DRAFT SENIOR SECONDARY CURRICULUM – GENERAL MATHEMATICS

Organisation

1. Overview of senior secondary Australian Curriculum

ACARA has developed draft senior secondary Australian Curriculum for English, Mathematics, Science and History according to a set of design specifications (see http://www.acara.edu.au/curriculum/development_of_the_australian_curriculum.html). The ACARA Board approved these specifications following consultation with state and territory curriculum, assessment and certification authorities.

Senior secondary Australian Curriculum will specify content and achievement standards for each senior secondary subject. Content refers to the knowledge, understanding and skills to be taught and learned within a given subject. Achievement standards refer to descriptions of the quality of learning (the depth of understanding, extent of knowledge and sophistication of skill) demonstrated by students who have studied the content for the subject.

The senior secondary Australian Curriculum for each subject has been organised into four units. The last two units are cognitively more challenging than the first two units. Each unit is designed to be taught in about half a 'school year' of senior secondary studies (approximately 50–60 hours duration including assessment). However, the senior secondary units have also been designed so that they may be studied singly, in pairs (that is, year-long), or as four units over two years. State and territory curriculum, assessment and certification authorities are responsible for the structure and organisation of their senior secondary courses and will determine how they will integrate the Australian Curriculum content and achievement standards into courses. They will also provide any advice on entry and exit points, in line with their curriculum, assessment and certification requirements.

States and territories, through their respective curriculum, assessment and certification authorities, will continue to be responsible for implementation of the senior secondary curriculum, including assessment, certification and the attendant quality assurance mechanisms. Each of these authorities acts in accordance with its respective legislation and the policy framework of its state government and Board. They will determine the assessment and certification specifications for their courses that use the Australian Curriculum content and achievement standards and any additional information, guidelines and rules to satisfy local requirements.

These draft documents should not, therefore, be read as proposed courses of study. Rather, they are presented as draft content and achievement standards that will provide the basis for senior secondary curriculum in each state and territory in the future. Once approved, the content and achievement standards would subsequently be integrated by states and territories into their courses.

2. Senior Secondary Mathematics subjects

The Senior Secondary Australian Curriculum: Mathematics consists of four subjects in mathematics. The subjects are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students. Each subject is organised into four units.

Essential Mathematics focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This subject provides the opportunity for students to prepare for post-school options of employment and further training.

General Mathematics focuses on using the techniques of discrete mathematics to solve problems in contexts that include financial modelling, network analysis, route and project planning, decision making, and discrete growth and decay. It provides an opportunity to analyse and solve a wide range of geometrical problems in areas such as measurement, scaling, triangulation and navigation. It also provides opportunities to develop systematic strategies based on the statistical investigation process for answering statistical questions that involve comparing groups, investigating associations and analysing time series.

Mathematical Methods focuses on the development of the use of calculus and statistical analysis. The study of calculus in Mathematical Methods provides a basis for an understanding of the physical world involving rates of change, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics in Mathematical Methods develops the ability to describe and analyse phenomena involving uncertainty and variation.

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers, matrices and recursive methods. Specialist Mathematics is the only mathematics subject that cannot be taken as a stand-alone subject.

3. Structure of General Mathematics

General Mathematics is structured over four units. The topics broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The units provide a blending of algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. The statistics topics are developed across all four units, ending with tools for students to evaluate sophisticated statistical information.

	Unit 1	Unit 2	Unit 3	Unit 4
General Mathematics	Financial mathematics 1: Basic principles Matrices Graphs and networks 1: Undirected graphs and their applications	Statistics 1: Comparisons Linear equations and their graphs Shape and measurement	Statistics 2: Associations Geometry and trigonometry Modelling discrete growth and decay	Financial mathematics 2: Investments, loans and asset revaluation Statistics 3: Time series Graphs and Networks 2: Directed graphs and their applications

Units

Unit 1 has three topics, 'Financial mathematics 1: Basic principles', 'Matrices', and 'Graphs and networks 1: undirected graphs and their applications'. The topic 'Financial mathematics 1' reviews the concepts of percentage change, simple and compound interest, and extends their use to a wide range of everyday financial situations. The topics 'Matrices' and 'Graphs and networks 1' will be new to students, but the mathematics involved follows naturally from the mathematics in the *Algebra and Number* strand of the F-10 curriculum. These topics find applications in practical situations that involve individual objects and their connections, for example, towns on a map and the roads between them, animals in a food web, or people in a social network.

Unit 2 has three topics, 'Statistics 1: Comparisons', 'Linear equations and their graphs', and 'Shape and measurement'. The topic 'Statistics 1' is concerned with developing students' ability to systematically compare two or more groups on a single statistical measure and to apply this knowledge in the context of conducting a statistical investigation. The topic 'Linear equations and their graphs' is concerned with the use of linear equations and straight-line graphs, as well as linear-piecewise and step graphs, to model and analyse practical situations. The 'Shape and measurement' topic builds on and extends the knowledge and skills students developed in the F-10 curriculum, with the concept of similarity and associated calculations involving simple geometric shapes. The emphasis in this topic is on applying these skills in a range of practical contexts, including those involving three-dimensional shapes.

Unit 3 has three topics, ‘Statistics 2: Associations’, ‘Geometry and trigonometry’, and ‘Modelling discrete growth and decay’. In the topic ‘Statistics 2’, students are introduced to some methods for identifying, analysing and describing associations between pairs of variables, including the use of the least-squares method as a tool for modelling and analysing linear associations. The content is to be taught within the framework of the statistical investigation process. The topic ‘Geometry and trigonometry’ focuses on solving practical problems involving both right-angled triangles and non-right angled triangles in two and three dimensions, including problems involving the use of angles of elevation and depression and bearings in navigation. The third topic, ‘Modelling discrete growth and decay’, is concerned with using recursion to generate sequences, including matrix sequences, that can be used to model and investigate patterns of growth and decay in discrete systems. These sequences find application in a wide range of practical situations ranging from modelling the growth of a bacterial population to the decrease in the value of a car over time. They are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4.

Unit 4 has three topics, ‘Financial mathematics 2: Investments, loans and asset revaluation’, ‘Statistics 3: Time series’, and ‘Graphs and networks 2: Directed graphs and their applications’. The topic ‘Financial mathematics 2’ aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage, making investments, or revaluing assets over time. The topic ‘Time series’ continues the students’ study of statistics by introducing them to the concepts and techniques of time series analysis. The topic ‘Graphs and networks 2’ extends the study of graphs and networks to include directed graphs, opening up a wide range of applications including ranking players in round robin tournaments, flow in networks, project scheduling and critical path analysis.

Organisation of achievement standards

The achievement standards have been organised into two dimensions, ‘Concepts and Techniques’ and ‘Reasoning and Communication’. These two dimensions reflect students’ understanding and skills in the study of mathematics.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in these subjects.

4. Links to F-10

The General Mathematics subject provides students with a breadth of mathematical experience that encompasses and builds on all three strands of the F-10 curriculum. In the *Number and Algebra* strand, the focus is on using the techniques of discrete mathematics to solve problems in contexts that include financial modelling, network analysis, route and project planning, decision making, and discrete growth and decay. In the *Measurement and Geometry* strand, the focus is on analysing and solving a wide range of geometrical problems in areas such as measurement, scaling, triangulation and navigation. In the *Probability and Statistics* strand, the focus is on acquiring systematic strategies based on the statistical investigation process for answering statistical questions that involve comparing groups, investigating associations and analysing time series. There is also an emphasis throughout the subject on the use and application of information and communication technologies.

5. Representation of General Capabilities

The seven general capabilities of *Literacy*, *Numeracy*, *Information and Communication Technology* (ICT) *capability*, *Critical and creative thinking*, *Personal and social capability*, *Ethical behaviour*, and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

General capabilities that are specifically covered in *General Mathematics* include *Literacy, Numeracy, Information and communication technology (ICT) capability, Critical and creative thinking and Ethical behaviour.*

Literacy is of fundamental importance in students' development of General Mathematics as they develop the knowledge, skills and dispositions to interpret and use language confidently for learning. Students will be taught to read, understand and gather information presented in a wide range of genres, modes and representations (including text, symbols, graphs and tables). They are taught to communicate ideas logically and fluently and to structure arguments.

Numeracy involves students recognising and understanding the role of mathematics in the world and to use mathematical knowledge and skills purposefully. General Mathematics has a central role in the development of numeracy in a manner that is more explicit and foregrounded than is the case in other learning areas. General Mathematics provides the opportunity to apply mathematical understanding and skills in a real world context.

Critical and creative thinking is inherent in General Mathematics. Students develop their critical and creative thinking as they learn to generate and evaluate knowledge, clarify concepts and ideas, seek possibilities, consider alternatives and solve problems. Critical and creative thinking is integral to activities that require students to think broadly and deeply using skills, behaviours and dispositions such as reason, logic, resourcefulness, imagination and innovation in all learning areas at school and their lives beyond school.

Ethical behaviour involves students exploring the ethics of their own and other others' actions. Students develop the capability to behave ethically as they identify and investigate the nature of ethical concepts, values, character traits and principles, and understand how reasoning can assist ethical judgement. There are opportunities in General Mathematics to explore, develop and apply ethical behaviour in a range of contexts.

Information and Communication Technology (ICT) is a key part of General Mathematics. Students develop ICT capability as they learn to use ICT effectively and appropriately to access, create and communicate information and ideas, solve problems, perform calculations, draw graphs, collect, analyse and interpret data. Digital technologies can engage students and promote the understanding of key concepts.

There are also opportunities within General Mathematics to develop the general capabilities of *Intercultural understanding* and *Personal and social capability*, with an appropriate choice of activities and contexts provided by the teacher.

6. Representation of Cross-curriculum priorities

The Cross-curriculum priorities of Aboriginal and Torres Strait Islander histories and cultures, Asia and Australia's engagement with Asia, and Sustainability, are not overtly evident in the content descriptions of the General Mathematics subject, but opportunities exist for teachers to reference them in the context of their teaching of relevant topics, especially those topics which use real data to develop mathematical and statistical concepts.

Aboriginal and Torres Strait Islander histories and cultures

Students will deepen their understanding of the lives of Aboriginal and Torres Strait Islander Peoples through the application of mathematical concepts in appropriate contexts. Teachers could develop statistical and mathematical learning opportunities based on information and data pertinent to Aboriginal and Torres Strait Islander histories and cultures.

Asia and Australia's engagement with Asia

In General Mathematics, the priority of Asia and Australia's engagement with Asia provides rich and engaging contexts for developing students' mathematical knowledge, skills and understanding.

In General Mathematics, students develop mathematical understanding in fields such as measurement and statistics by drawing on knowledge of and examples from the Asia region. Investigations involving data collection, representation and analysis can be used to examine issues pertinent to the Asia region.

Sustainability

In General Mathematics, the priority of sustainability provides rich, engaging and authentic contexts for developing students' abilities in number, measurement and statistics.

General Mathematics provides opportunities for students to develop problem solving and reasoning essential for the exploration of sustainability issues and their solutions. Mathematical understandings and skills are necessary to measure, monitor and quantify change in social, economic and ecological systems over time. Statistical analysis enables prediction of probable futures based on findings and helps inform decision making and actions that will lead to preferred futures.

In this learning area, students can observe, record and organise data collected from primary sources over time and analyse data relating to issues of sustainability from secondary sources. They can apply spatial reasoning, measurement, estimation, calculation and comparison to gauge local ecosystem health and can cost proposed actions for sustainability.

DRAFT SENIOR SECONDARY CURRICULUM – GENERAL MATHEMATICS

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways, to become the language now used to describe many aspects of the world in the twenty first century. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve practical problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

General Mathematics is designed for those students who want to extend their mathematical skills beyond Year 10 level but whose future studies or employment pathways do not require knowledge of calculus. The subject is designed for students who have a wide range of educational and employment aspirations, including continuing their studies at university or TAFE, as well as students wishing to undertake industry based traineeships or apprenticeships. The proficiency strands of the F-10 curriculum, understanding, fluency, problem solving and reasoning, are still relevant and are inherent in all aspects of this subject. Each of these proficiencies is essential and mutually reinforcing. Achieving a level of fluency with algorithmic processes is part of learning to become an effective and efficient problem solver. This fluency might include learning to perform routine calculations efficiently and accurately, or quickly recognising from a problem description the appropriate mathematical process or model to apply. Furthermore, understanding that a single mathematical process can be used in seemingly different situations helps students to see the connections between different areas of study and encourages the transfer of learning. This is an important part of learning the art of mathematical problem solving. In performing such analyses, reasoning is required at each decision-making step and in drawing appropriate conclusions. Presenting the analysis in a logical and clear manner to explain the reasoning used is also an integral part of the learning process.

The General Mathematics subject provides students with a breadth of mathematical experience that encompasses and builds on all three strands of the F-10 curriculum. In the *Number and Algebra* strand, the focus is on using the techniques of discrete mathematics to solve problems in contexts that include financial modelling, network analysis, route and project planning, decision making, and discrete growth and decay. In the *Measurement and Geometry* strand, the focus is on analysing and solving a wide range of geometrical problems in areas such as measurement, scaling, triangualtion and navigation. In the *Probability and Statistics* strand, the focus is on acquiring systematic strategies based on the statistical investigation process for answering statistical questions that involve comparing groups, investigating associations and analysing time series. There is also an emphasis throughout the subject on the use and application of information and communication technologies.

Aims

General mathematics aims to develop students':

- understanding of concepts and techniques drawn from discrete mathematics, geometry and trigonometry, and statistics
- ability to solve applied problems using concepts and techniques drawn from discrete mathematics, geometry and trigonometry, and statistics
- reasoning and interpretive skills in mathematical and statistical contexts
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately.

Unit 1

Unit Description

This unit has three topics, 'Financial mathematics 1: Basic principles'; 'Matrices'; and 'Graphs and networks 1: undirected graphs and their applications'.

The topic 'Financial mathematics 1' reviews the concepts of percentage change, simple and compound interest, and extends their use to a wide range of everyday financial situations. The topics 'Matrices and 'Graphs and networks 1' will be new to students but the mathematics involved follows naturally from the mathematics in the *Algebra and Number* strand of the F-10 curriculum. These topics find applications in practical situations that involve individual objects and their connections, for example, towns on a map and the roads between them, animals in a food web, or people in a social network.

Classroom access to the technology necessary to support the computational aspects of the topics in this unit is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in Financial mathematics, Matrices, and Graphs and networks
- apply reasoning skills and solve practical problems in Financial mathematics, Matrices, and Graphs and networks
- communicate their arguments and strategies when solving problems using appropriate mathematical language
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Financial mathematics 1: Basic principles

Percentage change:

- review the techniques for calculating percentages and percentage change
- apply percentage increases and decreases in contexts, such as: determining the effect of inflation on costs and wages, mark-ups and discounts, unit cost and savings made through buying in bulk, and costs associated with stamp duties and GST

Simple interest:

- review the concept of simple interest
- calculate the future value of a simple interest loan or investment and the total interest paid or earned
- calculate daily, monthly, quarterly and six monthly interest rates based on annual interest rates in contexts, such as: borrowing or investing money,
- determine the interest paid when multiple transactions are involved. For example*, a bank savings account that pays interest on the minimum monthly balance*

Compound interest:

- review the concept of compound interest
- calculate the future value of a compound interest loan or investment and the total interest paid or earned
- compare, numerically and graphically, the growth of simple interest and compound interest loans and investments, and solve related problems
- calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly

Topic 2: Matrices

Matrices and matrix arithmetic:

- use matrices for storing and displaying information that can be presented in rows and columns. For example, *data bases, links in social networks*
- recognise different types of matrices (row, column, square, zero, identity) and determine their size
- perform matrix addition, subtraction, multiplication by a scalar and matrix multiplication
- use technology with matrix arithmetic capabilities when appropriate
- use matrices to model and solve problems. For example, *costing or pricing problems*

The inverse of a matrix

- with the aid of technology, determine the inverse of a square matrix
- use matrices and their inverses to solve practical problems. For example, *coding and decoding messages*

Topic 3: Graphs and networks 1: Undirected graphs and their applications

The definition of a graph and associated terminology:

- recognise and explain the meanings of the terms: edge, loop, vertex, face, adjacent vertices, subgraph, simple graph, connected graph, complete graph, planar graph and tree
- identify practical situations that can be represented by a graph. For example, *a social network, a transport network, a food web, a family tree*
- determine the degree of a vertex
- construct an adjacency matrix
- apply Euler's rule to solve problems relating to planar graphs

Paths and cycles:

- identify paths and cycles in graphs and determine their lengths
- investigate and solve practical problems involving the use of Euler paths and Euler cycles. For example, *the Königsberg Bridge problem, planning an efficient garbage bin collection route*
- investigate and solve practical problems involving the use of Hamilton paths and Hamilton cycles. For example, *planning a sight-seeing tourist route around a city*
- identify (using trial and error methods only) the shortest path(s) in a weighted graph. For example, *solving simple cases of the travelling salesman problem*

Minimum connector problems:

- identify a minimum spanning tree in a connected weighted graph either by inspection or by using Prim's or Kruskal's algorithm
- use minimal spanning trees to solve minimal connector problems. For example, minimising the length of cable needed to provide power from a single power station to substations in several towns

Unit 2

Unit description

This unit has three topics, 'Statistics 1: Comparisons'; 'Linear equations and their graphs'; and 'Shape and measurement'. The topic 'Statistics 1' is concerned with developing students' ability to systematically compare two or more groups on a single statistical measure and to apply this knowledge in the context of conducting a statistical investigation. The topic 'Linear equations and their graphs' is concerned with the use of linear equations and straight-line graphs, as well as linear-piecewise and step graphs to model and analyse practical situations. The 'Shape and measurement' topic builds on and extends the knowledge and skills students developed in the F-10 curriculum with the concept of similarity and associated calculations involving simple geometric shapes. The emphasis in this topic is on applying these skills in a range of practical contexts, including those involving three-dimensional shapes.

Classroom access to the technology necessary to support the graphical, computational and statistical aspects of this unit is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques used in Statistics 1, Linear equations and their graphs, and Shape and measurement
- apply reasoning skills and solve practical problems in Statistics 1, Linear equations and their graphs, and Shape and measurement
- implement the statistical investigation process in contexts requiring the comparisons of data collected for two or more groups
- communicate their arguments and strategies when solving mathematical and statistical problems using appropriate mathematical or statistical language
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems and answers to statistical questions.

Content descriptions

Topic 1: Statistics 1: Comparisons

The statistical investigation process:

- review the statistical investigation process (identifying a problem and posing a statistical question, collecting or obtaining data, analysing the data, interpreting and communicating the results)

Making sense of data relating to a single numerical variable:

- classify numerical data as discrete (For example, *the number of rooms in a house*) or continuous (For example, *temperature in a room in °Celsius*)
- with the aid of an appropriate graphical display (chosen from dot plot, stem plot or histogram), describe the distribution of a numerical data set in terms of shape (single versus multimodal, symmetric versus skewed (positive or negative) location and spread and outliers and interpret this information in the context of the data
- determine the mean and standard deviation of a data set and use these statistics as measures of location and spread of a data distribution, being aware of their limitations

Comparing data for a numerical variable across two or more groups:

- construct and use parallel box plots (including the use of the 'Q1 – 1.5 x IQR' and 'Q3 + 1.5 x IQR' criteria for identifying possible outliers) to compare groups in terms of location (median), spread (IQR and range) and outliers and interpret and communicate the differences observed in the context of the data
- compare groups on a single numerical variable using medians, means, IQRs, ranges or standard deviations, as appropriate, interpret the differences observed in the context of the data and report the findings in a systematic and concise manner
- implement the statistical investigation process to answer questions that involve comparing the data for a numerical variable across two or more groups. For example, *are Year 10 students the fittest in the school?*

Topic 2: Linear equations and their graphs

Straight-line graphs and their applications:

- construct straight-line graphs both with and without the aid of technology
- determine the slope and intercepts of a straight-line graph from both its equation and its plot
- interpret, in context, the slope and intercept of a straight-line graph used to model and analyse a practical situation
- construct and analyse a straight-line graph to model a given linear relationship. For example, *modelling the level of water in a leaking water tank as a function of time*

Simultaneous linear equations and their applications:

- solve a pair of simultaneous linear equations using technology when appropriate
- solve practical problems that involve finding the point of intersection of two straight-line graphs. For example, *determining the break-even point where cost and revenue are represented by linear equations*

Piece-wise linear graphs and step graphs:

- sketch piece-wise linear graphs and step graphs, using technology when appropriate
- interpret piece-wise linear and step graphs used to model practical situations. For example, the change in the level of water in a tank over time when water is drawn off at different intervals and for different periods of time; the charging scheme for sending parcels of different weights through the post

Topic 3: Shape and measurement

Perimeters and area:

- solve practical problems requiring the calculation of perimeters and areas of circles, sectors of circles, triangles, rectangles, parallelograms and composites

Similar figures and scale factors:

- review the conditions for similarity of two-dimensional figures
- use the scale factor for two similar figures to solve linear scaling problems
- obtain measurements from scale drawings, such as: maps or building plans to solve problems
- construct scaled drawings of two-dimensional figures using a variety of techniques including the use of grid paper, internal and external point enlargements
- solve scaling problems involving the calculation of the areas of similar figures

Volume and surface area:

- calculate the volumes of standard three-dimensional objects such as spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations. For example, *the volume of water contained in a swimming pool*
- calculate the surface areas of standard three-dimensional objects such as spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations, For example, *the surface area of a cylindrical food container*
- find and apply scale factors to similar three-dimensional shapes to solve problems involving surface area and volume. For example, scaling up a model boat to determine the volume of water it will displace when constructed.

Achievement Standards Units 1 and 2

	Concepts and Techniques	Reasoning and Communication
A	The student: • understands and applies concepts and techniques to financial mathematics, matrices, graphs and networks, statistics linear equations and measurement • consistently and accurately applies multiple concepts and techniques, to solve a wide range of problem types, including non-standard problems • represents mathematical and statistical information accurately and precisely in numerical, graphical and symbolic form • uses digital technologies appropriately and skillfully to solve problems, to draw graphs, and to display and organise information	The student: • solves problems that require the synthesis of ideas from mathematics and statistics • communicates observations and mathematical and statistical arguments that are succinct, clear, reasoned, and evidenced to find solutions to non-standard problems • analyses and interprets results with comprehensive consideration of the validity and limitations of the use of models and evaluates the reasonableness of results and solutions to all problems • understands the relative strengths and weaknesses of the inter-relatedness of different representations of mathematical and statistical information
B	The student: • understands and applies most concepts and techniques to financial mathematics, matrices, graphs and networks, statistics linear equations, and measurement • consistently and accurately applies combinations of concepts and techniques to solve non-routine problems • represents mathematical and statistical information accurately in numerical, graphical and symbolic form • uses digital technologies appropriately and competently to solve problems, to draw graphs and to display and organise information	The student: • solves problems that require the interpretation of mathematical and statistical information • communicates observations and reasoned decisions to find solutions to problems • analyses and interprets results with consideration of the validity and limitations of the use of models and evaluates the reasonableness of results and solutions to problems • understands the relative strengths and weaknesses of different representations of mathematical and statistical information
C	The student: • understands and applies many concepts and techniques to financial mathematics, matrices, graphs and networks, statistics linear equations, and measurement • accurately applies combinations of some concepts and techniques to solve familiar problems • represents mathematical and statistical information in numerical, graphical and symbolic form • uses digital technologies appropriately to solve problems, to draw graphs and to display and organise information	The student: • solves familiar problems that require the interpretation of mathematical and statistical information • communicates methods for the solution of problems • analyses and interprets results with consideration of the limitations of the use of models and evaluates the reasonableness of results and solutions to familiar problems • recognises the different representations of mathematical and statistical information

	Concepts and Techniques	Reasoning and Communication
D	The student: • demonstrates limited application of some concepts and techniques to financial mathematics, matrices, graphs and networks, statistics linear equations and measurement • applies concepts and techniques to solve routine problems • represents mathematical and statistical information in limited forms • uses digital technologies to solve some problems and display information	The student: • solves routine problems that require the interpretation of mathematical and statistical information • communicates methods for the solution of some problems • recognises the reasonableness of results and solutions to routine problems • recognises some representations of mathematical and statistical information
E	The student: • demonstrates limited familiarity with financial mathematics, matrices, graphs and networks, statistics linear equations, and measurement • follows procedures to solve simple problems • uses digital technologies to perform simple calculations	The student: • communicates limited observations to the solutions of problems • recognises the solution to routine problems • recognises limited representations of mathematical and statistical information

Unit 3

Unit description

This unit has three topics, 'Statistics 2: Associations', 'Geometry and trigonometry', and 'Modelling discrete growth and decay'.

In the topic 'Statistics 2', students are introduced to some methods for identifying, analysing and describing associations between pairs of variables, including the use of the least-squares method as a tool for modelling and analysing linear associations. The content is to be taught within the framework of the statistical investigation process.

The topic 'Geometry and trigonometry' focuses on solving practical problems involving both right-angled triangles and non-right angled triangles in two and three dimensions, including problems involving the use of angles of elevation and depression and bearings in navigation.

The third topic, 'Modelling discrete growth and decay', is concerned with using recursion to generate sequences, including matrix sequences that can be used to model and investigate patterns of growth and decay in discrete systems. These sequences find application in a wide range of practical situations ranging from modelling the growth of a bacterial population to the decrease in the value of a car over time. They are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4.

Classroom access to technology to support the graphical and computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques used in Statistics 2, Geometry and trigonometry and Modelling discrete growth and decay
- apply reasoning skills and solve practical problems in Statistics 2, Geometry and trigonometry and Modelling discrete growth and decay
- implement the statistical investigation process in contexts requiring the analysis of associations
- communicate their arguments and strategies when solving mathematical and statistical problems using appropriate mathematical or statistical language
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems and answers to statistical questions.

Content descriptions

Topic 1: Statistics 2: Associations

Identifying and describing associations between two categorical variables:

- identify the response variable and the explanatory variable where appropriate
- construct two-way frequency tables and determine the associated row and column sums and percentages
- use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association
- describe an association in terms of differences observed in percentages across categories in a systematic and concise manner and interpret this in the context of the data

Identifying and describing associations between two numerical variables:

- identify the response variable and the explanatory variable
- construct a scatterplot to identify patterns in the data suggesting the presence of an association
- describe an association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak)
- calculate and interpret the correlation coefficient (r) to quantify the strength of a linear association

Fitting a linear model to numerical data:

- use a scatterplot to identify the nature of the relationship between the variables
- model a linear relationship by fitting a least-squares line to the data
- use a residual plot to assess the appropriateness of fitting a linear model to the data
- interpret the intercept and slope of the fitted line
- use the coefficient of determination to assess the strength of a linear association in terms of explained variation
- use the equation of a fitted line to make predictions
- distinguish between interpolation and extrapolation when using the fitted line to make predictions, recognising the potential dangers of extrapolation
- write-up the results of the analysis in a systematic and concise manner

Association and causation:

- recognise that an observed association between two variables does not necessarily mean that there is a causal relationship between the variables

- identify possible non-causal explanations for an association including coincidence and confounding due to common response to another variable, and communicate these explanations in a systematic and concise manner

The data investigation process:

- implement the statistical investigation process to answer questions that involve identifying, analysing and describing associations between two categorical variables or two numerical variables. For example; *Is there an association between attitude to capital punishment (agree with, no opinion, disagree with) and sex (male, female)? or, Is there an association between height and foot length?*

Topic 2: Geometry and trigonometry

Review of Pythagoras' theorem and the use of the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle

- determine the sine, cosine and tangent ratios for obtuse angles
- determine the area of a triangle given two sides and an included angle by using the rule Area = $\frac{1}{2}ab\sin C$, or three sides by using Heron's rule and solve related practical problems
- solve problems involving non-right angled triangles using the sine rule (ambiguous case excluded) and the cosine rule
- solve practical problems involving the trigonometry of right-angled and non-right angled triangles, including problems involving angles of elevation and depression and the use of bearings in navigation

Topic 3: Modelling discrete growth and decay

The arithmetic sequence:

- use recursion to generate an arithmetic sequence
- display the terms of an arithmetic sequence in both tabular and graphical form and demonstrate that arithmetic sequences can be used to model linear growth and decay in discrete situations
- deduce a rule for the nth term of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence and use this rule to make predictions
- use arithmetic sequences to model and analyse practical situations involving linear growth or decay. For example, calculating a taxi fare based on the flag fall and the charge per kilometre or using the straight-line method to depreciate the value of office furniture at the end of each year

The geometric sequence:

- use recursion to generate a geometric sequence

- display the terms of a geometric sequence in both tabular and graphical form and demonstrate that geometric sequences can be used to model exponential growth and decay in discrete situations
- deduce a rule for the *n*th term of a particular geometric sequence from the pattern of the terms in the sequence and use this rule to make predictions
- use geometric sequences to model and analyse (numerically or graphically only) practical problems involving exponential growth and decay in discrete situations. For example, *the growth of a bacterial population that doubles in size each hour; the decreasing height of the bounce of a bouncing ball at each bounce*

Sequences generated by first-order linear recurrence relations:

- use a general first-order linear recurrence relation to generate the terms of a sequence and display it in both tabular and graphical form
- recognise that a sequence generated by a first-order linear recurrence relation can have a long term increasing, decreasing or a steady-state solution
- use first-order linear recurrence relations to model and analyse (numerically or graphically only) practical problems involving growth and decay in discrete situations. For example, *the growth of a trout population in a lake recorded at the end of each year and where limited recreational fishing is permitted or the amount owing on a reducing balance loan after each payment is made*

Sequences generated by a first-order matrix recurrence relation:

- use a first-order linear matrix recurrence relation to generate the terms of a matrix sequence
- use a population projection matrix. For example, use the Leslie matrix, to model and analyse (numerically or graphically only), the growth of a population and changes in the age or life stage distribution within that population over time.

Unit 4

Unit description

This unit has three topics, 'Financial mathematics 2: Investments, loans and asset revaluation'; 'Statistics 3: Time series'; and 'Graphs and networks 2: Directed graphs and their applications'.

The topic 'Financial mathematics 2' aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage, making investments, or revaluing assets over time. The second topic, 'Time series', continues students' study of statistics by introducing them to the concepts and techniques of time series analysis. The final topic, 'Graphs and networks 2', extends the study of graphs and networks to include directed graphs, opening up a wide range of applications including ranking players in round robin tournaments, flow in networks, project scheduling and critical path analysis.

Classroom access to the technology necessary to support the graphical, computational and statistical aspects of this unit is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques used in Financial mathematics 2, Statistics 3, and Graphs and networks 2
- apply reasoning skills and solve practical problems in Financial mathematics 2, Statistics 3, and Graphs and networks 2
- implement the statistical investigation process in contexts requiring the analysis of time series data
- communicate their arguments and strategies when solving mathematical and statistical problems using appropriate mathematical or statistical language
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems and answers to statistical questions.

Content descriptions

Topic 1: Financial mathematics 2: Loans, investments and asset revaluation

Compound interest loans and investments:

- review the principles of simple interest and compound interest
- use a recurrence relation to model a compound interest loan or investment and investigate (numerically or graphically) the effect of the interest rate and the number of compounding periods on the future value of the loan or investment
- solve problems involving compound interest loans or investments. For example, *determining the future value of a loan*

Reducing balance loans (compound interest loans with periodic repayments):

- use a recurrence relation to model a reducing balance loan and investigate (numerically or graphically) the effect of the interest rate and repayment amount on the time taken to repay the loan
- with the aid of calculator or computer based financial software, solve problems involving reducing balance loans. For example, *determining the monthly repayments required to pay off a housing loan*

Annuities and perpetuities (compound interest investments with periodic payments made from the investment):

- use a recurrence relation to model an annuity and investigate (numerically or graphically) the effect of the amount invested, the interest rate and the payment amount on the duration of the annuity
- with the aid of calculator or computer based financial software, solve problems involving annuities (including perpetuities as a special case). For example, *determining the amount to be invested in an annuity to provide a regular monthly income of a certain amount*

Revaluing assets over time:

- adjust the book value of an asset to reflect its decreasing value over time using three methods: flat rate (straight line) depreciation, declining (reducing) balance depreciation and unit cost depreciation
- compare the three methods of depreciation to determine which produces the lowest book value of an asset after a given period of time
- adjust the prices of goods or services subject to inflation using the CPI or other indicators. For example, the annual cost of registering a car or determining the spending power of money that is devalued by inflation

Topic 2: Statistics 3: Time series analysis

Describing and interpreting patterns in time series data:

- construct time series plots
- describe time series plots by identifying features, such as: trend (long term direction), seasonality (systematic, calendar related movements) and irregular fluctuations (unsystematic, short term fluctuations) and recognise when there are outliers. For example, *one-off unanticipated events*

Analysing time series data:

- smooth time series data by using a simple moving average
- calculate seasonal indices by using the average percentage method
- deseasonalise a time series by using a seasonal index
- fit a least-squares line to model long term trends in time series data

The data investigation process:

- implement the statistical investigation process to answer questions that involve the analysis of time series data

Topic 3: Graphs and Networks 2: Directed graphs and their applications

The basic properties of directed graphs and their applications:

- review the terminology, properties and applications of non-directed graphs, such as: Euler path, Euler cycle, Hamilton path and Hamilton cycle
- extend the concept of a graph to include graphs with directed edges (arcs)
- construct an adjacency matrix A for a directed graph and, where there is a context, interpret the entries in A in terms of that context
- and use the matrix A^k (the k^{th} power of the matrix A) to determine the number of paths of length k between any pair of vertices and interpret the elements of A^k
- use directed graphs and their matrices to display and analyse practical situations (small scale only). For example, *food webs, one-way street systems or the results of a single round-robin sporting competition*

Project planning and scheduling using critical path analysis (CPA):

- construct a network to represent the durations and interdependencies of activities that must be completed during the project. For example, *preparing a meal*
- use forward and backward scanning to determine the earliest starting time (EST) and latest starting times (LST) of each activity in the project
- use ESTs and LSTs to locate the critical path(s) for the project
- use the critical path to determine the minimum time for a project to be completed
- calculate float times for non-critical activities

Flow networks:

- solve small scale network flow problems including the use of the maximum flow-minimum cut theorem. For example, determining the maximum volume of oil that can flow through a network of pipes from an oil storage tank (the source) to a terminal (the sink)

Achievement Standards Unit 3 & 4

	Concepts and Techniques	Reasoning and Communication
A	The student: • understands and applies concepts and techniques to statistics, geometry and trigonometry and modelling financial mathematics and graphs and networks • consistently and accurately applies multiple concepts, techniques and models, to solve a wide range of problem types, including non-standard problems • represents mathematical and statistical information accurately and precisely in numerical, graphical and symbolic form • uses digital technologies appropriately and skillfully to model and solve problems and to display and organise information	The student: • solves a wide range of problems, including non-standard problems, that require the synthesis of ideas from mathematics and statistics • precisely communicates observations and reasoned decisions based on information from varied situations to find solutions to non-standard problems • analyses and interprets the reasonableness of results and solutions to problems derived from diverse mathematical models and statistical information • analyses, interprets and communicates results with comprehensive consideration of the validity and limitations of the use of models • understands the relative strengths and weaknesses and the inter-relatedness of different representations of mathematical models and statistical information
B	The student: • understands and applies most concepts and techniques to statistics, geometry and trigonometry and modelling financial mathematics and graphs and networks • consistently and accurately applies a combination of concepts and procedures and models, to solve non-routine problems • represents mathematical and statistical information accurately in numerical, graphical and symbolic form • uses digital technologies appropriately and competently to model and solve problems and to display and organise information	The student: • solves non-routine problems that require the synthesis of mathematical models and statistical information and ideas • communicates observations and reasoned decisions to find the solutions to problems based on information from varied situations • analyses the reasonableness of results and solutions to problems derived from diverse mathematical models and statistical information • analyses and communicates results of investigations with comprehensive consideration of the validity and limitations of the use of models • understands the relative strengths and weaknesses and the inter-relatedness of different representations of mathematical models and statistical information
C	The student: • understands and applies some concepts and techniques to statistics, geometry and trigonometry and modelling financial mathematics and graphs and networks • accurately applies limited combinations of concepts and techniques to solve familiar problems • represents mathematical and statistical information in numerical, graphical and symbolic form • uses digital technologies appropriately to model and solve problems to display and organise information	The student: • solves familiar problems that require the use of mathematical models and statistical information • communicates observations and reasoned decisions to find the solutions to problems • recognises the reasonableness of results and solutions to problems derived from diverse mathematical models and statistical information • communicates results of investigations with limited consideration of the validity and limitations of the use of models • understands the inter-relatedness of different representations of mathematical models and statistical information

	Concepts and Techniques	Reasoning and Communication
D	The student: • demonstrates limited understanding and application of some concepts and techniques to statistics, geometry and trigonometry and modelling financial mathematics and graphs and networks • applies concepts and procedures to solve routine problems • represents mathematical and statistical information in limited forms • uses digital technologies to solve some problems	The student: • solves routine problems that require the use of mathematical models and statistical information • communicates some observations and makes decisions to find the solution to problems • recognises the reasonableness of results and solutions to problems • communicates results of investigations
E	The student: • demonstrates limited familiarity in statistics, geometry and trigonometry and modelling financial mathematics and graphs and networks • follows procedures to solve simple problems • uses digital technologies to represent information and to solve simple problems	The student: • follows procedures to solve routine problems that require the use of mathematical and statistical information • communicates some observations to find the solutions to problems • recognises the solutions to routine problems • recognises the representations of mathematical and statistical information

GLOSSARY ITEMS

Financial Mathematics

Annuity

An **annuity** is a compound interest investment from which payments are made on a regular basis for a fixed period of time. At the end of this time the investment has no residual value.

Book value

The **book value** is the value of an asset recorded on a balance sheet. The book value is based on the original cost of the asset less depreciation.

For example, if the original cost of a printer is $500 and its value depreciates by $100 over the next year, then its book value at the end of the year is $400.

There are three commonly used methods for calculating yearly depreciation in the value of an asset, namely, **reducing balance depreciation**, **flat rate depreciation** or **unit cost depreciation**.

CPI

The **Consumer Price Index** (CPI) is a measure of changes, over time, in retail prices of a constant basket of goods and services representative of consumption expenditure by resident households in Australian metropolitan areas.

Effective annual rate of interest

The **effective annual rate of interest** $i_{\text{effective}}$ is used to compare the interest paid on loans (or investments) with the same nominal annual interest rate i but with different compounding periods (daily, monthly, quarterly, annually, other)

If the number of compounding periods per annum is n, then $i_{\text{effective}} = (1+\frac{i}{n})^n - 1$

For example if the quoted annual interest rate for a loan is 9%, but interest is charged monthly, then the effective annual interest rate charged is $i_{\text{effective}} = \left(1+\frac{0.09}{12}\right)^{12} - 1 = 0.9416\ldots$, or around 9.4%.

Diminishing value depreciation see Reducing balance depreciation

Flat rate depreciation

In flat rate or straight-line depreciation the value of an asset is depreciated by a fixed amount each year. Usually this amount is specified as a fixed percentage of the original cost.

GST

The **GST** (Goods and Services Tax) is a broad sales tax of 10% on most goods and services transactions in Australia.

Straight-line depreciation

See: flat rate depreciation —

Compound interest

The interest earned by investing a sum of money (the principal) is compound interest if each successive interest payment is added to the principal for the purpose of calculating the next interest payment.

For example, if the principal *P* earns compound interest at the rate of *i* % per period, then after *n* periods the total amount accrued is $P(1+\frac{i}{100})^n$. When plotted on a graph, the total amount accrued is seen to grow exponentially.

Perpetuity

A **perpetuity** is a compound interest investment from which payments are made on a regular basis in perpetuity (forever). This is possible because the payments made at the end of each period exactly equal the interest earned during that period.

Reducing balance depreciation

In **reducing balance depreciation** the value of an asset is depreciated by a fixed percentage of its value each year.

Reducing balance depreciation is sometimes called diminishing value depreciation.

Reducing balance loan

A reducing balance loan is a compound interest loan where the loan is repaid by making regular payments and the interest paid is calculated on the amount still owing (the reducing balance of the loan) after each payment is made.

Simple interest

Simple interest is the interest accumulated when the interest payment in each period is a fixed fraction of the principal. For example, if the principle *P* earns simple interest at the rate of *i* % per period, then after *n* periods the accumulated simple interest is $nP\frac{i}{100}$

When plotted on a graph, the total amount accrued is seen to grow linearly.

Unit cost depreciation

In unit cost depreciation, the value of an asset is depreciated by an amount related to the number of units produced by the asset during the year.

Geometry and trigonometry

Angle of elevation

The angle a line makes above a plane.

Angle of depression

The angle a line makes below a plane.

Area of a triangle

The general rule for determining the area of a triangle is: $area = \frac{1}{2} base \times height$

Bearings (compass and true)

A **bearing** is the direction of a fixed point, or the path of an object, from the point of observation.

Compass bearings are specified as angles either side of north or south. For example a compass bearing of N50°E is found by facing north and moving through an angle of 50° to the East.

True (or three figure) bearings are measured in degrees from the north line. Three figures are used to specify the direction. Thus the direction of north is specified as 000°, east is specified as 090°, south is specified as 180° and north-west is specified as 315°.

Cosine rule

For a triangle of side lengths a, b and c and angles A, B and C, the cosine rule states that

$c^2 = a^2 + b^2 - 2ac \cos C$

Heron's rule is a rule for determining the area of a triangle given the lengths of its sides.

The area A of a triangle of side lengths a, b and c is given by $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$.

Similar figures

Two geometric figures are similar if they are of the same shape but not necessarily of the same size.

Sine rule

For a triangle of side lengths a, b and c and angles A, B and C, the sine rule states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Triangulation

The process of determining the location of a point by measuring angles to it from known points at either end of a fixed baseline, rather than measuring distances to the point directly. The point can then be fixed as the third point of a triangle with one known side and two known angles.

Scale factor

A **scale factor** is a number that scales, or multiplies, some quantity. In the equation $y = kx$, k is the scale factor for x.

If two or more figures are similar, their sizes can be compared. The scale factor is the ratio of the length of one side on one figure to the length of the corresponding side on the other figure. It is a measure of magnification, the change of size.

Graphs & networks

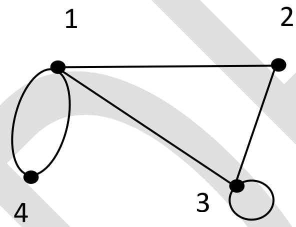
Adjacent (graph) see graph

Adjacency matrix

An **adjacency matrix** for a non-directed graph with n vertices is a $n \times n$ matrix in which the entry in row i and column j is the number of edges joining the vertices i and j. In an adjacency matrix, a **loop** is counted as 1 edge.

Example:

Non-directed graph Adjacency matrix

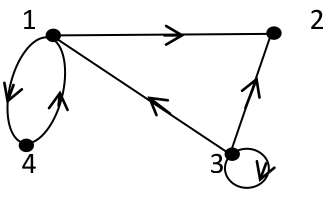


$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

For a directed graph the entry in row i and column j is the number of directed edges (arcs) joining the vertex i and j in the direction i to j.

Example:

Directed graph Adjacency matrix



$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Algorithm

An **algorithm** is a precisely defined routine procedure that can be applied and systematically followed through to a conclusion. An example is **Prim's algorithm** for determining a **minimum spanning tree** in a network.

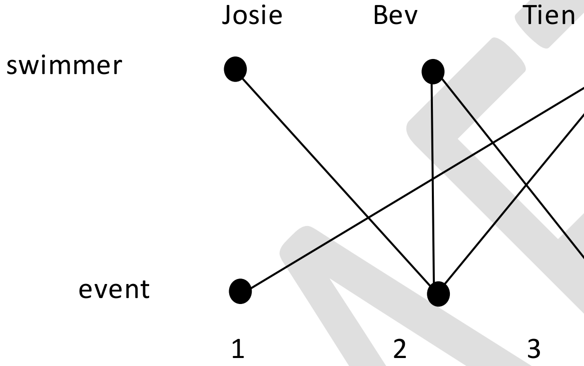
Arc

An arc is an edge in a directed graph.

Bipartite Graph

A bipartite graph is a graph whose set of vertices can be split into two distinct groups in such a way that each edge of the graph joins a vertex in the first group to a vertex in the second group.

Example:



Closed path see path

Complete graph

A **complete graph** is a **simple graph** in which every vertex is joined to every other vertex by an edge.

The complete graph with n vertices is denoted K_n .

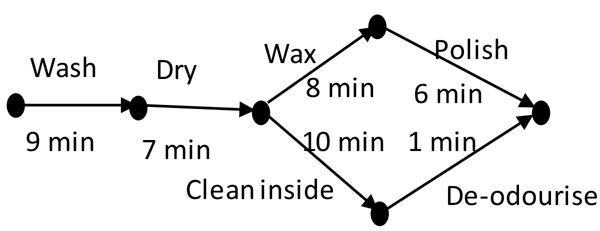
Connected graph

A graph is **connected** if there is a path between each pair of vertices.

Critical path analysis (CPA)

A project often involves many related activities some of which cannot be started until one or more earlier tasks have been completed. One way of scheduling such activities that takes this into account is to construct a network diagram.

The network diagram below can be used to schedule the activities of two or more individuals involved in cleaning and polishing a car. The completion times for each activity are also shown.



Critical path analysis is a method for determining the longest path (the **critical path**) in such a network and hence the minimum time in which the project can be completed. There may be more that one critical path in the network. In this project the critical path is 'Wash-Dry-Wax-Polish' with a total completion time of 30 minutes.

The **earliest starting time (EST)** of an activity 'Polish' is 24 minutes because activities 'Wash', 'Dry' and 'Wax' must be completed first. The process of systematically determining earliest starting times is called **forward scanning.**

The shortest time that the project can be completed is 30 minutes. Thus, the **latest starting time (LST)** for the activity 'De-odourise' is 29 minutes. The process of systematically determining latest starting times is called **backward scanning**.

Float or slack

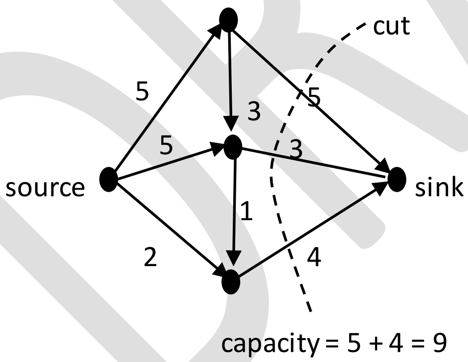
Is the amount of time that a task in a project network can be delayed without causing a delay to subsequent tasks. For example, the activity 'De-odourise' is said to have a **float** of 3 minutes because its earliest EST (26 minutes) is three minutes before its LST (29 minutes). As a result this activity can be started at any time between 26 and 29 minutes after the project started. All activities on a critical path have zero floats.

Cut (in a flow network)

In a flow network, a **cut** is a partition of the vertices of a graph into two separate groups with the **source** in one group and the **sink** in the other.

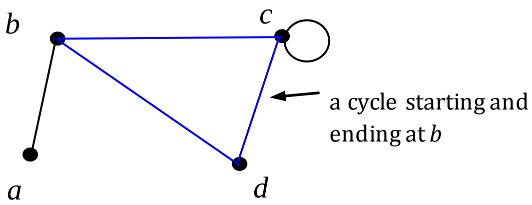
The capacity of the **cut** is the sum of the capacities of the cut edges directed from source to sink. Cut edges directed from sink to source are ignored.

Example:



Cycle

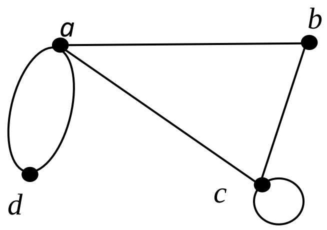
A cycle is a closed **path**. If a, b, c and d are the vertices of a graph, the closed **path** *bcdb* that starts and ends at vertex b (shown in blue) an example of a cycle.



Degree of a vertex (graph)

In a graph, the **degree of a vertex** is the number of edges incident with the vertex, with loops counted twice. It is denoted deg v.

In the graph below, deg a = 4, deg b = 2, deg c = 4 and deg d=2.

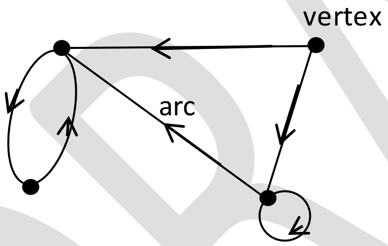


Digraph

See directed graph

Directed graph

A **directed graph** is a diagram comprising points, called vertices, joined by directed edges called **arcs**. The directed graphs are sometimes called **digraphs**.



Earliest starting time (EST)

See Critical Path Analysis

Edge

See graph

Euler cycle

An **Euler cycle** is a **cycle** that includes each **edge** in a **graph** once only. An Euler cycle may include repeated vertices.

Euler's formula

For a connected planar graph, **Euler's rule** states that

$$v + f = e + 2$$

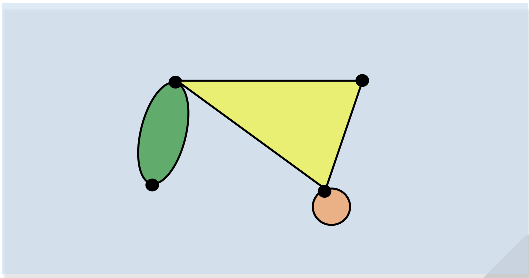
where v is the number vertices, e the number of edges and f is the number of faces.

Euler path

An **Euler path** is a **path** that includes each **edge** in a **graph** once only. An Euler path may include repeated vertices.

Face

The faces of a planar graph are the regions bounded by the edges including the outer infinitely large region. The planar graph shown has four faces.



Float time

See Critical Path Analysis

Flow network

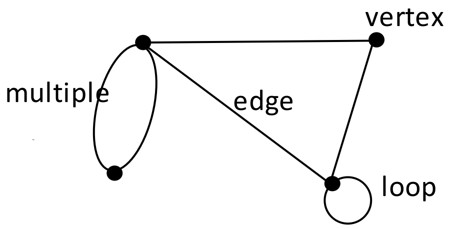
A **flow network** is a directed graph where each edge has a capacity (e.g. 100 cars per hour, 800 litres per minute, etc) and each edge receives a flow. The amount of flow on an edge cannot exceed the capacity of the edge. A flow must satisfy the restriction that the amount of flow into a node equals the amount of flow out of it, except when it is a **source**, which has more outgoing flow, or a **sink**, which has more incoming flow. A flow network can be used to model traffic in a road system, fluids in pipes, currents in an electrical circuit, or any situation in which something travels through a network of nodes.

Food web

A **food web** (or food chain) depicts feeding connections (who eats whom) in an ecological community.

Graph

A **graph** is a diagram that consists of a set of points, called **vertices** that are joined by a set of lines called **edges**. Each edge joins two vertices. A **loop** is an edge in a graph that joins a **vertex** in a **graph** to itself. Two vertices are **adjacent** if they a joined by an edge. Two or more edges connect the same vertices are called **multiple edges**.



Hamiltonian cycle

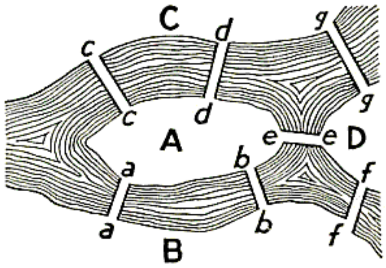
A **Hamiltonian cycle** is a **cycle** that includes each **vertex** in a **graph** once only. There are no repeated edges in a Hamilton cycle.

Hamiltonian path

A **Hamilton path** is path that includes each **vertex** in a **graph** once only. There are no repeated edges in a Hamilton path.

Königsberg bridge problem

The Königsberg bridge problem asks: Can the seven bridges of the city of Königsberg all be traversed in a single trip that starts and finishes at the same place?



Kruskal's algorithm

An **algorithm** for determining a **minimum spanning tree** in a connected weighted graph.

See also **Prim's algorithm**.

Latest starting time (LST)

See Critical Path Analysis

Length (of a path or cycle)

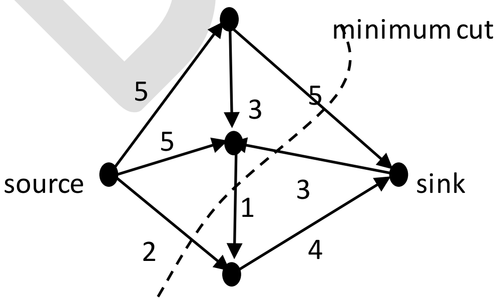
The **length** of a path or cycle is the number of edges it includes.

Minimum cut-maximum flow theorem

The **maximum flow–minimum cut theorem** states that in a flow network, the maximum flow from the **source** to the **sink** is equal to the capacity of the **minimum cut**.

In everyday language, the minimum cut involves identifies the 'bottle-neck' in the system.

Example:



capacity = 5+1+2 = 8

Minimum spanning tree

For a given connected **weighted graph**, the **minimum spanning tree** is the **spanning tree** of minimum length.

Multiple edges see graph

Network

The word network is frequently used in everyday life, e.g. television network, rail network, etc. In this topic, it also has a more specialised meaning where it also refers to any graph or digraph that carries numerical information.

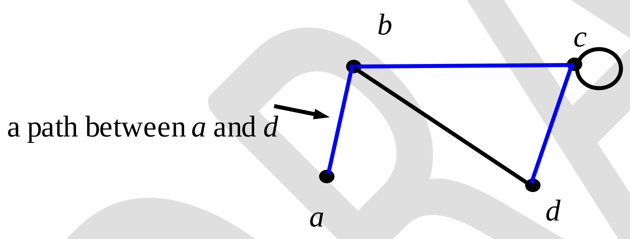
Open path

See path

Path (in a graph)

A **path** in a graph is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence. A path that starts and finishes at different vertices is said to be open, while a path that stats and finishes at the same vertex is said to be closed.

If a and d are the vertices of a graph, a walk from a to d along the edges coloured blue is a path. Depending on the graph, there may be multiple paths between the same two vertices, as is the case here.



Note: A more formal definition of a path does not allow for repeated edges and vertices. Such a path is sometimes called a **simple path**.

Planar graph

A **planar graph** is a graph that can be drawn in the plane. A planar graph can always be drawn so that no two edges cross.

Prim's algorithm

An **algorithm** for determining a **minimum spanning tree** in a connected weighted graph.

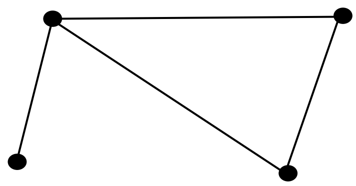
See also Kruskal's algorithm.

Round-robin sporting competition

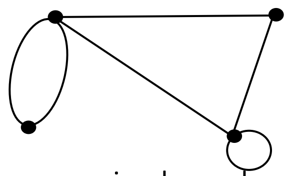
A single round robin sporting competition is a competition in which each competitor plays each other competitor once only.

Simple graph

A simple graph has no loops or multiple edges.



simple graph



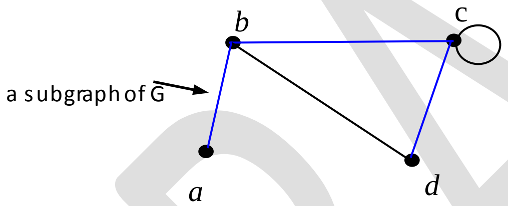
non-simple graph

Simple path

See path

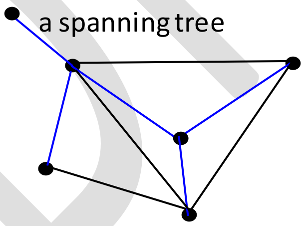
Subgraph

When the vertices and edges of a graph *A* (shown in blue) are the vertices and edges of the graph *G*, graph *A* is said to be a **subgraph** of graph *G*.



Spanning tree

A **spanning tree** is a **subgraph** of a **connected graph** that connects all vertices and is also a **tree.**



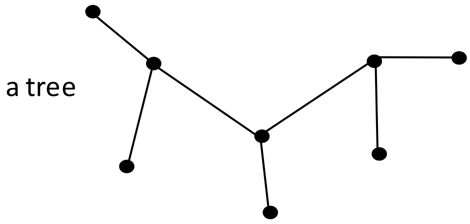
The travelling salesman problem

The travelling salesman problem can be described as follows: Given a list of cities and the distance between each city, find the shortest possible route that visits each city exactly once.

While in simple cases this problem can be solved by systematic identification and testing of possible solutions, no there is no known efficient method for solving this problem.

Tree

A **tree** is a connected graph with no circuits.

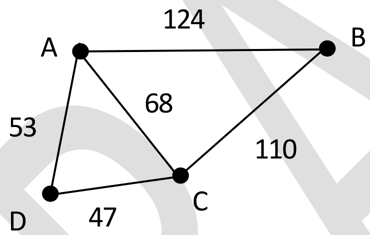


Vertex

See graph

Weighted graph

A weighted graph is a graph in which each edge is labelled with a number used to represent some quantity associated with the edge. For example, if the vertices represent towns, the weights on the edges may represent the distances in kilometres between the towns.



Growth and decay in sequences

Arithmetic sequence

An **arithmetic sequence** is a sequence of numbers such that the difference between any two successive members of the sequence is constant.

For example, the sequence

$2, 5, 8, 11, 14, 17, \ldots$

is an arithmetic sequence with first term 2 and common difference 3.

By inspection of the sequence, the rule for the nth term t_n of this sequence is:

$t_n = 2 + (n - 1)\,3 = 3n - 1 \quad n \geq 1$

If t_n is used to denote the nth term in the sequence, then a recursion relation that will generate this sequence is: $t_1 = 2,\ t_n = t_{n-1} + 3 \quad n \geq 2$

Break-even point

The **break-even point** is the point at which revenue begins to exceed the cost of production.

First-order linear recurrence relation

A **first-order linear recurrence relation** is defined by the rule: $t_0 = a$, $t_n = bt_{n-1} + c$ for $n \geq 1$

For example, the rule: $t_0 = 10$, $t_n = 5t_{n-1} + 1$ for $n \geq 1$ is a first-order recurrence relation.

The sequence generated by this rule is: $10, 51, 256, \ldots$ as shown below.

$t_0 = 10$, $t_1 = 5t_0 + 1 = 5 \times 10 + 1 = 51$, $t_2 = 5t_1 + 1 = 5 \times 51 + 1 = 256, \ldots$

Geometric growth or decay (sequence)

A sequence displays geometric growth or decay when each term is some constant multiple (greater or less than one) of the preceding term. A multiple greater than one corresponds to growth. A multiple less than one corresponds to decay.

For example, the sequence:

$1, 2, 4, \ldots$ displays geometric growth because each term is double the previous term.

$100, 10, 0.1, \ldots$ displays geometric decay because each term is one tenth of the previous term.

Geometric growth is an example of exponential growth in discrete situations.

Geometric sequence

A **geometric sequence**, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed non-zero number called the **common ratio**. For example, the sequence

$2, 6, 18, \ldots$

is a geometric sequence with first term 2 and common ratio 3.

By inspection of the sequence, the rule for the *n*th term of this sequence is:

$t_n = 2 \times 3^{n-1} \quad n \geq 1$

If t_n is used to denote the *n*th term in the sequence, then a recursion relation that will generate this sequence is: $t_1 = 2$, $t_n = 3t_{n-1} \quad n \geq 2$

Linear growth or decay (sequence)

A sequence displays linear growth or decay when the difference between successive terms is constant. A positive constant difference corresponds to linear growth while a negative constant difference corresponds to decay.

Examples:

The sequence, 1, 4, 7, … displays linear growth because the difference between successive terms is 3.

The sequence, 100, 90, 80, … displays linear decay because the difference between successive terms is –10. By definition, arithmetic sequences display linear growth or decay.

Population projection matrix

A **population projection matrix *P*** contains information about reproduction rates and survival rates of the female members of some population in which individuals can be classified as belonging to different age or life-stage groups, eg. individuals in population of insects might be classified as eggs, juveniles and adults.

Using a column matrix $\mathbf{S}_n$ to represent the state of the population at the *n*th time step, the state of the population at the *n*+1th time step can be determined using the matrix **recurrence relation**

$S_{n+1} = LS_n$ or $S_n = L^n S_0$ where S_0 represents the initial state of the population.

Commonly used examples of population projection matrices are the Leslie matrix and the Lefkovitch matrix.

Recursion

See recurrence relation

Recurrence relation

A **recurrence relation** is an equation that recursively defines a sequence; that is, once one or more initial terms are given, each further term of the sequence is defined as a function of the preceding terms.

Sequence

A **sequence** is an ordered list of numbers (or objects).

For example $1, 3, 5, 7$ is a sequence of numbers that differs from the sequence $3, 1, 7, 5$ as order matters.

A sequence maybe finite, for example, $1, 3, 5, 7$ (the sequence of the first four odd numbers), or infinite, for example, $1, 3, 5, \ldots$ (the sequence of all odd numbers).

Linear equations (relations) and graphs

Linear equation

A linear equation in one variable *x* is an equation of the form $ax + b = 0$, e.g. $3x + 1 = 0$

A linear equation in two variables x and y is an equation of the form $ax + by + c = 0$,

e.g. $2x - 3y + 5 = 0$

Linear graph

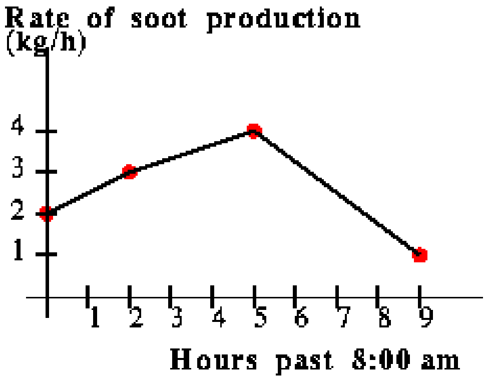
A **linear graph** is a graph of a linear equation with two variables. If the linear equation is written in the form $y = a + bx$, then a represents the y-intercept and b represents the slope (or gradient) of the linear graph.

Piecewise-linear graph

A graph consisting of one or more none overlapping line segments.

Sometimes called a line segment graph.

Example:



Slope (gradient)

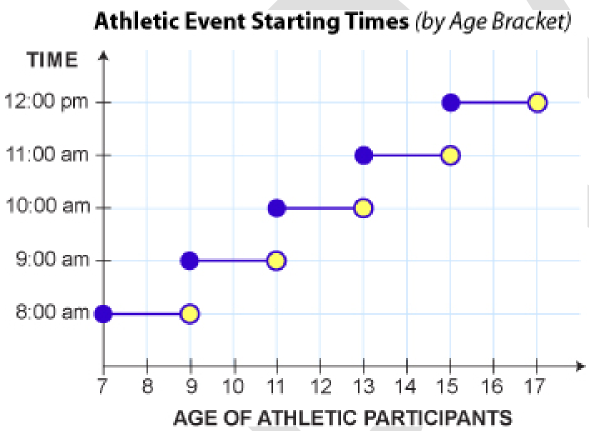
The **slope** or **gradient** of a line describes its steepness, incline, or grade.

Slope is normally described by the ratio of the "rise" divided by the "run" between two points on a line.

See also linear graph.

Step graph

A graph consisting of one or more non-overlapping horizontal line segments that follow a step-like pattern.



Matrices

Addition of matrices

If ***A*** and ***B*** are matrices of the same size (order) and the elements of **A** are a_{ij} and the elements of **B** are b_{ij} then the elements of ***A* + *B*** are $a_{ij} + b_{ij}$

For example if ***A*** = $\begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and ***B*** = $\begin{bmatrix} 5 & 1 \\ 2 & 1 \\ 1 & 6 \end{bmatrix}$ then

$$\boldsymbol{A} + \boldsymbol{B} = \begin{bmatrix} 7 & 2 \\ 2 & 4 \\ 2 & 10 \end{bmatrix}$$

Elements (Entries) of a matrix

The symbol a_{ij} represents the (i,j) element occurring in the i^{th} row and the j^{th} column.

For example a general 3×2 matrix is:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$ where a_{32} is the element in the third row and the second column

Identity matrix

A multiplicative **identity matrix** is a square matrix in which all of the elements in the leading diagonal are 1s and the remaining elements are 0s. Identity matrices are designated by the letter ***I***.

For example,

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ are both identity matrices.

.

There is an identity matrix for each size (or order) of a square matrix. When clarity is needed, the order is written with a subscript: $\boldsymbol{I}_n$

Inverse of a square matrix

The **inverse of a square matrix *A*** is written as $\boldsymbol{A}^{-1}$ and has the property that

$$\boldsymbol{A}\boldsymbol{A}^{-1} = \boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{I}$$

Not all square matrices have an inverse. A matrix that has an inverse is said to be **invertible**.

Inverse of a 2 × 2 matrix

The **inverse** of the matrix $\boldsymbol{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Is

$$\boldsymbol{A}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ provided $ad - bc \neq 0$

Leading diagonal

The leading diagonal of a square matrix is the diagonal that runs from the top left corner to the bottom right corner of the matrix.

Matrix (matrices)

A **matrix** is a rectangular array of elements or entities displayed in rows and columns.

For example,

$\boldsymbol{A} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}$ are both matrices with six elements.

Matrix ***A*** is said to be a 3×2 matrix (three rows and two columns) while ***B*** is said to be a 2 × 3 matrix (two rows and three columns).

A **square matrix** has the same number of rows and columns.

A **column matrix** (or vector) has only one column.

A **row matrix** (or vector) has only one row.

Matrix multiplication

Matrix multiplication is the process of multiplying a matrix by another matrix.

For example, forming the product

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 25 \\ 11 & 45 \end{bmatrix}$$

The multiplication is defined by $1 \times 2 + 8 \times 0 + 0 \times 4 = 2$

$$1 \times 1 + 8 \times 3 + 0 \times 4 = 25$$

$$2 \times 2 + 5 \times 0 + 7 \times 1 = 11$$

$$2 \times 1 + 5 \times 3 + 7 \times 4 = 45$$

This is an example of the process of matrix multiplication.

The product ***AB*** of two matrices ***A*** and ***B*** of size $m \times n$ and $p \times q$ respectively is defined if $n = p$.

If $n = p$ the resulting matrix has size $m \times q$.

If $\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$ then

$$\boldsymbol{AB} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{bmatrix}$$

Order (of a matrix)

see **size** (of a matrix)

Scalar multiplication (matrices)

Scalar multiplication is the process of multiplying a matrix by a scalar (number).

For example, forming the product

$$10\begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 20 & 10 \\ 0 & 30 \\ 10 & 40 \end{bmatrix}$$

is an example of the process of scalar multiplication.

In general for the matrix ***A*** with elements a_{ij} the elements of *k**A*** are ka_{ij}.

Singular matrix

A matrix is singular if det ***A*** = 0. A singular matrix does not have a multiplicative inverse.

Size (of a matrix)

Two matrices are said to have the same **size** (or **order**) **i**f they have the same number of rows and columns. A matrix with *m* rows and *n* columns is said to be a $m \times n$ matrix.

For example, the matrices

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \text{and} \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

have the same size. They are both 2×3 matrices.

Zero matrix

A zero matrix is a matrix if all of its entries are zero. For example:

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ are zero matrices.

Statistics

Association

A general term used to describe the relationship between two (or more) variables. The term **association** is often used interchangeably with the term **correlation**. The latter tends to be used when referring to the strength of a linear relationship between two numerical variables.

Average percentage method

In the **average percentage method** for calculating a **seasonal index**, the data for each 'season' are expressed as percentages of the average for the year. The percentages for the corresponding 'seasons' for different years are then averaged using a mean or median to arrive at a seasonal index.

Categorical variable

A **categorical variable** is a variable whose values are categories.

Examples include blood group (A, B, AB or O) or house construction type (brick, concrete, timber, steel, other).

Categories may have numerical labels, eg. the numbers worn by player in a sporting team, but these labels have no numerical significance, they merely serve as labels.

Categorical data

Data associated with a **categorical variable** is called categorical data.

Causation

A relationship between an explanatory and a response variable is said to be causal if the change in the explanatory variable actually causes a change in the response variable. Simply knowing that two variables are associated, no matter how strongly, is not sufficient evidence by itself to conclude that the two variables are causally related.

Possible explanations for an observed association between an explanatory and a response variable include:

- the **explanatory variable** is actually causing a change in the response variable
- there may be causation, but the change may also be caused by one or more uncontrolled variables whose effects cannot be disentangled from the effect of the response variable. This is known as **confounding**.
- there is no causation, the association is explained by at least one other variable that is associated with both the explanatory and the response variable . This is known as a **common response**.
- the **response variable** is actually causing a change in the explanatory variable

Coefficient of determination

In a linear model between two variables, the coefficient of determination (R^2) is the proportion of the total variation that can be explained by the linear relationship existing between the two variables, usually expressed as a percentage. For two variables only, the coefficient of determination is numerically equal to the square of the correlation coefficient (r^2).

Example

A study finds that the correlation between the heart weight and body weight of a sample of mice is $r = 0.765$. The coefficient of determination = $r^2 = 0.765^2 = 0.5852$ ... or approximately 59%

From this information, it can be concluded that approximately 59% of the variation in heart weights of these mice can be explained by the variation in their body weights.

Note: The coefficient of determination has a more general and more important meaning in considering relationships between more than two variables, but this is not a school level topic.

Common response
See Causation

Confounding

See Causation

Continuous data

Data associated with a **continuous variable** is called continuous data.

Continuous variable

A **continuous variable** is a **numerical variable** that can take any value that lies within an interval. In practice, the values taken are subject to accuracy of the measurement instrument used to obtain these values.

Examples include height, reaction time and systolic blood pressure.

Correlation

Correlation is a measure of the strength of the linear relationship between two variables. See also **association**.

Correlation coefficient (*r*)

The correlation coefficient (r) is a measure of the strength of the liner relationship between a pair of variables. The formula for calculating r is given below.

For variables x and y, and computed for n cases, the formula for r is:

$$r = \frac{1}{n-1}\sum\left(\frac{x_i-\bar{x}}{s_x}\right)\left(\frac{y_i-\bar{y}}{s_y}\right)$$

Discrete data

Discrete data is data associated with a discrete variable. Discrete data is sometimes called count data.

Discrete variable

A **discrete variable** is a **numerical variable** that can take only integer values.

Examples include the number of people in a car, the number of decayed teeth in 18 year-old males, etc.

Explanatory variable

When investigating relationships in bivariate data, the **explanatory variable** is the variable used to explain or predict a difference in the **response variable**.

For example, when investigating the relationship between the temperature of a loaf of bread and the time it has spent in a hot oven, *temperature* is the response variable and *time* is the explanatory variable.

Extrapolation

In the context of fitting a linear relationship between two variables, extrapolation occurs when the fitted model is used to make predictions using values of the **explanatory variable** that are outside the range of the original data. Extrapolation is a dangerous process as it can sometimes lead to quite erroneous predictions.

See also interpolation.

Five-number summary

A **five-number summary** is a method of summarising a set of data using the minimum value, the lower or first-quartile (Q_1), the median, the upper or third-quartile (Q_3) and the maximum value. Forms the basis for a boxplot.

Interpolation

In the context of fitting a linear relationship between two variables, interpolation occurs when the fitted model is used to make predictions using values of the **explanatory variable** that lie within the range of the original data.

See also extrapolation.

Irregular variation or noise (time series)

Irregular variation or noise is erratic and short-term variation in a time series that is the product of chance occurrences.

Least-squares line

In fitting a straight-line $y = a + bx$ to the relationship between a response variable y and an explanatory variable x, the **least-squares line** is the line for which the sum of the squared **residuals** is the smallest.

The formula for calculating the slope (b) and the intercept (a) of the least squares line is given below.

For variables x and y computed for n cases, the slope (b) and intercept (a) of the least-squares line are given by:

$$b = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} \quad \text{and} \quad a = \bar{y} - b\bar{x}$$

Location

The notion of central or 'typical value' in a sample distribution.

See also **mean**, **median** and **mode**.

Mean

The arithmetic mean of a list of numbers is the sum of the data values divided by the number of values in the list.

In everyday language, the arithmetic mean is commonly called the average.

For example, for the following list of five numbers 2, 3, 3, 6, 8 the mean equals

$$\frac{2+3+3+6+8}{5} = \frac{22}{5} = 4.4$$

In more general language, the mean of n observations $x_1, x_2, \ldots, x_n$ is

$$\bar{x} = \frac{\Sigma x_i}{n}$$

Median

The **median** is the value in a set of ordered set of data values that divides the data into two parts of equal size. When there are an odd number of data values, the median is the middle value. When there is an even number of data values, the median is the average of the two central values.

Mode

The **mode** is the most frequently occurring value is a data set.

Moving average

In a time series, a simple moving average is a method used to **smooth** the time series whereby each observation is replaced by a simple average of the observation and its near neighbours. This process reduces the effect of non-typical data and makes the overall trend easier to see.

Note: There are times when it is preferable to use a weighted average rather simple average, but this is not required in the current curriculum.

Outlier

An outlier in a set of data is an observation that appears to be inconsistent with the remainder of that set of data. An outlier is a surprising observation.

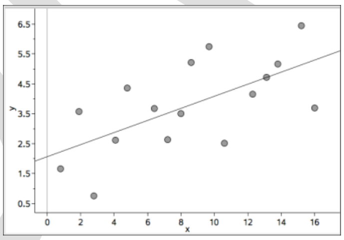
Residual values

The difference between the observed value and the value predicted by a statistical model (e.g., by a least-squares line)

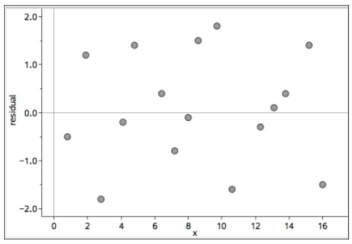
Residual plot

A residual plot is a **scatterplot** with the **residual values** shown on the vertical axis and the **explanatory variable** shown on the horizontal axis. Residual plots are useful in assessing the fit of the statistical model (e.g., by a least-squares line).

When the least-squares line captures the overall relationship between the response variable *y* and the explanatory variable *x*, the residual plot will have no clear pattern (be random) see opposite. This is what is hoped for.



scatterplot with least squares line



residual plot

If the least-squares line fails to capture the overall relationship between a response variable and an explanatory variable, a residual plot will reveal a pattern in the residuals. A residual plot will also reveal any outliers that may call into question the use of a least-squares line to describe the relationship. Interpreting patterns in residual plots is a skilled art and is not required in this curriculum.

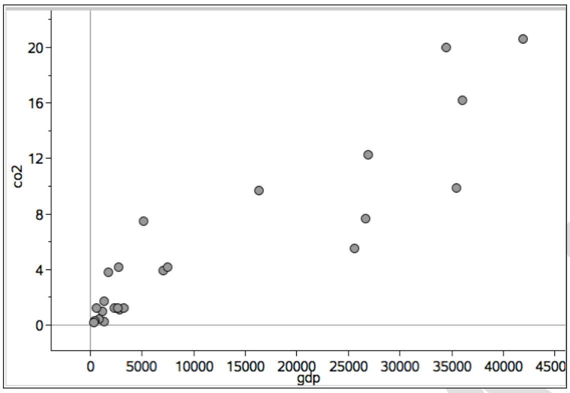
Response variable

See Explanatory variable

Scatterplot

A two-dimensional data plot using Cartesian co-ordinates to display the values of two variables in a bivariate data set.

For example the scatterplot below displays the CO_2 emissions in tonnes per person (*co2*) plotted against Gross Domestic Product per person in $US (*gdp*) for a sample of 24 countries in 2004. In constructing this scatterplot, gdp has been used as the explanatory variable.



Seasonal adjustment (adjusting for seasonality)

A term used to describe a time series from which periodic variations due to seasonal effects have been removed.

See also seasonal index.

Seasonal index

The seasonal index can be used to remove seasonality from data. An index value is attached to each period of the time series within a year. For the seasons of the year (Summer, Autumn, Winter, Spring) there are four separate seasonal indices; for months, there are 12 separate seasonal indices, one for each month, and so on. There are several methods for determining seasonal indices.

Seasonal variation

A regular rise and fall in the time series that recurs each year.

Seasonal variation is measured in terms of a **seasonal index**.

Smoothing (time series)see moving average

Standard deviation

The standard deviation is a measure of the variability or spread of a data set. It gives an indication of the degree to which the individual data values are spread around their **mean**.

The standard deviation of *n* observations $x_1, x_2, \ldots, x_n$ is

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$$

Statistical investigation process

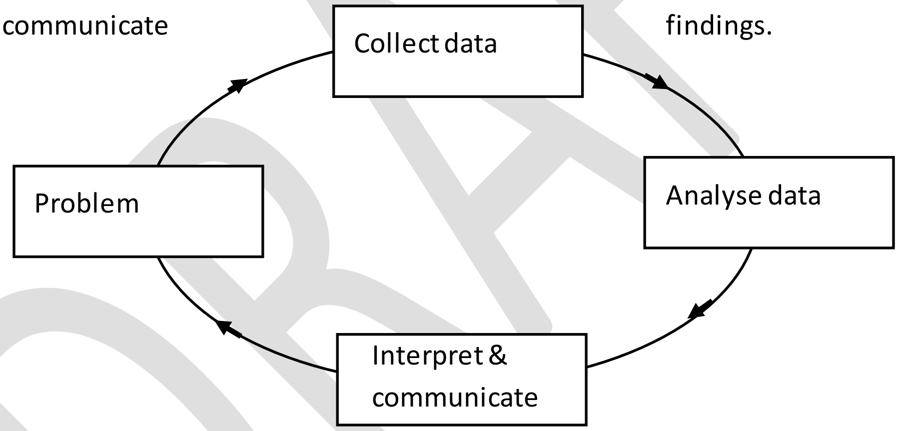
The statistical investigation process is a cyclical process that begins with the need to solve a real world problem and aims to reflect the way statisticians work. One description of the statistical investigation process in terms of four steps is as follows.

Step 1. Clarify the problem and formulate one or more questions that can be answered with data.

Step 2. Design and implement a plan to collect or obtain appropriate data.

Step 3. Select and apply appropriate graphical or numerical techniques to analyse the data.

Step 4. Interpret the results of this analysis and relate the interpretation to the original question; communicate findings.



Time series

Values of a variable recorded, usually at regular intervals, over a period of time. The observed movement and fluctuations of many such series comprise long-term **trend**, **seasonal variation**, and **irregular variation** or **noise**.

Time series plot

The graph of a **time series** with time plotted on the horizontal axis.

Trend (time series)

Trend is the term used to describe the general direction of a time series (increasing/ decreasing) over a long period of time.

Two-way frequency table

A two-way frequency table is commonly used for displaying the two-way **frequency distribution** that arises when a group of individuals or objects are categorised according to two criteria.

For example, the two-way table below displays the frequency distribution that arises when 27 children are categorised according to *hair type* (straight or curly) and *hair colour* (red, brown, blonde, black).

Hair colour	Hair type		Total
	Straight	Curly	
red	1	1	2
brown	8	4	12
blonde	1	3	4
black	7	2	9
Total	17	10	27

The row and column totals represent the total number of observations in each row and column and are sometimes called **row sums** or **column sums**.

If the table is 'percentaged' using row sums the resulting percentages are called **row percentages**. If the table is 'percentaged' using row sums the resulting percentages are called **column percentages**.