Ku-Ring-Gai Creative Arts High School

4 unit mathematics

Trial HSC Examination 1989

1. **(i)** Find **(a)** $\int \frac{x+4}{(x-2)(x+1)}\,dx$ **(b)** $\int \frac{\sec^2 x}{9+\tan^2 x}\,dx$
(ii) Evaluate the following: **(a)** $\int_0^{\frac{\pi}{2}} \sin^3 x\,dx$ **(b)** $\int_0^2 x \ln x\,dx$

2. **(i)** **(a)** Find the cube roots of unity and hence or otherwise show that if ω is a complex cube root of unity $1+\omega+\omega^2=0$.
(b) If ω is a complex cube root of unity, show that $\frac{1}{1-x}+\frac{\omega}{\omega-x}+\frac{\omega^2}{\omega^2-x}=\frac{3}{1-x^3}$.
(ii) The complex number z and its conjugate $\overline{z}$ satisfy the equation $(z-\overline{z})^2+8a(z+\overline{z})=16a^2$ where a is a positive real number. If Z represents z in the Argand Diagram
(a) Find the equation of the locus of Z
(b) Describe fully the locus of Z
(c) Sketch the locus of Z
(iii) If $z_1=5+12i$ and $|z_2|=5$. Find the greatest value of $|z_1+z_2|$
(iv) In the complex plane the points Z_1, Z_2, Z_3 represent the complex numbers z_1, z_2, z_3 respectively. If Z_1', Z_2', Z_3' represent the complex numbers $z_2+z_3, z_3+z_1, z_1+z_2$ respectively prove that $\triangle Z_1Z_2Z_3 \equiv \triangle Z_1'Z_2'Z_3'$.

3. **(i)** **(a)** Write down an expanded expression for $(\sum_{i=1}^{n}\alpha_i)^2$.
(b) If all the roots α_i of the equation $x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots+a_n=0$ are real, prove that $a_1^2-2a_2\geq 0$
(ii) If one root of the equation $x^3+lx^2+mx+n=0$ is the sum of the other two roots, prove that $l^3-4lm+8n=0$
(iii) Find the relation between p and q if the equation $x^n-px^{n-1}+q=0$ has a multiple root.

4. **(i)** **(a)** Given that $\sin x \sin y=\frac{1}{2}(\cos A-\cos B)$. Find A and B in terms of x and y.
(b) Hence prove that for any positive interger n,
$\sin x+\sin 3x+\sin 5x+\cdots+\sin(2n-1)x=\frac{\sin^2 nx}{\sin x}$
(ii) An aeroplane is flying at a constant height of h metres and at a constant speed of v metres/second so that its path passes directly over a searchlight.
(a) Derive a formula giving the rate in degrees/second at which the searchlight must be turned so that the aeroplane will be in the beam when its angle of elevation, as observed from the searchlight, is Θ degrees.
(b) Find the rate at which the searchlight must be turned in degrees/second if a plane is flying at an altitude of 2000 metres with a speed of 160π km/hour at the moment the angle of elevation of the plane is $60°$.

5 (i) (a) Find the general solutions of the equation $\cos 2x - \sin 2x = \cos x - \sin x - 1$
(b) And hence write down the least positive value of x that satisfies the equation.
(ii) ABC is a triangle right-angled at A and lying in a horizontal plane. $AB = 8,\ AC = 6$. P is a point vertically above A and $P\hat{B}A = 60°$. Find exact expressions for
(a) the length PC
(b) the angle of inclination of the plane BPC.
(iii) If $y = \sin^{-1} x$ show that $(1 - x^2)\frac{d^2y}{dx^2} - \frac{x\ dy}{dx} = 0$.

6. (i) For the function $y = 4x^2 - x^4$
(a) Sketch the graph of the function over a domain adequate enough to show its essential features.
(b) The region enclosed by the curve in the first quadrant is rotated about the y-axis to form a solid of revolution. Use the method of "cylindrical shells" to determine the volume of the solid. Leave your answer in exact form.
(ii) P, Q are variable points $(ct_1, \frac{c}{t_1}), (ct_2, \frac{c}{t_2})$ on the hyperbola $xy = c^2$.
(a) Find the equations of the tangents at P and Q.
(b) Show that if $t_1 t_2$ is constant, the locus of the point of intersection of the tangents at P and Q is a straight line passing through the origin.

7. (i) (a) Prove the trigonometric identity $\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$
(b) If $f(x)$ has a primitive function $F(x)$ show that $\int_0^a f(x)\ dx = \int_0^a f(a - x)\ dx$
(c) Hence deduce that $\int_0^{\frac{\pi}{4}} \frac{1-\sin 2x}{1+\sin 2x}\ dx = \int_0^{\frac{\pi}{4}} \tan^2 x\ dx$ and hence evaluate the integral, leaving your answer in exact form.
(ii) It is given that x and y are unequal positive numbers, by first expanding $(\sqrt{x} - \sqrt{y})^2$
(a) Prove that $x + y > 2\sqrt{xy}$
(b) Hence if x, y, z are unequal positive numbers prove that
$(x + y)(y + z)(z + x) > 8xyz$
(c) Hence prove that $\frac{x+y}{z} + \frac{y+z}{x} + \frac{x+z}{y} > 6$

8. (i) A particle of unit mass moves in a straight line under the action of a constant propelling force F against a resistance kv, where v is its speed and k is a positive constant. Show that if the speed increases from u to $2u$ over a time interval t then $F = \frac{ku(2e^{kt}-1)}{e^{kt}-1}$.
(ii) A smooth ring of mass m is threaded on a light inextensible string of length 1.5 metres. The ends of the string are fixed to two points A and B in the same vertical line such that AB is 1 metre in length. The string and the ring are made to rotate about AB as axis so that the string is taut and one part of the string is horizontal. Find
(a) the tension of the string
(b) the period of rotation of the system.