots for different data sets
- complete a five number summary for different data sets
- construct box plots using a five number summary
- compare the characteristics of the shape of histograms using symmetry, skewness and bimodality

Topic 2: Percentages

Percentage calculations:

- review calculating a percentage of a given amount
- review one amount expressed as a percentage of another

Applications of percentages:

- apply percentage increases and decreases in contexts, such as: markups and discounts, unit cost and savings made through buying in bulk, costs associated with stamp duties and GST
- determine the overall change in a quantity following repeated percentage changes. For example*, an increase of 10% followed by a decrease of 10%.*
- calculate simple interest for different rates and periods

Topic 3: Rates and ratios:

Ratios:

- demonstrate an understanding of the elementary ideas and notation of ratio
- understand the relationship between fractions and ratio
- express a ratio in simplest form
- find the ratio of two quantities
- divide a quantity in a given ratio
- use ratio to describe simple scales

Rates:

- review identifying common usage of rates such as, km/h
- convert between units for rates. For example, km/h *to* m/s, mL/min to L/h
- complete calculations with rates, including solving problems involving direct proportion in terms of rate. For example: *If a person works for 3 weeks at a rate of $300 per week how much do they earn?*
- use rates to make comparisons

Topic 4: Time and motion

Time:

- use of units of time, conversions between units, fractional, digital and decimal representations
- represent time using 12 hour and 24 hour clocks
- calculate time intervals, such as: time between, time ahead, time behind
- interpret timetables, such as: bus, train and ferry timetables
- use of several timetables and electronic technologies to plan the most time efficient routes
- interpret complex timetables, such as: tide charts, sunrise charts and moon phases
- compare time travelled by car with other modes of transport

Distance:

- use scales to find distances on maps, such as: road maps, street maps, bushwalking maps, online maps and cadastral maps
- optimise distances through trial and error and systematic methods, such as: shortest path, routes to visit all towns and routes to use all roads

Speed:

- identify the appropriate units for different activities, such as: walking, running, swimming and flying
- calculate speed, distance or time using the formula *speed = distance/time*
- calculate the time or costs for a journey from distances estimated from maps
- interpret distance versus time graphs
- calculate and interpret the average speed, such as: a 4 hour trip covering 250 km

Unit 2 - Examples in Context

Topic 1: Representing and comparing data

- analysing and interpreting a range of statistical information related to car theft, car accidents and driver behaviour
- using statistics and graphs to find the number of people in each blood type given the population percentages of blood types in different countries
- using blood usage statistics to predict the amount of blood needed at different times of the year
- using blood donation statistics to predict when and how much blood is needed

Topic 2: Percentages

- calculating stamp duty costs involved in buying a car using percentages and tables
- calculating depreciation of a vehicle over time
- using statistics and graphs to find the number of people in each blood type given the population percentages of blood types in different countries

Topic 3: Rates and ratios

Rates – for example

- using rates to find fuel consumption for different vehicles under different driving conditions
- calculating heart rates as beats per minute given the number of beats and different time periods
- calculating the total number of heart beats over varying times under different conditions
- calculating the amount of blood pumped by the heart over given periods of time under different exercise conditions
- applying rates to calculate the energy used in various activities over different time periods.

Ratios – for example

- analysing and interpreting tables and graphs that compare body ratios such as hip height versus stride length, foot length versus height

Topic 4: Time and motion

Time – for example

- calculating reaction times through experiments

Distance – for example

- calculating distances travelled to school and the time taken to get from home to school considering different average speeds

Speed – for example

- calculating stopping distances for different speeds through use of formula for different conditions such as road type, tyre conditions and vehicle type

Achievement Standards Unit 1 & 2

	Concepts and Techniques	Reasoning and Communication
A	The student: • understands and applies concepts and techniques in calculations, rates and ratios, measurement, data displays, algebra and graphs • consistently and accurately applies a variety of multiple concepts and techniques, to solve a wide range of problem types, including non-standard problems • represents mathematical and statistical information accurately and precisely in numerical, graphical and symbolic form • uses digital technologies appropriately and skillfully to solve problems, and to display and organise information	The student: • solves problems that require the synthesis of ideas from mathematical and statistical information • precisely communicates observations and reasoned decisions to find solutions to non-standard problems • analyses and interprets the reasonableness of results and solutions to a wide range of problem types • analyses and interprets results with comprehensive consideration of the validity and limitations of the use of models • understands the relative strengths and weaknesses and the inter-relatedness of different representations of mathematical and statistical information
B	The student: • understands and applies most concepts and techniques in calculations, rates and ratios, measurement, data displays, algebra and graphs • consistently and accurately applies a variety of combinations of concepts and techniques to solve non-routine problems • represents mathematical and statistical information accurately in numerical, graphical and symbolic form • uses digital technologies appropriately and competently to solve problems and to display and organise information	The student: • solves problems that require the interpretation of mathematical and statistical information • communicates observations and reasoned decisions to find solutions to problems • analyses the reasonableness of results and solutions to problems • analyses and interprets results with consideration of the validity and limitations of the use of models • understands the relative strengths and weaknesses of different representations of mathematical and statistical information
C	The student: • understands and applies some concepts and techniques in calculations, rates and ratios, measurement, data displays, algebra and graphs • accurately applies combinations of some concepts and techniques to solve familiar problems • represents mathematical and statistical information in numerical, graphical and symbolic form • uses digital technologies appropriately to solve problems and to display and organise information	The student: • solves familiar problems that require the interpretation of mathematical and statistical information • communicates observations to the solution of problems • analyses the reasonableness of results and solutions to familiar problems • analyses and interprets results with consideration of the limitations of the use of models • recognises the different representations of mathematical and statistical information

D	The student: • demonstrates limited application of some concepts and techniques in calculations, rates and ratios, measurement, data displays, algebra and graphs • applies concepts and techniques to solve routine problems • represents mathematical and statistical information in limited forms • uses digital technologies to solve some problems	The student: • solves routine problems that require the interpretation of mathematical and statistical information • communicates some observations to the solution of problems • recognises the reasonableness of results and solutions to routine problems • recognises some representations of mathematical and statistical information
E	The student: • demonstrates limited familiarity with calculations, rates and ratios, measurement, data displays, algebra and graphs • follows procedures to solve simple problems • uses digital technologies to represent information to solve simple problems	The student: • communicates limited observations to the solutions of problems • recognises the solution to routine problems • recognises limited representations of mathematical and statistical information

Unit 3

Unit Description

This unit provides students with the mathematical skills and understanding to solve problems related to measurement, scales, plans and models, drawing and interpreting graphs and data collection. Teachers are encouraged to apply the content of Topics 1, 2, 3 and 4 in a context which is meaningful and of interest to the students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit to achieve this goal are Mathematics and design and Mathematics and medicine. However these contexts may not be relevant for all students, and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

It is assumed that students will be taught this subject with an extensive range of technological applications and techniques. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques used in measurement, scales, plans and models, graphs and data collection
- apply reasoning skills and solve practical problems in measurement, scales, plans and models, graphs and data collection
- communicate their arguments and strategies when solving mathematical and statistical problems using appropriate mathematical or statistical language
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Measurement

Linear measure:

- review of metric units of length, their abbreviations, conversions between them, estimation of lengths and appropriate choices of units
- calculate perimeters of familiar shapes, such as: triangles, squares, rectangles, polygons, circles, arc lengths, and composites of these

Area measure:

- review of metric units of area, their abbreviations and conversions between them
- estimation of areas and appropriate choices of units
- use formulas to calculate areas of regular shapes, such as: triangles, squares, rectangles, parallelograms, trapeziums, circles and sectors
- find the area of irregular figures by decomposition into regular shapes
- find the surface area of familiar solids, such as: cubes, rectangular and triangular prisms, spheres and cylinders
- find the surface area of pyramids, such as: rectangular and triangular based pyramids
- use addition of area of the faces of solids to find the surface area of irregular solids

Mass:

- review of metric units of mass (and weight), their abbreviations, conversions between them, and appropriate choices of units
- recognise the need for milligrams
- convert between grams and milligrams

Volume and capacity:

- review of metric units of volume, their abbreviations, conversions between them, and appropriate choices of units
- recognise relations between volume and capacity, recognising that $1\text{cm}^3 = 1\text{mL}$ and $1\text{m}^3 = 1\text{kL}$
- estimate volume and capacity of various solids
- use formulas to find the volume and capacity of regular objects, such as: cubes, rectangular and triangular prisms and cylinders
- use formulas to find the volume of pyramids and spheres

Topic 2: Scales, plans and models

Geometry:

- recognise the properties of common two dimensional geometric shapes and three dimensional solids
- interpret different forms of two-dimensional representations of three-dimensional objects, including nets and perspective diagrams
- use symbols and conventions for the representation of geometric information. For example, *point, line, ray, angle, diagonal, edge, curve, face and vertex*

Interpret scale drawings:

- interpret commonly used symbols and abbreviations in scale drawings
- find actual measurements from scale drawings, such as: lengths, perimeters and areas
- estimate and compare quantities, materials and costs using actual measurements from scale drawings, such as: packaging, clothes, painting, bricklaying and landscaping

Creating scale drawings:

- understand and apply drawing conventions of scale drawings, such as: scales in ratio, clear indications of dimensions and clear labelling
- construct scale drawings by hand and by using software packages

Three dimensional objects:

- interpret plans and elevation views of models
- sketch elevation views of different models
- interpret diagrams of three-dimensional objects

Right-angled triangles:

- apply Pythagoras' theorem to solve problems
- apply the tangent ratio to find unknown angles and sides in right-angled triangles
- work with the concepts of angle of elevation and angle of depression
- apply the cosine and sine ratios to find unknown angles and sides in right-angle triangles
- solve problems involving bearings

Topic 3: Graphs

Cartesian plane:

- demonstrate familiarity with Cartesian co-ordinates in two dimensions by plotting points on the Cartesian plane
- generate tables of values for linear functions, including for negative values of x
- graph linear functions for all values of x with pencil and paper, and with graphing software

Using graphs:

- interpret and use graphs in practical situations, including travel graphs and conversion graphs
- draw graphs from given data to represent practical situations

Straight line graphs:

- interpret and obtain the equation of a straight line graph in the form $y = mx + c$
- determine the equation of a straight line parallel to a given line.
- interpret the point of intersection and other important features of given graphs of two linear functions drawn from practical contexts. For example, *'break-even' point*

Topic 4: Data collection

Census:

- investigate the procedure for conducting a census
- investigate the advantages and disadvantages of conducting a census

Surveys:

- understand the purpose of sampling to provide an estimate of population values when a census is not used
- investigate the different kinds of samples, such as: systematic samples, self-selected samples, simple random samples
- investigate the advantages and disadvantages of these kinds of samples. For example, *compare simple random samples with self-selected samples*

Simple survey procedure:

- identify the target population to be surveyed
- investigate questionnaire design principles, such as: simple language, unambiguous questions, consideration of number of choices, issues of privacy and ethics, freedom from bias

Sources of bias:

- describe the faults in the collection of data process
- describe sources of error in surveys, such as: sampling error and measurement error
- investigate possible misrepresentation of the results of a survey from misunderstanding the procedure, or misunderstanding the reliability of generalising the survey findings to the entire population
- errors and misrepresentation in surveys including: examples of media misrepresentations of surveys

Bivariate scatterplots:

- describe the patterns and features of bivariate data
- describe the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength(strong/moderate/weak)

Line of best fit:

- identify the dependent and independent variable
- find a line of best fit by eye
- use technology to find the line of best fit
- interpret relationships in terms of the variables
- use technology to find the correlation coefficient (an indicator of the strength of linear association)
- use the line of best fit to make predictions, both by interpolation and extrapolation
- recognise the dangers of extrapolation
- distinguish between causality and correlation through examples

Unit 3 - Examples In Context

Topic 1: Measurement

- calculating and interpreting dosages for children and adults from dosage panels on medicines given age or weight
- calculating and interpreting dosages for children from adults' medication using various formulas (Fried, Young, Clark)

Topic 2: Scales, plans and models

- drawing scale diagrams of everyday 2 dimensional shapes
- interpreting common symbols and abbreviations used on house plans
- using the scale on a plan to calculate actual external or internal dimensions, the lengths of the house and particular rooms
- translating two- dimensional house plans to three- dimensional buildings using technology
- creating landscape designs using technology

Topic 3: Graphs

- interpreting graphs showing growth ranges for children (height or weight or head circumference versus age)
- interpreting hourly hospital charts showing temperature and pulse
- interpreting graphs showing life expectancy with different variables .

Topic 4: Data collection

- analysing data obtained from medical sources including bivariate data

Unit 4

Unit Description

This unit provides students with the mathematical skills and understanding to solve problems related to probability, earth geometry and time zones, and loans and compound interest. Teachers are encouraged to apply the content of Topics 1, 2, and 3 in a context which is meaningful and of interest to the students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit to achieve this goal are *Earning and managing money and Mathematics of travelling*. However these contexts may not be relevant for all students, and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

It is assumed that students will be taught this subject with an extensive range of technological applications and techniques. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques used in probability and relative frequencies, earth geometry and time zones, loans and compound interest
- apply reasoning skills and solve practical problems in probability and relative frequencies, earth geometry and time zones, loans and compound interest
- communicate their arguments and strategies when solving mathematical problems using appropriate mathematical or statistical language
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Probability and relative frequencies

Probability expressions:

- interpret commonly used probability statements, such as: 'possible', 'probable', 'likely', 'certain'
- describe ways of expressing probabilities formally using fractions, decimals, ratios, percentages

Simulations:

- perform simulations of experiments with the use of technology
- recognise that repetitions of chance events are likely to produce different results
- identify relative frequency as probability
- identify factors that could complicate the simulation of real world events

Simple probabilities:

- construct a sample space for an experiment
- use a sample space to determine the probability of outcomes for an experiment
- use arrays or tree diagrams to determine the outcomes and the probabilities for experiments

Probability applications

- determine the probabilities associated with simple games
- determine the probabilities of simple traffic light problems

Topic 2: Earth geometry and time zones

Location:

- locate positions on the Earth's surface given latitude and longitude using GPS, a globe, an atlas, and digital technologies
- find distances between two places on earth on the same longitude
- find distances between two places on earth using appropriate technology

Time:

- understand the link between longitude and time
- solve problems involving time zones in Australia and its neighbours including allowances for daylight saving
- solve problems involving Greenwich Mean Time and the International Date Line
- find the time differences between two places on the earth
- solve problems associated with time zones, such as: internet and phone usage
- solve problems relating to travelling east and west and incorporating time zone changes

Topic 3: Loans and compound interest

Compound interest:

- review the principles of simple interest
- understand the concept of compound interest as a recurrence relation
- consider other similar problems involving compounding, such as: population growth
- use technology to calculate the future value of a compound interest loan or investment and the total interest paid or earned
- use technology to compare, numerically and graphically the growth of simple interest and compound interest loans and investments
- use technology to investigate the effect of the interest rate and the number of compounding periods on the future value of a loan or investment

Reducing balance loans (compound interest loans with periodic repayments):

- use technology and a recurrence relation to model a reducing balance loan
- investigate the effect of the interest rate and repayment amount on the time taken to repay a loan

Unit 4 - Examples in Context

Topic 1: Probability and relative frequency

- calculating from data the relative frequencies of the different countries of origin of visitors to a particular tourist venue or country
- calculating from data the relative frequencies of the amounts of household expenditure is made

Topic 3: Loans and compound interest

- using formula, graphs and spreadsheets to calculate the outcomes of investment accounts with compound interest.
- using percentages, rates and spreadsheets to investigate personal loan calculations.
- calculating and analysing the costs, hidden traps. advantages and disadvantages for payment plans with interest free periods using rates and percentages

Achievement Standards Unit 3 & 4

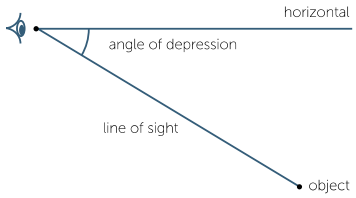
	Concepts and Techniques	Reasoning and Communication
A	The student: • understands and applies concepts and techniques in measurement and geometry, probability and statistics, graphs and financial mathematics • consistently and accurately applies multiple concepts and techniques, to solve a wide range of problem types, including non-standard problems • represents mathematical and statistical information accurately and precisely in numerical, graphical and symbolic form • uses digital technologies appropriately and skillfully to solve problems and to display and organise information	The student: • solves a wide range of problems, including non-standard problems, that require the synthesis of ideas from mathematical and statistical information • precisely communicates observations and reasoned decisions based on information from varied situations to find solutions to non-standard problems • analyses and interprets the reasonableness of results and solutions to problems derived from diverse mathematical and statistical information • analyses, interprets and communicates results of investigations with comprehensive consideration of the validity and limitations of the use of models • understands the relative strengths and weaknesses and the inter-relatedness of different representations of mathematical and statistical information
B	The student: • understands and applies most concepts and techniques in measurement and geometry, probability and statistics, graphs and financial mathematics • consistently and accurately applies a variety of concepts and techniques, to solve non-routine problems • represents mathematical and statistical information accurately in numerical, graphical and symbolic form • uses digital technologies appropriately and competently to solve problems and to display and organise information	The student: • solves non-routine problems that require the synthesis of mathematical and statistical information and ideas • communicates observations and reasoned decisions to find the solutions to problems based on information from varied situations • analyses the reasonableness of results and solutions to problems derived from diverse mathematical and statistical information • analyses and communicates the results of investigations with comprehensive consideration of the validity and limitations of the use of models • understands the relative strengths and weaknesses and the inter-relatedness of different representations of mathematical and statistical information
C	The student: • understands and applies some concepts and techniques in measurement and geometry, probability and statistics, graphs and financial mathematics • accurately applies limited combinations of concepts and techniques to solve familiar problems • represents mathematical and statistical information in numerical, graphical and symbolic form • uses digital technologies appropriately to solve problems to display and organise information	The student: • solves familiar problems that require the use of mathematical and statistical information • communicates observations and reasoned decisions to find the solutions to problems • recognises the reasonableness of results and solutions to problems derived from diverse mathematical and statistical information • communicates the results of investigations with limited consideration of the validity and limitations of the use of models • understands the inter-relatedness of different representations of mathematical and statistical information

D	The student: • demonstrates limited understanding and application of some concepts and techniques in measurement and geometry, probability and statistics, graphs and financial mathematics • applies concepts and techniques to solve routine problems • represents mathematical and statistical information in limited forms • uses digital technologies to solve some problems	The student: • solves routine problems that require the use of mathematical and statistical information • communicates some observations and makes decisions to find the solution to problems • recognises the reasonableness of results and solutions to problems • communicates results of investigations
E	The student: • demonstrates limited familiarity in measurement and geometry, probability and statistics, graphs and financial mathematics • follows procedures to solve simple problems • uses digital technologies to represent information and to solve simple problems	The student: • follows procedures to solve routine problems that require the use of mathematical and statistical information • communicates some observations to find the solutions to problems • recognises the solutions to routine problems • recognises the representations of mathematical and statistical information

GLOSSARY ITEMS

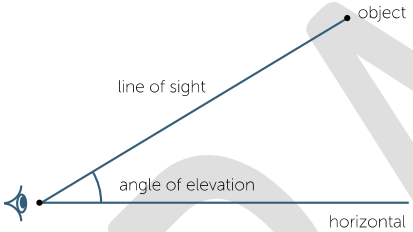
Angle of depression

When an observer looks at an object that is lower than 'the eye of' the observer', the angle between the line of sight and the horizontal is called the angle of depression.



Angle of elevation

When an observer looks at an object that is higher than 'the eye of' the observer', the angle between the line of sight and the horizontal is called the **angle of elevation**.



Average speed

Average speed is the total distance travelled divided by the total time taken.

Back-to back stem plots

A **back-to-back stem-and-leaf plot** is a method for comparing two data distributions by attaching two sets of 'leaves' to the same 'stem' in a **stem-and-leaf plot**.

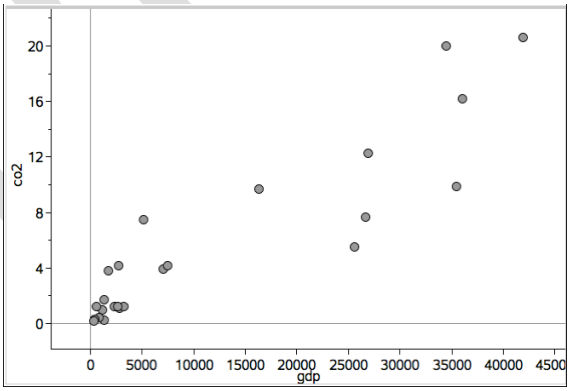
For example, the stem-and-leaf plot below displays the distribution of pulse rates of 19 students before and after gentle exercise.

pulse rate		
before		after
9 8 8 8	6	
8 6 6 4 1 1	7	
8 8 6 2	8	6 7 8 8
6 0	9	0 2 2 4 5 8 9 9
4	10	0 4 4
0	11	8
	12	4 4
	13	

Bivariate data scatter plot

A two-dimensional data plot using Cartesian co-ordinates to display the values of two variables in a bivariate data set.

For example the scatterplot below displays the CO2 emissions in tonnes per person (*co2*) plotted against Gross Domestic Product per person in $US (*gdp*) for a sample of 24 countries in 2004. In constructing this scatterplot, gdp has been used as the explanatory variable.



Capacity

Capacity versus volume. Volume refers to the space taken up by an object itself, while capacity refers to the amount of a liquid or other pourable substance a container can (or does) hold.

Categorical data

Data associated with a **categorical variable** is called categorical data.

Categorical variable

A **categorical variable** is a variable whose values are categories.

Examples include blood group (A, B, AB or O) or house construction type (brick, concrete, timber, steel, other).

Categories may have numerical labels, eg. the numbers worn by player in a sporting team, but these labels have no numerical significance, they merely serve as labels.

Census

A **population** is the complete set of individuals, objects, places, etc, that we want information about.

A **census** is an attempt to collect information about the whole population.

Compound interest

The interest earned by investing a sum of money (the principal) is compound interest if each successive interest payment is added to the principal for the purpose of calculating the next interest payment.

For example, if the principal P earns compound interest at the rate of i % per period, then after n periods the total amount accrued is $P(1+\frac{i}{100})^n$

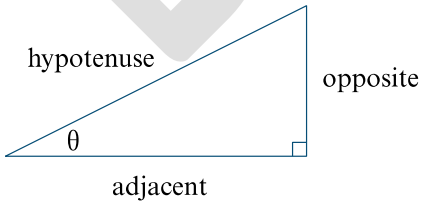
Correlation

Correlation is a measure of the strength of the linear relationship between two variables.

Cosine ratio

In any right-angled triangle,

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \text{where} \quad 0^\circ < \theta < 90^\circ$$



Correlation coefficient (*r*)

The correlation coefficient (*r*) is a measure of the strength of the liner relationship between a pair of variables. The formula for calculating *r* is given below.

For variables *x* and *y*, and computed for *n* cases, the formula for *r* is:

$$r = \frac{1}{n-1}\sum\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

Extrapolation

In the context of fitting a linear relationship between two variables, extrapolation occurs when the fitted model is used to make predictions using values of the **explanatory variable** that are outside the range of the original data. Extrapolation is a dangerous process as it can sometimes lead to quite erroneous predictions.

Five-number summary

A **five**-number summary is a method of summarising a set of data using the minimum value, the lower or first-quartile (Q_1), the median, the upper or third-quartile (Q_3) and the maximum value. Forms the basis for a boxplot.

Frieds' formula Young's formula Clarks formula Drip rates

Frieds' formula

Dosage for children 1-2 years = (age (in months) x adult dosage) /150

Young's formula

Dosage for Children 1-12 years = (weight in kg x adult dosage)/ (age of child (in years) + 12)

Clarks formula

Dosage for children (general formula) = (weight in kg x adult dosage) /70

GST

The **GST** (Goods and Services Tax) is a broad sales tax of 10% on most goods and services transactions in Australia.

Interquartile range

The **interquartile range** (IQR) is a measure of the spread within a numerical data set. It is equal to the upper quartile (Q_3) minus the lower quartiles (Q_1); that is, $IQR = Q_3 - Q_1$

The IQR is the width of an interval that contains the middle 50% (approximately) of the data values. To be exactly 50%, the sample size must be a multiple of four.

kWh (kilowatt hour)

The kilowatt hour, or kilowatt-hour, is a unit of energy equal to 1000 watt hours or 3.6 megajoules The kilowatt hour is most commonly known as a billing unit for energy delivered to consumers by electric utilities.

MJ (Megajoule)

A **joule** is the SI unit of work. The megajoule (MJ) is equal to one million joules

Mean

The arithmetic mean of a list of numbers is the sum of the data values divided by the number of values in the list.

In everyday language, the arithmetic mean is commonly called the average.

For example, for the following list of five numbers 2, 3, 3, 6, 8 the mean equals

$$\frac{2+3+3+6+8}{5} = \frac{22}{5} = 4.4$$

In more general language, the mean of n observations $x_1, x_2, \ldots, x_n$ is

$$\bar{x} = \frac{\sum x_i}{n}$$

Median

The **median** is the value in a set of ordered set of data values that divides the data into two parts of equal size. When there are an odd number of data values, the median is the middle value. When there is an even number of data values, the median is the average of the two central values.

Mode

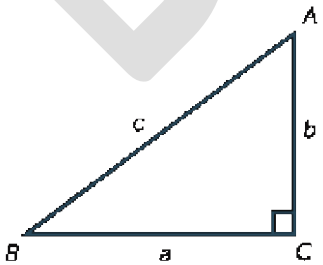
The **mode** is the most frequently occurring value is a data set.

Outlier

An outlier in a set of data is an observation that appears to be inconsistent with the remainder of that set of data. An outlier is a surprising observation.

Pythagoras' theorem

- For a right-angled triangle
- The square of the hypotenuse of a right-angled triangle equals the sum of the squares of the lengths of the other two sides.
- In symbols, $c^2 = a^2 + b^2$.



The converse

If $c^2 = a^2 + b^2$ in a triangle ABC, then $\angle C$ is a right angle.

Range

The **range** is the difference between the largest and smallest observations in a data set.

The range can be used as a measure of spread in a data set, but it is extremely sensitive to the presence of outliers and should only be used with care.

Reaction time

The time a person takes to react to a situation (pressing the brake) requiring them to stop

Simple interest

Simple interest is the interest accumulated when the interest payment in each period is a fixed fraction of the principal. For example, if the principle P earns simple interest at the rate of i % per period, then after n periods the accumulated simple interest is $\frac{Pni}{100}$.

Stopping distances

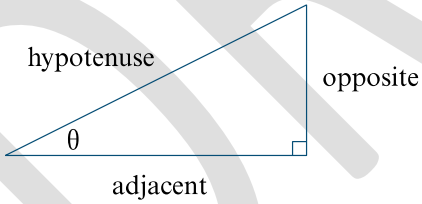
The distance a car travels after the driver has applied the brake given speed of the vehicle and/or conditions of the road which can be found using formula or tables.

Stopping distance = braking distance + reaction time(secs) Xspeed

Sine ratio

In any right-angled triangle,

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, where $0^\circ < \theta < 90^\circ$



Tangent ratio

In any right-angled triangle,

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$, where $0^\circ < \theta < 90^\circ$.