9 Balfour Street
Greenwich
NSW 2065
Friday 14th May 2010

Mr Tom Alegounarias
President, New South Wales Board of Studies
Office of the Board of Studies NSW
GPO Box 5300
Sydney NSW 2001

Dear Mr Alegounarias,

Structure and Content in the Draft K–10 Mathematics National Curriculum

I am writing to recommend that the NSW Board of Studies not accept the Draft K–10 Mathematics Syllabus as proposed by ACARA for the National Curriculum. Mathematics in NSW schools presently has excellent syllabuses and an excellent teaching tradition, both of which are the result of many decades of careful discussion and evolution. This excellence would no longer be possible if the *Draft* were adopted in NSW.

I will discuss first the need for a differentiated course structure in Years 9–10 Mathematics, and then explain in some detail why the *Draft's* content is uninspiring for students and teachers, unbalanced and incoherent in its presentation of topics, lacking in the natural interrelationships amongst topics, and without proper foundation for those proceeding to calculus in Years 11–12. I will also make some concluding remarks about the developing situation in Years 11–12. There are serious problems with almost every other aspect of the *Draft*, and I know that many other submissions will go into details about these things.

The differentiated courses in NSW for Years 9–10

By the time students enter secondary school, there is enormous divergence in their ability to assimilate mathematics. This divergence is so great that it is unwise, both for their intellectual development and their self-respect, for all students to be taught the same course. NSW has devised, with its Stages, a most effective solution to this problem.

- The single Stage 4 course targeted for Years 7–8 gives all students, whatever their primary school experience, two years to find their performance level in secondary school. The fact that it is a 'Stage' rather than a fixed course allows teachers to tailor the course to their needs, reviewing whatever Stage 3 material is necessary and pacing the course appropriately.
- Calculus is difficult, and requires considerable specific understanding and training in various topics — algebra, geometry, mensuration, trigonometry, coordinate geometry, graphs, and the arithmetic of surds, indices and logs. The three flavours of the present Stage 5.3 course allow students aiming for calculus in Years 11–12 to be given as much training in these areas as they can cope with, and even the 5.2 course, with judicious additions from 5.3 and perseverance from students, allows progression at least to the 2 Unit calculus course in Years 11–12.
- On the other hand, large numbers of Years 9–10 students struggling with mathematics cannot possibly cope with the material of the 5.3 or 5.2 courses. Giving them their own 5.1 course protects them from unreasonable demands that would only undermine their self-confidence and their whole approach to school. Moreover, the assessment structure, Byzantine as it is, helps to preserve their dignity so that they are not simply cannon-fodder for the higher grades of those attempting the harder courses.

The *Draft* would be a disaster for this carefully structured NSW system in Years 9–10. Considering first those students proceeding to calculus in Years 11–12, the *Draft's* Year 9 course omits six major topics that form the backbone of the Year 9 challenge for our present 5.3 students:

- surds,
- trigonometry,
- coordinate geometry,
- quadratics,
- algebraic fractions,
- fractional indices.

The result is that the *Draft's* Year 9 course makes no significant advance on Year 8 at a crucial stage of schooling. (Most of us are assuming that the mention of trigonometry in an Elaboration of Year 9 'Pythagoras' is one of the document's many typos.)

The *Draft's* Year 10 and Year 10A courses together present most of the present NSW Stage 5.3 course, to be taught in just one year. This is ludicrously out of balance, and could not possibly be used as a programme for any classroom, in NSW or elsewhere.

Each topic in the list above is difficult and needs to be presented over two years — right-angled trigonometry in Year 9 followed by the sine and cosine rules in Year 10, gradient and simple equations of lines in Year 9 followed by review, circles and more complex problems in Year 10, factoring quadratics in Year 9 followed by their graphs and symmetry in Year 10, and so forth. Thus the present 5.3 course in NSW prepares students effectively for Years 11–12 calculus, but the *Draft* does not.

Secondly, the least able 20%–30% could not cope even with the *Draft's* Year 9 course, let alone its Year 10 course. How will teachers of these students proceed? Do they design their own cut-down course, or will the States do it for them? Will the assessment structures force students to sit for tests on topics that they have not covered in class? Will all students be ranked on a common scale? These issues have been thought through extremely carefully in NSW, but have been ignored by ACARA.

A possible solution for NSW: At the meeting of the Professional Teacher's Council on Monday 3rd May, I asked whether it would be possible for NSW to retain the present 5.1, 5.2 and 5.3 course-structure in Years 9–10 while implementing the *Draft*. You gave the only reasonable reply, that while it may be legally possible, it would hardly be consistent with 'implementing the National Curriculum'.

A constructive solution may be possible along the following lines:

- ACARA write the Years 9–10 Mathematics course as a single two-year course.
- ACARA acknowledge the spread of abilities that inevitably occurs in Years 9–10, including the fact that many students do not achieve the performance standards of the Year 8, Year 7 or even Year 6 course by the beginning of Year 9.
- ACARA explicitly leave any further division of this single Years 9–10 course into years, and into differentiated levels, to the States.

This would allow NSW to continue its tradition of differentiated courses in Years 9–10, a tradition that has served it so well for so many years. It would also allow common-sense solutions to the same problem in other States, where the NSW solution may well be inappropriate because of different school structures and different traditions of teaching mathematics.

If no arrangement can be made that allows NSW to keep its differentiated courses, then NSW cannot possibly adopt the National Curriculum in Years 9–10 Mathematics.

Content in the Draft

The content in the *Draft* lacks balance and coherence, omits natural interrelationships amongst the topics, and is uninspiring for teachers and students. Neither does it constitute a satisfactory preparation for calculus for those proceeding to calculus in Years 11–12.

Unfortunately, discussion is hampered not only by the lack of clarity in the *Draft's* prose, but by its confusing policy of adding supposedly non-mandatory 'Elaborations' to each statement of content. What does ACARA mean when they say that the Elaborations are non-mandatory? Most of the statements of content are too vague to define what is to be taught unless they are interpreted by details in the Elaborations — which themselves are often too vague for satisfactory discussion.

1. The over-emphasis on statistics

There is far too much statistics in all Years K–10, and its presence seriously distorts the courses. These matters have been argued interminably, and the *Draft's* statistics content has received little support in NSW. I simply make the following points.

- The statistics in the *Draft* is descriptive and non-rigorous, understandably, because no more is possible in these years. It will do little to prepare students for the proper study of statistics. Its major effect will be to waste valuable time in mathematics classes.
- The main sequence of mathematical foundation for advanced students in Years K–10 is arithmetic, algebra, geometry, coordinate geometry and trigonometry, in preparation for calculus in Years 11–12. The large amount of statistics in the *Draft* will take time away from these central aspects of mathematics, so that students will be less secure in the foundations of calculus.
- Any serious study of statistics requires an extensive understanding of calculus. The time wasted by the *Draft* in Years K–10 on descriptive statistics will detract from the necessary preparation for calculus and make the difficult subject of statistics even more difficult to learn.
- Elementary probability, on the other hand, fits well with the other topics, like fractions, powers and binomial expansions. The problems of complicated mathematics arise when probability is combined with statistics.
- The writers of the *Draft's* statistics strand seem to have little idea of the maturity required for some ideas. Year 5 is too early to explain the median, and Year 6 is too early for stem-and-leaf plots.
- It is extremely difficult to make descriptive statistics anything but endlessly boring in Years 7–10. Projects, tabulation, graphs, and dodgy arguments are not the way to engender interest in statistics. If the aim is to 'train statisticians for a future Australia', don't go down the path of the *Draft*, because that will only reduce the number of young people interested in statistics.
- Despite all the dull, repetitive statistics content, standard deviation has inexplicably been omitted from the *Draft*. Standard deviation is one statistical object that is very well suited to Year 10, because it involves degree 2 algebra and so fits well with quadratics. Standard deviation is ubiquitous in any statistical discussion, but sadly remains a mystery to most adults, despite the fact that it can quite easily be explained in Year 10.

A possible solution for NSW: As with the differentiated courses, it would not be possible for NSW to cut the statistics back to something reasonable and claim that it is 'implementing the National Curriculum'. The only solution is therefore for ACARA to apply some common sense and do this job itself. The present time-on-task in Years K–10 Mathematics in NSW is already too restricted in most schools for effective study of the arithmetic-algebra-geometry-trigonometry foundation necessary for calculus in Years 11–12.

I make the following general suggestions for statistics in a revised National Curriculum.

- Statistics should not be written as a 'third strand', because it should not make up any more than a minor part of the curriculum. It should certainly not start in Kindergarten.
- The excellent suggestion has often been made to teach descriptive statistics in subjects where statistics is clearly useful and makes sense to the students. For example, discussing the numbers and types of shops in a shopping strip, and the traffic flowing past them at different times of day, could form part of an urban study course in Geography, in which various tools of descriptive statistics — tables, charts, graphs, statistical measures of the data, box plots of the relationships — would make immediate sense to the students because of the context. Similarly in Science, an activity using data on the spacing, height and diversity of native bushes would clearly demonstrate the usefulness of these tools.

  Students learn things like descriptive statistics far more quickly when they are useful within some other task that they are committed to. Descriptive statistics cannot come alive in a mathematics lesson, where there is no context and no commitment, and where the teacher probably has little expertise, either in statistics or in the subject from which the data is drawn.
- All that should remain in the Years K–10 mathematics course should be the important mathematical ideas of frequency tables, cumulative frequency tables (missing from the *Draft*), relative frequency tables and their associated graphs, the mean, median and mode, the interquartile range, and standard deviation (missing from the *Draft*).

  At this descriptive statistics level, examples of the relationship between two statistical variables should be taught in other subjects where the concepts will make sense in context — the mathematics of 'least squares' is too difficult for Years K–10.

Similar remarks about context apply to the topic of 'location' — latitude and longitude and time zones — which has crept into the *Draft* for reasons that remain unclear. These things are important school topics, but they should not be part of a mathematics course. They naturally form part of a Geography course, where each latitude and longitude is associated with stories of the particular countries, cultures and economies at that place, and where different time zones can be explained in the context of their effects on daily life.

2. Topics are unfinished or omitted

No teacher should introduce unexplained algorithms to children, because mathematics is all about logic, reasoning and understanding — when an algorithm is taught as a 'black box' method, one of the essential structures of mathematics is lost. On the other hand, the explanation of each algorithm is very much in the hands of the teacher, and will depend on the maturity and progress of the class members. It is always a difficult judgement for syllabus writers, who should suggest, but should not specify in too much detail, how each algorithm is to be developed and explained.

The *Draft*, while tending to give too much detail about how each topic is to be taught, often falls into the clear error of failing to specify that students should have any proper algorithm at the end of the learning process (although the muddiness of the *Draft's* language means that its intention is not always clear). Progress in mathematics requires that each successive process of calculation be finally mastered in a clear and easily-performed algorithm that can quickly be inserted into the more sophisticated reasoning processes required in a later topic.

Competing algorithms are not the issue here — for example, there is sharp debate over the merits of competing algorithms for subtraction and for factoring quadratics — the issue is that students should eventually have an algorithm that is generally accepted as quick and efficient.

Other topics have been truncated before they come to their natural conclusion, so that they remain fragments, lacking a clear purpose in themselves and lacking their proper interrelationships with other topics in the course. I have summarised below some details in various areas of Years K–10.

- ADDITION AND SUBTRACTION: Does the convoluted language of Year 3 'Addition and subtraction' and its Elaborations mean that every student should be able to add a column of numbers of arbitrary size, and subtract two numbers of arbitrary size, without a calculator?
- MULTIPLICATION: Is a student eventually intended to have an efficient written algorithm for multiplying two numbers of arbitrary size? We can all argue about whether the 'double distribution' method described in an Elaboration of Year 5 'Multiplication and Addition' may or may not be helpful as a stage in learning, but it is certainly not an efficient algorithm for multiplying large numbers.
- DIVISION: Long division is part of the NSW Years 7–8 Syllabus, but the language of the *Draft* is too ambiguous to know whether or not it also contains long division. Year 7 'Calculation' seem at first to stipulate a review of all four written algorithms, but its Elaborations seem to undermine this. May a student choose a calculator algorithm instead of a written algorithm?
- FRACTIONS: While it is good to see fractions introduced in Primary school, the lack of any specific fraction content in Years 7–8 of the *Draft* suggests that students can perform the impossible feat of mastering all fraction algorithms in primary school. Most children starting Year 7 need further careful explanation even of the meaning of a fraction, and almost all of them need a thorough re-teaching, from the start, of the algorithms used in their arithmetic. Very few come into Year 7, for example, able to subtract mixed numerals efficiently.
- FACTORING: Students proceeding to calculus need to be fluent in factoring by the end of Year 10 — even the beginnings of differential calculus in Years 11–12 require the constant use of factoring. Yet the *Draft* gives no clear overview of factoring and is unclear as to what the extent of factoring should be. Are non-monic quadratics to be factored? Is grouping to be taught, or differences of cubes or of higher powers? I suspect that the intended answers are all 'No'.
- ALGEBRAIC FRACTIONS: The writers seem unaware of the need to master algebraic fractions before learning calculus, and their omission in the *Draft* would cause problems in Years 11–12. Algebraic fractions should begin in Years 7–8, and as each method of factoring is developed, it should be applied to the corresponding calculations with algebraic fractions.
- FRACTIONAL INDICES: Mastering fractional indices needs considerable time and effort, yet their only mention in the *Draft* is that the solution to an example in an Elaboration requires fractional indices — that is, if one corrects the typo in the obvious way (Year 10A 'Surds', Elaboration 5). Calculus can't proceed without fractional indices.
- THE POINT–GRADIENT FORM FOR A LINE: Calculus in Years 11–12 requires students to find the equations of tangents quickly using the point–gradient form $y-y_1 = m(x-x_1)$ rather than clumsily substituting back into $y = mx + b$ to find the value of b. The point–gradient form, however, is missing from the *Draft*, and the coordinate geometry is too undeveloped.
- CONGRUENCE AND SIMILARITY: It is unfortunately typical of the *Draft* that congruence and similarity are introduced, but applied to almost nothing specific. The theorem on the base angles of an isosceles triangle (Year 10A 'Geometry') and its converse, should be proven as soon as congruence is introduced so that the power of congruence is clearly demonstrated. More generally, the central role of congruence in the systematic development of geometry should be clear in any Years 7–10 syllabus, otherwise it will seem pointless.

  Similarity is not applied to anything, not even to the midpoints of the sides of a triangle.

- SPECIAL QUADRILATERALS: The *Draft* mentions special quadrilaterals, but doesn't give any clear statement of tests for them, let alone of the proofs of those tests. Even the simple fact that a rhombus is a parallelogram seems to have been omitted. Apparently we should also not care how they are defined — 'rhombus', 'rectangle' and 'parallelogram', for example, seem to be defined neither in the *Draft* nor in the glossary on ACARA's website. It's hard to do calculus, vectors and complex numbers in the coordinate plane when you can't identity a special quadrilateral, particularly when you don't even know what it is!
- TANGENTS: The geometry of circles using Euclidean methods, and of non-linear simultaneous equations using algebraic methods, are both terminated quite unnaturally without any discussion of tangents, which seem to be missing entirely from the *Draft*. This is ludicrous in a course whose principal purpose for the more able students is preparation for calculus. Students must have experience with tangents to circles, handled by Euclidean methods, and tangents at least to the graphs of quadratic and reciprocal functions, handled by algebraic methods, before embarking on the study of tangents handled by the limiting methods of calculus. Dealing with these two matters separately:
- CIRCLE GEOMETRY, TANGENTS AND SIMILARITY: The *Draft's* Circle geometry stops before it achieves a unified view of Euclidean geometry by combining circles with tangents and similarity. This is always one of the most exciting parts of the NSW Stage 5.3 syllabus. What is needed are: angles in a semicircle, angles in a cyclic quadrilateral, tangents and radii, tangents from an external point, the alternate segment theorem, intersecting chords, intersecting secants, and the square on a tangent. (The converse circle theorems — conditions for four points to be concyclic — are conceptually too tricky for Year 10, but should be in Years 11–12, where they are important for complex numbers in particular.)
- TANGENTS AND NON-LINEAR SIMULTANEOUS EQUATIONS: The work on non-linear simultaneous equations in Year 10 'Equations' in the *Draft* needs considerable development. The possibilities of multiple solutions and no solutions, and particularly of a tangent, need explicit mention, because the ideas are so new and dramatic. The equations should also involves reciprocal functions, and cubics and higher powers, where the solutions can be obtained by the factoring methods introduced so far.
- POLYNOMIALS: NSW at present breaks the ground with an excellent short course in polynomials, their factorings and their graphs — another source of drama. Cubics and quartics are among the first functions analysed with differential calculus, and students in the 5.3 course should have the opportunity in Year 10, if there is time, to handle them by algebraic methods.
- THE YEARS 7–10A COURSES ARE TOO WEAK TO BE PREPARATION FOR CALCULUS: The *Draft's* Years 7–10A courses are too weak for those proceeding to calculus, either at advanced or standard level. This is the combined result of far too much irrelevant statistics, and of topics not being properly completed, or being confused, or being omitted. The failure to complete topics satisfactorily has also had the effect of partitioning the Years 7–10A courses into unrelated fragments, contrary to the unified view of mathematics necessary for calculus. It has also made the courses incoherent and very dull for students and their teachers.

A possible solution for NSW: Some of these things could possibly be remedied by clarifications, additions and options from the NSW Board of Studies. It would be far preferable, however, for ACARA to design its mathematics courses with proper attention to the unified and interrelated nature of mathematics, to the coherence and liveliness of each topic, and to the role of these courses as a preparation for calculus for many students. It must also sort out its difficulties with the function of its 'Elaborations' and with the clarity of its mathematical prose.

3. Logarithms should not be replaced by sequences

Besides powers of x, there are only two groups of functions in school calculus courses — trigonometric functions and their inverses, and the mutually inverse exponential and logarithmic functions. If a student is to master $y = e^x$ and $y = \log_e x$ in Years 11–12, it is desirable to break the ground in Year 10 with such things as $\log_2 8$, $\log_3 \frac{1}{3}$ and $\log_{25} 5$.

Moreover, logarithmic functions and exponential functions are mutually inverse, so the learning of logarithms is an essential part of learning indices and exponential functions, and in particular, their graphs are closely related and this relationship illustrates significant ideas about graphs in general.

The *Draft* has inexplicably omitted logarithms and instead inserted a dull, basic and directionless course in sequences. Sequences add almost nothing to the Years 10 course, and confuse the new idea of 'function' because they are functions of whole numbers rather than of real numbers, and their notation is contrary to the normal function notation.

This is yet another case of a topic — in this case indices and exponential functions — not being allowed to reach its natural conclusion.

A possible solution for NSW: Over the last two decades, as the time available for mathematics has decreased, many NSW schools have unfortunately failed to teach logarithms in Year 10. Nevertheless, teachers seem agreed on the appropriateness of logarithms in Year 10. The situation will not be sorted out until the hours of mathematics teaching are better defined, but if ACARA insists on including sequences at the expense of logarithms, this will only make the situation worse.

4. Calculators and other technology

It is well known that there is agreement amongst neither teachers nor academics on when calculators should be used. Many experienced teachers, myself included, believe that the use of calculators before they are required for trigonometry in Year 9 prevents the consolidation of mental and written arithmetic, detracts from the intuitive understanding of numbers and fractions, and fails to reflect the nature of mathematics as a discipline based on logic, in which each new topic and algorithm is understood with total clarity. Other teachers believe that early introduction of calculators can give confidence to less able students, and that some experimentation with technology helps intuition.

In this situation, ACARA cannot mandate technology before Year 9. The *Draft's* ambiguous language strongly recommends technology from an early stage, and seems to suggest that technology is mandatory — everything hangs on the interpretation of the words 'and' and 'including':

- '... using efficient written and calculator strategies ... ' (Year 2 'Addition and subtraction')
- '... using efficient mental and written strategies and calculator ... ' (Year 2 Multiplication and division)
- '... including using digital technology ... ' (Year 2 'Transformations')
- '... including using ICT ... ' (Year 4 'Data investigation', Year 4 'Data representation', and Year 4 'Chance')
- '... such as constructing spinners using technology' (Year 4 'Unequal outcomes')
- '... including identifying equivalence transformations using ICT' (Year 6 'Transformations and symmetry')
- '... including using technology' (Year 6 'Decimals')

These examples are taken from the statements of content, not from their Elaborations. They are mostly only the first of successive mentions of technology within a given topic area.

A possible solution for NSW: This could possibly be remedied by clarifications from the NSW Board of Studies, but it would be far better for ACARA to drop its fascination with technology.

The problems of the proposed calculus courses in Years 11–12

ACARA has not yet released its official drafts of the senior courses, but what it has so far revealed of its intentions for the Years 11–12 calculus courses are quite alarming for NSW:

- There will be no 3 Unit calculus course corresponding to the NSW Extension 1 course.
- The 4 Unit calculus course will be 4 units for both years, not just for the HSC year as is the case for the NSW Extension 2 course. It will be harder than the NSW Extension 2 course.
- The 2 Unit calculus course will be much harder, and duller, than the NSW 2 Unit calculus course (and will contain sampling theory way beyond the abilities of our present 2 Unit candidates).

Last year NSW had $5460 + 3170 = 8630$ school-leavers taking 3 units or 4 units of Mathematics — these are the school-leavers regarded as capable of taking serious mathematics at university. If ACARA's course structure is introduced, with no 3 Unit course, the number of school-leavers capable of serious mathematics at university could thus drop to a little over 3000 — a 60% 'dumbing down'. That would ruin the present strength and diversity of the universities in NSW.

The fact that ACARA can seriously put forward such a proposal into the public domain surely indicates how unwise it would be for NSW to follow their leadership. NSW schools produce more than 40% of the school-leavers across Australia capable of serious university mathematics — and do so with great enthusiasm from these students and corresponding pride and morale amongst their teachers. The excellence in NSW school mathematics is the result of decades of careful development of sequential, differentiated and interesting courses in Years K–6, Years 7–10, and Years 11–12. It would be unthinkable to compromise such clear excellence for the sake of some vaguely expressed benefit of national uniformity.

Yours sincerely,

Dr Bill Pender
Australian Mathematical Sciences Institute

Cc: Mr Peter Osland
Chief Inspector of Mathematics
Office of the Board of Studies NSW
GPO Box 5300
Sydney NSW 2001

Mr Rob Randall
General Manager Curriculum
ACARA
Level 10, 255 Pitt Street
Sydney NSW 2000