DRAFT SENIOR SECONDARY CURRICULUM – MATHEMATICAL METHODS

Organisation

1. Overview of senior secondary Australian Curriculum

ACARA has developed draft senior secondary Australian Curriculum for English, Mathematics, Science and History according to a set of design specifications (see http://www.acara.edu.au/curriculum/development_of_the_australian_curriculum.html). The ACARA Board approved these specifications following consultation with state and territory curriculum, assessment and certification authorities.

Senior secondary Australian Curriculum will specify content and achievement standards for each senior secondary subject. Content refers to the knowledge, understanding and skills to be taught and learned within a given subject. Achievement standards refer to descriptions of the quality of learning (the depth of understanding, extent of knowledge and sophistication of skill) demonstrated by students who have studied the content for the subject.

The senior secondary Australian Curriculum for each subject has been organised into four units. The last two units are cognitively more challenging than the first two units. Each unit is designed to be taught in about half a 'school year' of senior secondary studies (approximately 50–60 hours duration including assessment). However, the senior secondary units have also been designed so that they may be studied singly, in pairs (that is, year-long), or as four units over two years. State and territory curriculum, assessment and certification authorities are responsible for the structure and organisation of their senior secondary courses and will determine how they will integrate the Australian Curriculum content and achievement standards into courses. They will also provide any advice on entry and exit points, in line with their curriculum, assessment and certification requirements.

States and territories, through their respective curriculum, assessment and certification authorities, will continue to be responsible for implementation of the senior secondary curriculum, including assessment, certification and the attendant quality assurance mechanisms. Each of these authorities acts in accordance with its respective legislation and the policy framework of its state government and Board. They will determine the assessment and certification specifications for their courses that use the Australian Curriculum content and achievement standards and any additional information, guidelines and rules to satisfy local requirements.

These draft documents should not, therefore, be read as proposed courses of study. Rather, they are presented as draft content and achievement standards that will provide the basis for senior secondary curriculum in each state and territory in the future. Once approved, the content and achievement standards would subsequently be integrated by states and territories into their courses.

2. Senior Secondary Mathematics subjects

The Senior Secondary Australian Curriculum: Mathematics consists of four subjects in mathematics. The subjects are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students. Each subject is organised into four units.

Essential Mathematics focuses on using mathematics effectively, efficiently and critically, to make informed decisions. It provide students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This subject provides the opportunity for students to prepare for post-school options of employment and further training.

General Mathematics focuses on using the techniques of discrete mathematics to solve problems in contexts that include financial modelling, network analysis, route and project planning, decision making, and discrete growth and decay. It provides an opportunity to analyse and solve a wide range of geometrical problems in areas such as measurement, scaling, triangulation and navigation. It also provides opportunities to develop systematic strategies based on the statistical investigation process for answering statistical questions that involve comparing groups, investigating associations and analysing time series.

Mathematical Methods focuses on the development of the use of calculus and statistical analysis. The study of calculus in Mathematical Methods provides a basis for an understanding of the physical world involving rates of change, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics in Mathematical Methods develops the ability to describe and analyse phenomena involving uncertainty and variation.

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers, matrices and recursive methods. Specialist Mathematics is the only mathematics subject that cannot be taken as a stand-alone subject.

3. Structure of Mathematical Methods

Mathematical Methods is structured over four units. The topics broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The units provide a blending of algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. The statistics topics are developed across all four units, ending with tools for students to evaluate sophisticated statistical information.

	Unit 1	Unit 2	Unit 3	Unit 4
Mathematical Methods	Algebra functions and graphs 1 Calculus 1 Probability	Algebra functions and graphs 2 Calculus 2 Discrete Random Variables	Calculus 3 Calculus 4 Continuous Random Variables	Interval estimates for proportions and means Calculus 5

Units

Unit 1 begins with a review of basic algebraic concepts and techniques required for a successful introduction to the study of calculus. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These are reinforced numerically, by calculating difference quotients, geometrically, as slopes of chords and tangents, and algebraically. The statistics strand begins in this unit with a review of the fundamentals of probability, and the introduction of the concepts of conditioning and independence. The algebra section of this unit focuses on trigonometric functions, exponentials and logarithms. Their graphs are examined and their applications in a wide range of settings are explored.

In Unit 2, the study of calculus focuses on the derivatives of polynomial functions, with simple applications of the derivative to curve sketching, calculating slopes and equations of tangents, determining instantaneous velocities and solving optimisation problems. In the statistics strand, discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. The purpose here is to develop a framework for statistical inference.

In Unit 3, the study of calculus continues with the derivatives of exponential and trigonometric functions and their applications, and some basic differentiation techniques. It concludes with integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. The statistics strand deals with continuous random variables and their applications. Probabilities associated with continuous distributions will be calculated using definite integrals.

In Unit 4, the calculus strand deals with derivatives of logarithmic functions, and continues with the concept of a second derivative, its meaning and applications. It concludes with applications of standard calculus techniques applied to a wide range of functions. The aim is to demonstrate to students the beauty and power of calculus and the breadth of its applications. The statistics strand in this unit is the culmination of earlier work on probability and random variables. It introduces students to one of the most important parts of statistics, namely statistical inference, where the goal is to estimate an unknown parameter associated with a population using a sample of that population. In this unit, inference is restricted to estimating means of continuous distributions and proportions in two-outcome populations. Students will already be familiar with many examples of these types of populations, and if they master the basic concepts of inference in these settings, they will be well prepared for studying other types of statistical inference.

Organisation of achievement standards

The achievement standards have been organised into two dimensions, 'Concepts and Techniques' and 'Reasoning and Communication'. These two dimensions reflect students' understanding and skills in the study of mathematics.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in these subjects.

4. Links to F-10

In Mathematical Methods, there is a strong emphasis on mutually reinforcing proficiencies in understanding, fluency, problem solving and reasoning. Students will gain fluency in a variety of mathematical and statistical skills, including algebraic manipulations, constructing and interpreting graphs, calculating derivatives and integrals, applying probabilistic models, estimating probabilities and parameters from data, and using appropriate technologies. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem solving. In studying Mathematical Methods, it is desirable that students complete topics from 10A. The knowledge and skills from the content descriptions ACMNA264, ACMNA269, ACMSP278 from 10A are highly recommended as preparation for Mathematical Methods.

5. Representation of General Capabilities

The seven general capabilities of *Literacy*, *Numeracy*, *Information and Communication technology (ICT) capability*, *Critical and creative thinking*, *Personal and social capability, Ethical behaviour,* and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

General capabilities that are specifically covered in *Mathematical Methods* include *Literacy, Numeracy, Information and communication technology (ICT) capability, Critical and creative thinking and Ethical behaviour.*

Literacy is of fundamental importance in students' development of Mathematical Methods as they develop the knowledge, skills and dispositions to interpret and use language confidently for learning. Students will be taught to read, understand and gather information presented in a wide range of genres, modes and representations (including text, symbols, graphs and tables). They are taught to communicate ideas logically and fluently and to structure arguments.

Numeracy involves students recognising and understanding the role of mathematics in the world and to use mathematical knowledge and skills purposefully. Mathematical Methods provides the opportunity to apply mathematical understanding and skills in real world contexts. The twenty-first century world is information driven and through statistical analysis, students can interpret data and make informed judgements about events involving chance

Critical and creative thinking is inherent in Mathematical Methods. Students develop their critical and creative thinking as they learn to generate and evaluate knowledge, clarify concepts and ideas, seek possibilities, consider alternatives and solve problems. Critical and creative thinking is integral to activities that require students to think broadly and deeply using skills, behaviours and dispositions such as reason, logic, resourcefulness, imagination and innovation in all learning areas at school and their lives beyond school.

Ethical behaviour involves students exploring the ethics of their own and other others' actions. Students develop the capability to behave ethically as they identify and investigate the nature of ethical concepts, values, character traits and principles, and understand how reasoning can assist ethical judgement. There are opportunities in Mathematical methods to explore, develop and apply ethical behaviour in a range of contexts.

Information and communication technology (ICT) is a key part of Mathematical Methods. Students develop ICT capability as they learn to use ICT effectively and appropriately to access, create and communicate information and ideas, solve problems, perform calculations, draw graphs, analyse and interpret data. Digital technologies can engage students and promote the understanding of key concepts.

There are also opportunities within Mathematical Methods to develop the general capabilities of *Intercultural understanding* and *Personal and social capability*, with an appropriate choice of activities and contexts provided by the teacher.

6. Representation of Cross-curriculum priorities

The Cross-Curriculum priorities of Aboriginal and Torres Strait Islander histories and cultures, Asia and Australia's engagement with Asia, and Sustainability, are not overtly evident in the content descriptions of the Mathematical Methods subject. However opportunities exist for teachers to reference them in the context of their teaching of relevant topics, especially those topics which use real data to develop mathematical and statistical concepts.

Aboriginal and Torres Strait Islander histories and cultures

Students will deepen their understanding of the lives of Aboriginal and Torres Strait Islander Peoples through the application of mathematical concepts in appropriate contexts. Teachers could develop statistical and mathematical learning opportunities based on information and data pertinent to Aboriginal and Torres Strait Islander histories and cultures.

Asia and Australia's engagement with Asia

In Mathematical Methods, the priority of Asia and Australia's engagement with Asia provides rich and engaging contexts for developing students' mathematical knowledge, skills and understanding.

In Mathematical Methods, students develop mathematical understanding by drawing on knowledge of and examples from the Asia region. Investigations involving data collection, representation and analysis can be used to examine issues pertinent to the Asia region.

Sustainability

In Mathematical Methods, the priority of sustainability provides rich, engaging and authentic contexts for students learning.

Mathematical Methods provides opportunities for students to develop problem solving and reasoning essential for the exploration of sustainability issues and their solutions. Mathematical understandings and skills are necessary to measure, monitor and quantify change in social, economic and ecological systems over time. Statistical analysis enables prediction of probable futures based on findings and helps inform decision making and actions that will lead to preferred futures.

In Mathematical Methods , students can analyse data relating to issues of sustainability from secondary sources. They can apply reasoning, calculation and comparison to gauge local ecosystem health and can cost proposed actions for sustainability.

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Rationale

The subject Mathematical Methods is designed for students whose future pathways may involve mathematics and statistics and their applications in a range of disciplines at the tertiary level. Mathematics is the study of order, relation and pattern. From its origins in counting and measuring, it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and problems in the presence of uncertainty. Together mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

The major themes of the subject are calculus and statistics. They include as necessary prerequisites studies of algebra, functions and their graphs, and probability. They are developed systematically over the four units of the subject, with increasing levels of sophistication and complexity. Calculus is essential for an understanding of the physical world because many of the laws of science are relationships involving rates of change. Statistics is used to describe and analyse phenomena involving uncertainty and variation. For these reasons this subject provides a foundation for further studies in disciplines in which mathematics and statistics have important roles, such as economics, teaching, computer science, and all branches of science and engineering. It is also advantageous for further studies in the health and social sciences.

In calculus, understanding begins with the concept of a rate of change. It includes the notion of a derivative as a function, the connections between derivatives and integrals, and the use of functions and their derivatives and integrals in modelling physical processes. In the statistics section of this subject, the key concepts include using probability to quantify uncertainty, the notion of a random variable and its distribution, and the nature of inference based on a random sample from a population.

In Mathematical Methods, there is a strong emphasis on mutually reinforcing proficiencies in understanding, fluency, problem solving and reasoning. Students will gain fluency in a variety of mathematical and statistical skills, including algebraic manipulations, constructing and interpreting graphs, calculating derivatives and integrals, applying probabilistic models, estimating probabilities and parameters from data, and using appropriate technologies. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem solving. Because both calculus and statistics are widely applicable as models of the world around us, there is ample opportunity for problem solving throughout this subject. There is also a sound logical basis to the subject, and in mastering the subject students will develop logical reasoning skills to a high level.

Aims

Mathematical Methods aims to develop students':

- understanding of concepts and techniques drawn from algebra, functions, calculus, probability and inferential statistics
- ability to solve applied problems using concepts and techniques drawn from algebra, functions, calculus, probability and inferential statistics
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language

Unit 1

Unit description

The unit begins with a review of basic algebraic concepts and techniques required for a successful introduction to the study of calculus. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These are reinforced numerically, by calculating difference quotients, geometrically, as slopes of chords and tangents, and algebraically. The study of statistics begins in this unit with a review of the fundamentals of probability, and the introduction of the concepts of conditioning and independence. Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in algebra, functions, graphs, calculus and probability
- solve problems in algebra, functions, graphs, calculus and probability
- apply reasoning skills in algebra, functions, graphs, calculus and probability
- communicate arguments and strategies when solving problems
- interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems.

Content descriptions

Topic 1: Algebra, functions and graphs 1

Review of linear relationships:

- examples of direct proportion and linearly related variables
- features of the graph of $y = mx + c$ including its linear shape, its intercepts and its slope or gradient
- find the equation of a straight line given sufficient information
- solve linear equations

Review of quadratic relationships:

- examples of quadratically related variables
- features of the graphs of $y = x^2$, $y = a(x-b)^2 + c$, and $y = a(x-b)(x-c)$, including their parabolic shapes, turning points, axes of symmetry and intercepts
- solve quadratic equations using the quadratic formula and by completing the square
- find the equation of a quadratic given sufficient information
- turning points and zeros of quadratics and the role of the discriminant
- features of the graph of the general quadratic $y = ax^2 + bx + c$

Inverse proportion:

- examples of inverse proportion
- features of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x-b}$, including their hyperbolic shapes, and their asymptotes

Powers and polynomials:

- features of the graphs of $y = x^n$ for $n \in \boldsymbol{N}$, $n = -1$ and $n = ½$, including shape, and behaviour as $x \to \infty$ and $x \to -\infty$
- coefficients and the degree of a polynomial
- expand quadratic and cubic polynomials from factors
- features of the graphs of $y = x^3$, $y = a(x-b)^3 + c$ and $y = k(x-a)(x-b)(x-c)$, including shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$
- factorise cubic polynomials in cases where a linear factor is easily obtained
- solve cubic equations using technology and algebraically in cases where a linear factor is easily obtained

Absolute value:

- the notation $|x|$ for the absolute value for the real number x
- $|x-y|$ as the distance between real numbers x and y on the real line
- features of the graphs of $y=|x|$ and $y=a|x-b|$, including the absence of a tangent at the 'corner'.

Graphs of relations:

- features of the graphs of $x^2+y^2=r^2$ and $(x-a)^2+(y-b)^2=r^2$, including their circular shapes, their centres and their radii
- features of the graph of $y^2=x$ including its parabolic shape and its axis of symmetry

Functions:

- functions as mappings between sets, and as rules or formulas that define one variable quantity in terms of another
- function notation, domain and range, independent and dependent variables
- the graph of a function
- translations and the graphs of $y=f(x)+a$ and $y=f(x+b)$
- dilations and the graphs of $y=cf(x)$ and $y=f(dx)$
- the distinction between functions and relations and the vertical line test

Binomial theorem:

- expansions of $(x+y)^n$ for small positive integers n
- Pascal's triangle and its properties to aid binomial expansions
- the notation $\binom{n}{r}$ for the number of combinations of r objects taken from a set of n distinct objects, and the formula $\binom{n}{r}=\frac{n!}{r!(n-r)!}$
- the numbers $\binom{n}{r}$ as binomial coefficients, (as coefficients in the expansion of $(x+y)^n$, and the binomial theorem)

Topic 2: Calculus 1

Rates of change:

- the difference quotient $\frac{f(x+h)-f(x)}{h}$ as the average rate of change of a function f
- the Leibniz notation δx and δy for changes or increments in the variables x and y
- the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $y = f(x)$
- interpretation of the ratios $\frac{f(x+h)-f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y = f(x)$
- variable rates of change of non-linear functions

The concept of the derivative:

- informal introduction to limits, restricted to the behaviour of $\frac{f(x+h)-f(x)}{h}$ as $h \to 0$
- the derivative $f'(x)$ defined as $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
- the Leibniz notation for the derivative: $\frac{dy}{dx} = \lim_{\delta x\to 0}\frac{\delta y}{\delta x}$ and the correspondence $\frac{dy}{dx} = f'(x)$ where $y = f(x)$
- the derivative as the instantaneous rate of change
- the derivative as the slope or gradient of a tangent line of the graph of $y = f(x)$

Computation of derivatives:

- numerical estimates of the value of a derivative, for simple power functions
- establish the formula $\frac{d}{dx}(x^n) = nx^{n-1}$ for positive integers n by expanding $(x+h)^n$

Topic 3: Probability

Language of events and sets:

- review the concepts and language of outcomes, sample spaces and events as sets of outcomes
- the complement $\overline{A}$ (or A') of an event A as the event 'A does not occur'
- the intersection $A \cap B$ as the event 'both A and B occur' and the union $A \cup B$ as the event 'at least one of A and B occurs'
- mutually exclusive events
- use everyday occurrences to illustrate set descriptions and representations of events, and set operations

Review of the fundamentals of probability:

- probability as a measure of 'the likelihood of occurrence' of an event
- the probability scale: $0 \leq P(A) \leq 1$ for each event A, with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certainty
- the rules: $P(\overline{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- use relative frequencies obtained from data as point estimates of probabilities

Conditioning and independence:

- the notion of a conditional probability and the use of language, such as: 'given that', 'knowing that', 'if', and so on, to indicate conditionality
- the notation $P(A|B)$ for the probability of event A given that event B has occurred
- the formula $P(A \cap B) = P(A|B)P(B)$
- independence of an event A from an event B defined by $P(A|B) = P(A)$
- symmetry of independence and the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B
- use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events

Unit 2

Unit description

The algebra section of this unit focuses on trigonometric functions, exponentials and logarithms. Their graphs are examined and their applications in a wide range of settings are explored. The study of calculus focuses on the derivatives of polynomial functions, with simple applications of the derivative to curve sketching, calculating slopes and equations of tangents, determining instantaneous velocities and solving optimization problems. In statistics, discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. The purpose here is to develop a framework for statistical inference. Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in algebra, functions, graphs, calculus and statistics
- solve problems in algebra, functions, graphs, calculus and statistics
- apply reasoning skills in algebra, functions, graphs, calculus and statistics
- communicate arguments and strategies when solving problems
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.

Content descriptions

Topic 1: Algebra, functions & graphs 2

Circular measure:

- radian measure and its relationship with degree measure
- extended angle measure
- lengths of arcs in circles

Trigonometric functions:

- review of sine, cosine and tangent as ratios of side lengths in right-angled triangles
- sine and cosine rules
- the unit circle definition of $\cos\theta$, $\sin\theta$ and $\tan\theta$ and periodicity
- exact values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ at integer multiples of $\pi/6$ and $\pi/4$
- the graphs of $y=\sin x$, $y=\cos x$, and $y=\tan x$ on extended domains
- amplitude changes and the graphs of $y=a\sin x$ and $y=a\cos x$
- period changes and the graphs of $y=\sin bx$, $y=\cos bx$ and $y=\tan\ bx$
- phase changes and the graphs of $y=\sin(x+c)$, $y=\cos(x+c)$ and
- $y=\tan\,(x+c)$ and the relationships $\sin\left(x+\frac{\pi}{2}\right)=\cos x$ and $\cos\left(x-\frac{\pi}{2}\right)=\sin x$
- identify contexts suitable for modelling by trigonometric functions
- solve equations involving trigonometric functions using technology and algebraically in simple cases
- use trigonometric functions to solve practical problems

Indices and the index laws:

- review of indices (including fractional indices) and the index laws
- radicals and conversions to and from fractional indices
- scientific notation and significant figures

Exponential functions:

- algebraic properties of exponential functions
- qualitative features of the graph of $y = a^x$ $(a > 0)$ including asymptotes, and of its translations ($y = a^x + b$ and $y = a^{x+c}$)
- identify contexts suitable for modelling by exponential functions
- solve equations involving exponential functions using technology and algebraically in simple cases
- use exponential functions to solve practical problems

Logarithmic functions:

- logarithms defined as indices: $a^x = b$ is equivalent to $x = \log_a b$ i.e. $a^{\log_a b} = b$
- algebraic properties of logarithms
- the inverse relationship between logarithms and exponentials: $y = a^x$ is equivalent to $x = \log_a y$
- logarithmic scales such as decibels in acoustics, Richter Scale for earthquake magnitude, octaves in music, pH in chemistry
- solve equations involving indices using logarithms
- qualitative features of the graph of $y = \log_a x$ $(a > 1)$ including asymptotes, and of its translations $y = \log_a x + b$ and $y = \log_a(x + c)$
- algebraic and graphical solution of simple equations involving logarithmic functions
- identify contexts suitable for modelling by logarithmic functions
- algebraic and graphical solution of simple equations involving logarithmic functions
- use logarithmic functions to solve practical problems

Topic 2: Calculus 2

Properties of derivatives:

- the concept of the derivative as a function
- linearity properties of the derivative
- derivatives of polynomials and other linear combinations of power functions

Applications of derivatives:

- find instantaneous rates of change
- find the slope of a tangent and the equation of the tangent
- construct and interpret position such as time graphs, with velocity as the slope of the tangent
- sketch curves associated with simple polynomials $p(x)$, find stationary points, and local and global maxima and minima, and examine behaviour as $x \rightarrow \infty$ and $x \rightarrow -\infty$
- solve optimisation problems involving simple polynomials on finite interval domains

Topic 3: Discrete random variables

Random variables for discrete data:

- the concepts of a discrete random variable and its associated probability function, and their use in modelling count data
- use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable
- uniform discrete random variables and their applications in modelling random phenomena with equally likely outcomes
- simple examples of non-uniform discrete random variables
- the mean or expected value of a discrete random variable as a measure of its location, evaluated in simple cases
- the variance and standard deviation of a discrete random variable as measures of its spread, evaluated in simple cases
- use discrete random variables and associated probabilities to solve practical problems

Bernoulli distributions:

- a Bernoulli random variable as a model for two-outcome situations
- identify contexts and data sets suitable for modelling by Bernoulli random variables
- the mean p and variance $p(1-p)$ of the Bernoulli distribution with parameter p
- use Bernoulli random variables and associated probabilities to model data and solve practical problems

Binomial distributions:

- Bernoulli trials and the binomial random variable (as a count variable) as the number of 'successes' in n independent Bernoulli trials, with the same probability of success p in each trial
- identify contexts and data sets suitable for modelling by binomial random variables,
- the probabilities $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$ associated with the binomial distribution with parameters n and p, evaluated using technology or manually in simple cases
- note the mean np and variance $np(1-p)$ of a binomial distribution
- use binomial distributions and associated probabilities to solve practical problems

Achievement Standards Unit 1 & 2

	Concepts and Techniques	Reasoning and Communication
A	The student: • understands and applies concepts and techniques in algebra, functions, graphs, calculus, probability and statistics to solve a wide range of problems including non-routine problems • uses digital technologies appropriately to solve a range of non-routine problems, and to graph, display and organise mathematical and statistical information effectively • represents varied functions and relations, accurately and precisely in numerical, graphical and symbolic form and uses differential calculus efficiently and effectively • translates efficiently and effectively between practical problems and their mathematical or statistical model in a variety of situations including unfamiliar contexts	The student: • sythesises mathematical techniques, results and ideas creatively to solve problems • determines the solutions to problems, that require the application of multi-step mathematical reasoning and analyses and interprets the reasonableness of the results and solutions • evaluates and interprets results with comprehensive consideration of the validity and limitations of the use of any mathematical or statistical models • communicates observations, judgments and decisions which are succinct, clear, reasoned, and evidenced, as needed • evaluates and communicates the inter-relatedness of different representations of mathematical and statistical information
B	The student: • understands and applies most concepts and techniques in algebra, functions, graphs, calculus, probability and statistics to solve a wide range of problems • uses digital technologies appropriately to solve a range of problems, and to graph, display and organise mathematical and statistical information effectively • represents functions and relations, accurately in numerical, graphical and symbolic form and uses differential calculus competently • translates appropriately between practical problems and their mathematical or statistical model in a variety of situations	The student: • solves problems that require the interpretation of mathematical and statistical information • determines the solutions to problems and analyses and interprets the reasonableness of the results and solutions • analyses results with comprehensive consideration of the validity and limitations of the use of any mathematical or statistical models • communicates observations, judgments and decisions which are clear and reasoned • communicates the inter-relatedness of different representations of mathematical and statistical information
C	The student: • understands and applies some concepts and techniques in algebra, functions, graphs, calculus, probability and statistics to solve familiar problems • uses digital technologies to solve problems, and to graph, display and organise mathematical and statistical information effectively • represents functions and relations in numerical, graphical and symbolic form • solves practical problems using a mathematical or statistical model	The student: • solves familiar problems that require the interpretation of mathematical and statistical information • analyses and interprets the reasonableness of the results and solutions to familiar problems • analyses results with consideration of the validity and limitations of the use of any mathematical or statistical models • communicates observations and decisions which are clear • recognises the inter-relatedness of different representations of mathematical and statistical information

	Concepts and Techniques	Reasoning and Communication
D	The student: • demonstrates limited understanding of concepts and techniques in algebra, functions, graphs, calculus, probability and statistics to solve standard problems • uses digital technologies to graph, display and organise mathematical and statistical information • represents functions and relations in limited forms • solves some practical problems using limited mathematical or statistical information	The student: • solves routine problems that require the interpretation of familiar mathematical and statistical information • interprets the reasonableness of the results and solutions to routine problems • recognises results of the use of any mathematical or statistical models • communicates some observations clearly • recognises the inter-relatedness of some different representations of mathematical and statistical information
E	The student: • demonstrates limited familiarity of concepts in algebra, functions, graphs, calculus, probability and statistics • uses digital technologies for arithmetic calculations and the representation of statistical information • represents mathematical and statistical information in limited forms • solves some routine problems set in context	The student: • recognises the solution to routine mathematical and statistical problems • makes reasonable observations based on mathematical and statistical information • recognises the representations of mathematical and statistical information

Unit 3

Unit description

The study of calculus continues in this unit with the derivatives of exponential and trigonometric functions and their applications, and some basic differentiation techniques. It concludes with integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. In statistics, continuous random variables and their applications are introduced. Probabilities associated with continuous distributions will be calculated using definite integrals. Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus and statistics
- solve problems in calculus and statistics
- apply reasoning skills in calculus and statistics
- communicate arguments and strategies when solving problems
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.

Content descriptions

Topic 1: Calculus 3

Exponential functions:

- estimate the limit of $\frac{a^h - 1}{h}$ as $h \to 0$ using technology, for various values of $a > 0$
- the number e defined as the unique number a for which the above limit is 1
- establish the formula $\frac{d}{dx}(e^x) = e^x$
- use exponential functions and their derivatives to solve practical problems

Trigonometric functions:

- establish the formulas $\frac{d}{dx}(\sin x) = \cos x$, and $\frac{d}{dx}(\cos x) = -\sin x$ by numerical estimations of the limits and informal proofs based on geometric constructions
- establish the formula $\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x}$ using the quotient rule
- use trigonometric functions and their derivatives to solve practical problems

Differentiation rules:

- the product and quotient rules
- the notion of composition of functions and the use of the chain rule for determining the derivatives of composite functions
- application of the product, quotient and chain rule to differentiate functions such as xe^x, $\tan x$, $\frac{1}{x^n}$, $x \sin x$, $e^{-x} \sin x$ and $f(ax + b)$

Topic 2: Calculus 4

Anti-differentiation:

- anti-differentiation as the reverse of differentiation
- the notation $\int f(x)dx$ for anti-derivatives, primitives or indefinite integrals
- establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$ for $n \neq -1$
- establish and use the formula $\int e^x dx = e^x + c$
- establish and use the formulas $\int \sin x \, dx = -\cos x + c$ and $\int \cos x \, dx = \sin x + c$
- linearity of anti-differentiation
- indefinite integrals of the form $\int f(ax + b)dx$

- identify families of curves with the same derivative function
- determine $f(x)$, given $f^{-}(x)$ and an initial condition $f(a) = b$
- determine displacement given velocity in linear motion problems

Definite integrals:

- the area problem, and the use of sums of the form $\sum_i f(x_i)\, \delta x_i$ to estimate the area under the curve $y = f(x)$
- the definite integral $\int_a^b f(x)dx$ as area under the curve $y = f(x)$ if $f(x) > 0$
- the definite integral $\int_a^b f(x)dx$ as a limit of sums of the form $\sum_i f(x_i)\, \delta x_i$
- $\int_a^b f(x)\,dx$ as a sum of signed areas
- additivity and linearity of definite integrals

Fundamental theorem:

- the concept of the signed area function $F(x) = \int_a^x f(t)\,dt$
- the theorem: $F^{-}(x) = \frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$, and a geometric illustration of its proof
- the formula $\int_a^b f^{-}(x)dx = f(b) - f(a)$ and its use for calculating definite integrals

Applications of integration:

- areas between curves in simple cases
- total change as the integral of instantaneous or marginal rate of change

Topic 3: Continuous Random Variables

Random variables for continuous data:

- use relative frequencies and histograms obtained from given or collected data to estimate probabilities associated with intervals of a continuous random variable
- the concept of a probability density function and probabilities in models of a continuous random variable as integrals
- simple examples of continuous random variables, such as: uniform and triangular, with applications in appropriate contexts
- the mean or expected value of a continuous random variable as a measure of its location, evaluated in simple cases
- the variance and standard deviation of a continuous random variable as measures of its spread, evaluated in simple cases
- the effects of linear changes of scale and origin on the mean and the standard deviation

Exponential distributions:

- the probability density function, $f(t) = \lambda e^{-\lambda t}$ for $t \geq 0$, of the exponential random variable with parameter $\lambda > 0$, and its graph
- identify contexts and data sets suitable for modelling by exponential random variables
- the probability $P(X > t) = e^{-\lambda t}$ of the exponential distribution, and its interpretation in relation to a waiting time
- the median $\frac{\ln 2}{\lambda}$ of the exponential distribution, and $\frac{\ln 2}{M}$ as an estimate of the parameter λ, where M is the median of a data set
- use exponential random variables and associated probabilities and quantiles to model data and solve practical problems

Normal distributions:

- identify data sets and contexts such as naturally occurring variation that are suitable for modelling by normal random variables
- the probability density function of the normal distribution with mean μ and standard deviation σ, and its graph
- the standard normal distribution
- quantiles for the standard normal distribution
- calculate probabilities and quantiles associated with a given normal distribution using technology
- use normal distributions and associated probabilities and quantiles to solve practical problems

Unit 4

Unit description

The calculus strand in this unit deals with derivatives of logarithmic functions, and continues with the concept of a second derivative, its meaning and applications. It concludes with applications of standard calculus techniques applied to a wide range of functions. The aim is to demonstrate to students the beauty and power of calculus and the breadth of its applications. The study of statistics in this unit is the culmination of earlier work on probability and random variables. It introduces students to one of the most important parts of statistics, namely statistical inference, where the goal is to estimate an unknown parameter associated with a population using a sample of that population. In this unit inference is restricted to estimating means of continuous distributions and proportions in two-outcome populations. Students will already be familiar with many examples of these types of populations, and if they master the basic concepts of inference in these settings, they will be well prepared for studying other types of statistical inference. Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus and statistics
- solve problems in calculus and statistics
- apply reasoning skills in calculus and statistics
- communicate arguments and strategies when solving problems
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.

Content descriptions

Topic 1: Interval estimates for proportions and means

Random sampling:

- the concept of a random sample of values of a random variable
- discuss sources of bias in samples and procedures to ensure randomness
- use graphs of simulated data to investigate the variability of random samples from various types of distributions, including uniform, exponential, normal and Bernoulli

Sample proportions:

- the sample proportion $\hat{p}$ as the average number of 'successes' in a sample, of a fixed size *n*, from a 'two-outcome' situation, in which the outcomes are typically labelled 'success' and 'failure', and the probability of success is the parameter p
- the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples
- the mean p and standard deviation $\sqrt{(p(1-p)/n}$ of the distribution of $\hat{p}$
- the approximate normality of the distribution of $\hat{p}$ for large samples
- simulate repeated random sampling, for a variety of values of *p* and a range of sample sizes, to illustrate the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p}-p}{\sqrt{(\hat{p}(1-\hat{p})/n}}$ where the closeness of the approximation depends on both *n* and *p*

Confidence intervals for proportions:

- the concept of an interval estimate for a parameter associated with a random variable
- the approximate confidence interval $\left(\hat{p}-z\sqrt{(\hat{p}(1-\hat{p})/n},\ \hat{p}+z\sqrt{(\hat{p}(1-\hat{p})/n}\right)$, as an interval estimate for p, where z is the appropriate quantile for the standard normal distribution
- the approximate margin of error $E=z\sqrt{(\hat{p}(1-\hat{p})/n}$ and the trade-off between margin of error and level of confidence
- the approximate conservative 95% confidence interval $\left(\hat{p}-1/\sqrt{n},\ \hat{p}+1/\sqrt{n}\right)$ for p
- use the approximate conservative 95% confidence interval to suggest the sample size $n\approx 1/E^2$ to estimate p with a desired restriction on the margin of error.

- use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain p
- collect data and construct an approximate confidence interval to estimate a proportion, and report on survey procedures and data quality
- critically analyse survey procedures, data quality and the information provided in surveys and opinion polls giving estimates of proportions

Sample means:

- the concept of the sample mean $\bar{X}$ as a random variable whose value varies between samples
- simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate properties of the distribution of $\bar{X}$ across samples of a fixed size n, including its mean μ, its standard deviation $\sigma/\sqrt{n}$ (where μ and σ are the mean and standard deviation of X), and its approximate normality if n is large
- simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate the approximate standard normality of $\frac{\bar{X}-\mu}{s/\sqrt{n}}$ for large samples ($n \geq 30$), where s is the sample standard deviation.

Confidence intervals for means:

- the approximate confidence interval $\left(\bar{X} - zs/\sqrt{n},\ \bar{X} + zs/\sqrt{n}\right)$, as an interval estimate for μ where z is the appropriate quantile for the standard normal distribution,
- the approximate margin of error $E = zs/\sqrt{n}$ and the trade-off between precision and level of confidence
- use an approximate confidence interval to suggest the sample size $n = (\frac{z\sigma}{E})^2$ to estimate μ with a desired restriction on the margin of error E, where a value is assumed for the population standard deviation σ
- use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain μ
- use $\bar{x}$ and s to estimate μ and σ, to obtain approximate intervals covering desired proportions of values of a normal random variable and compare with an approximate confidence interval for μ
- collect data and construct an approximate confidence interval to estimate a mean, and report on survey procedures and data quality

Topic 2: Calculus 5

Logarithmic functions:

- the natural logarithm $\ln x = \log_e x$
- the inverse relationship of the functions $y = e^x$ and $y = \ln x$
- establish and use the formula $\frac{d}{dx}(\ln x) = \frac{1}{x}$ via implicit differentiation
- establish and use the formula $\int \frac{1}{x} dx = \ln|x| + c$
- use logarithmic functions and their derivatives to solve practical problems

Applications of derivatives:

- implicit differentiation
- optimisation over finite intervals
- the increments formula: $\delta y \cong \frac{dy}{dx} \times \delta x$
- related rates as instances of the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

The second derivative:

- the second derivative as the rate of change of the first derivative function
- acceleration as the second derivative of position
- concavity and points of inflection, and applications in curve sketching
- the second derivative test for local maxima and minima
- solve optimisation problems using first and second derivatives

Achievement Standards Unit 3 & 4

	Concepts and Techniques	Reasoning and Communication
A	The student: • understands and applies concepts and techniques in calculus and statistics to solve a wide range of problems including non-routine problems • uses digital technologies appropriately to solve a range of non-routine problems, and to graph, display and organise mathematical and statistical information effectively • represents varied functions and relations, accurately, precisely and effectively in numerical, graphical and symbolic form and uses differential and integral calculus effectively • translates efficiently and effectively between contextual problems and their mathematical or statistical model in a variety of unfamiliar situations	The student: • synthesises mathematical and statistical techniques, results and ideas creatively to interpret the solutions to problems • determines the solutions to a wide range of problems, that require the application of multi-step mathematical and statistical reasoning and analysis and interpretation of the reasonableness of the results and solutions • evaluates and interprets mathematical and statistical results with comprehensive consideration of the validity and limitations of the use of any models • communicates observations, judgments and decisions which are succinct, clear, reasoned, and evidenced using appropriate mathematical and statistical language • evaluates and communicates the inter-relatedness of different representations of mathematical and statistical information
B	The student: • understands and applies most concepts and techniques in calculus and statistics to solve a wide range of problems • uses digital technologies appropriately to solve a range of problems, and to graph, display and organise mathematical and statistical information • represents functions and relations, accurately in numerical, graphical and symbolic form and uses differential and integral calculus competently • solves a variety of contextual problems using a mathematical or statistical model	The student: • analyses mathematical and statistical techniques, results and ideas to interpret the solutions to problems • determines the solutions to problems, that require the application of multi-step mathematical and statistical analysis and interpretation of the reasonableness of the results and solutions • evaluates mathematical and statistical results with consideration of the validity and limitations of the use of any models • communicates observations, judgments and decisions which are clear and reasoned using appropriate mathematical and statistical language • describes the inter-relatedness of different representations of mathematical and statistical information
C	The student: • understands and applies some concepts and techniques in calculus and statistics • uses digital technologies to graph, display and organise mathematical and statistical information • represents functions and relations in numerical, graphical and symbolic form • solves contextual problems using a mathematical or statistical model	The student: • analyses mathematical and statistical techniques and results to interpret the solutions to familiar problems • determines the solutions to problems, that require the application of mathematical and statistical analysis and interpretation of the results and solutions • evaluates mathematical and statistical results with consideration of the limitations of the use of any models • uses appropriate mathematical and statistical language to communicate observations and judgments • recognises the inter-relatedness of different representations of mathematical and statistical information

	Concepts and Techniques	Reasoning and Communication
D	The student: • demonstrates limited understanding of concepts and techniques in calculus and statistics • uses digital technologies to display some mathematical and statistical information • represents functions and relations in limited form • solves routine problems using mathematical or statistical model	The student: • describes mathematical and statistical techniques to interpret the solutions to familiar problems • interprets the solutions to problems, • recognises the mathematical and statistical results and solutions to routine problems • uses some mathematical and statistical language to communicate observations and judgments • recognises the inter-relatedness of mathematical and statistical information
E	The student: • demonstrates limited familiarity of concepts in calculus and statistics • uses digital technologies for arithmetic calculations and to display limited mathematical and statistical information • represents mathematical and statistical information in limited form • solves simple problems using mathematical or statistical information	The student: • interprets the solutions to familiar problems • recognises representation of mathematical and statistical information • uses simple mathematical and statistical language to make observations • recognises mathematical and statistical information

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GLOSSARY ITEMS FOR MATHEMATICAL METHODS

Unit 1

Algebra functions and graphs 1

Asymptote

A line is an **asymptote** to a curve if the distance between the line and the curve approaches zero as they 'tend to infinity'. For example, the line with equation $x = \pi/2$ is a vertical asymptote to the graph of $y = \tan x$, and the line with equation $y = 0$ is a horizontal asymptote to the graph of $y = 1/x$.

Binomial distribution

The expansion $(x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{r}x^{n-r}y^r + \cdots + y^n$ is known as the **binomial theorem**. The numbers $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n\times(n-1)\times\cdots\times(n-r+1)}{r\times(r-1)\times\cdots\times 2\times 1}$ are called binomial coefficients.

Completing the square

The quadratic expression $ax^2 + bx + c$ can be rewritten as $a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$. Rewriting it in this way is called **completing the square**.

Discriminant

The **discriminant** of the quadratic expression $ax^2 + bx + c$ is the quantity $b^2 - 4ac$

Function

A **function** f is a rule that associates with each element x in a set S a unique element $f(x)$ in a set T. We write $x \mapsto f(x)$ to indicate the mapping of x to $f(x)$. The set S is called the **domain** of f and the set T is called the **codomain**. The subset of T consisting of all the elements $f(x): x \in S$ is called the **range** of f. If we write $y = f(x)$ we say that x is the **independent variable** and y is the **dependent variable.**

Graph of a function

The **graph of a function** f is the set of all points (x, y) in Cartesian plane where x is in the domain of f and $y = f(x)$

Pascal's triangle

Pascal's triangle is a triangular arrangement of binomial coefficients. The n^{th} row consists of the **binomial coefficients** $\binom{n}{r}$, for $0 \le r \le n$, each interior entry is the sum of the two entries above it, and sum of the entries in the n^{th} row is 2^n

Quadratic formula

If $ax^2 + bx + c = 0$ with $a \neq 0$, then $x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$. This formula for the roots is called the **quadratic formula**.

Stationary point

A **stationary point** on the graph of $y = f(x)$ is a point $(x, f(x))$ where $f'(x) = 0$. A **turning point** is a **stationary point** where the derivative $f'(x)$ changes sign. A turning point is either a **local maximum** or a **local minimum**.

Vertical line test

A relation between two real variables x and y is a function and $y = f(x)$ for some function f, if and only if each vertical line, i.e. each line parallel to the $y -$ axis, intersects the graph of the relation in at most one point. This test to determine whether a relation is, in fact, a function is known as the **vertical line test**.

Calculus 1

Gradient (Slope)

The **gradient** of the straight line passing through points (x_1, y_1) and (x_2, y_2) is the ratio $\frac{y_2 - y_1}{x_2 - x_1}$. **Slope** is a synonym for **gradient**.

Secant

A **secant** of the graph of a function is the straight line passing through two points on the graph. The line segment between the two points is called a **chord**.

Tangent line

The **tangent line** (or simply the **tangent**) to a curve at a given point P can be described intuitively as the straight line that "just touches" the curve at that point. At P the curve meet, the curve has "the same direction" as the tangent line. In this sense it is the best straight-line approximation to the curve at the point P.

Probability

Conditional probability

The probability that an event A occurs can change if it becomes known that another event B occurs. The new probability is known as a **conditional probability** and is written as $P(A|B)$. If B has occurred, the sample space is reduced by discarding all outcomes that are not in the event B. The new sample space, called **the reduced sample space**, is B. The conditional probability of event A is given by $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

Independent events

Two events are **independent** if knowing that one occurs tells us nothing about the other. The concept can be defined formally using probabilities in various ways: events A and B are independent if $P(A \cap B) = P(A)P(B)$, if $P(A|B) = P(A)$ or if $P(B) = P(B|A)$. For events A and B with non-zero probabilities, any one of these equations implies any other.

Mutually exclusive

Two events are **mutually exclusive** if there is no outcome in which both events occur.

Point and interval estimates

In statistics estimation is the use of information derived from a sample to produce an estimate of an unknown probability or population parameter. If the estimate is a single number, this number is called a **point estimate.** An **interval estimate** is an interval derived from the sample that, in some sense, is likely to contain the parameter.

A simple example of a point estimate of the probability p of an event is the relative frequency f of the event in a large number of Bernoulli trials. An example of an interval estimate for p is a confidence interval centred on the relative frequency f.

Relative frequency

If an event E occurs r times when a chance experiment is repeated n times, the **relative** frequency of E is r/n.

Unit 2

Algebra, functions & graphs 2

Circular measure

Circular measure is the measurement of angle size in radians.

Radian measure

The **radian measure** θ of an angle in a sector of a circle is defined by $\theta = \ell/r$, where r is the radius and ℓ is the arc length. Thus an angle whose degree measure is 180 has radian measure π.

Extended angle measure

Extended angle measure: An angle greater than 2π $(360°)$ corresponds to an anti-clockwise rotation through more than one full circle, and a negative angle corresponds to a clockwise rotation.

Length of an arc

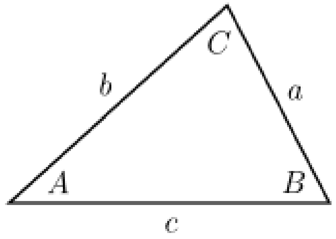
The **length of an arc in a circle** is given by $\ell = r\theta$, where ℓ is the arc length, r is the radius and θ is the angle subtended at the centre, measured in radians. This is simply a rearrangement of the formula defining the radian measure of an angle.

Sine rule and cosine rule

The lengths of the sides of a triangle are related to the sines of its angles by the equations

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

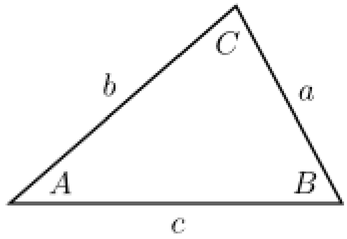
This is known as the **sine rule**.



The lengths of the sides of a triangle are related to the cosine of one of its angles by the equation

$$c^2 = a^2 + b^2 - 2ab\cos C$$

This is known as the **cosine rule**.



Sine and cosine functions

In the unit circle definition of cosine and sine, $\cos\theta$ and $\sin\theta$ are the x and y coordinates of the point on the unit circle corresponding to the angle θ

Period of a function

The period of a function $f(x)$ is the smallest positive number p with the property that $f(x+p) = f(x)$ for all x. The functions $\sin x$ and $\cos x$ both have period 2π and $\tan x$ has period π

Index laws

The index laws are the rules: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, and $(ab)^x = a^x b^x$, for any real numbers x, y, a and b, with $a > 0$ and $b > 0$

Algebraic properties of exponential functions

The algebraic properties of exponential functions are the index laws: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, for any real numbers x, y, and a, with $a > 0$

Algebraic properties of logarithms

The algebraic properties of logarithms are the rules: $\log_a(xy) = \log_a x + \log_a y$, $\log_a \frac{1}{x} = -\log_a x$, and $\log_a 1 = 0$, for any positive real numbers x, y and a

Calculus 2

linearity property of the derivative

The **linearity property of the derivative** is summarized by the equations:

$\frac{d}{dx}(ky) = k\frac{dy}{dx}$ for any constant k

and $\frac{d}{dx}(y_1 + y_2) = \frac{dy_1}{dx} + \frac{dy_2}{dx}$

Local and global maximum and minimum

A **stationary point** on the graph $y = f(x)$ of a differentiable function is a point where $f'(x) = 0$.

We say that $f(x_0)$ is a **local maximum** of the function $f(x)$ if $f(x) \le f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a **global maximum** of the function $f(x)$ if $f(x) \le f(x_0)$ for all values of x in the domain of f.

We say that $f(x_0)$ is a **local minimum** of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a **global minimum** of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x in the domain of f.

Discrete random variables

Random variable

A **random variable** is a numerical quantity whose value depends on the outcome of a chance experiment. Typical examples are the number of people who attend an AFL grand final, the proportion of heads observed in 100 tosses of a coin, and the number of tonnes of wheat produced in Australia in a year.

A **discrete random variable** is one whose possible values are the counting numbers $0,1,2,3,\cdots$, or form a finite set, as in the first two examples.

A **continuous random variable** is one whose set of possible values are all of the real numbers in some interval.

Probability distribution

The **probability distribution** of a discrete random variable is the set of probabilities for each of its possible values.

Uniform discrete random variable

A **uniform discrete random variable** is one whose possible values have equal probability of occurrence. If there are n possible values, the probability of occurrence of any one of them is $1/n$.

Expected value

The **expected value** $E(X)$ of a random variable X is a measure of the central tendency of its distribution.

If X is discrete, $E(X) = \sum_i p_i x_i$, where the x_i are the possible values of X and

$p_i = P(X = x_i)$.

If X is continuous, $E(x) = \int_{-\infty}^{\infty} xp(x)\,dx$, where $p(x)$ is the probability density function of X

Mean of a random variable

The **mean** of a random variable is another name for its expected value.

Variance of a random variable

The **variance** $Var(X)$ of a random variable X is a measure of the 'spread' of its distribution.

If X is discrete, $Var(X) = \sum_i p_i(x_i - \mu)^2$, where $\mu = E(X)$ is the expected value.

If X is continuous, $Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)dx$

Standard deviation of a random variable

The **standard deviation** of a random variable is the square root of its variance.

Effect of linear change

The **effects of linear changes of scale and origin** on the mean and variance of a random variable are summarized as follows:

If X is a random variable and $Y = aX + b$, where a and b are constants, then

$E(Y) = aE(X) + b$ and $Var(Y) = a^2Var(X)$

Bernoulli random variable

A Bernoulli random variable has two possible values, namely 0 and 1. The parameter associated with such a random variable is the probability p of obtaining a 1.

Bernoulli trial

A **Bernoulli trial** is a chance experiment with possible outcomes, typically labeled 'success' and failure'.

Unit 3

Calculus 3

Euler's number

Euler's number e is an irrational number whose decimal expansion begins

$$e = 2.7182818284590452353602874713527 \cdots$$

It is the base of the natural logarithms, and can be defined in various ways including:

$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$ and $e = \lim_{n \to \infty} (1 + \frac{1}{n})^n$.

Product rule

The **product rule** relates the derivative of the product of two functions to the functions and their derivatives.

If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$,

and in Leibniz notation: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$

Quotient rule

The **quotient rule** relates the derivative of the quotient of two functions to the functions and their derivatives

If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

and in Leibniz notation: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Composition of functions

If $y = g(x)$ and $z = f(y)$ for functions f and g, then z is a composite function of x. We write $z = f \circ g(x) = f(g(x))$. For example, $z = \sqrt{x^2 + 3}$ expresses z as a composite of the functions $f(y) = \sqrt{y}$ and $g(x) = x^2 + 3$

Chain rule

The **chain rule** relates the derivative of the composite of two functions to the functions and their derivatives.

If $h(x) = f \circ g(x)$ then $(f \circ g)'(x) = f'\big(g(x)\big)g'(x)$,

and in Leibniz notation: $\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$

Calculus 4

Antidifferntiation

An **anti-derivative**, **primitive** or **indefinite integral** of a function $f(x)$ is a function $F(x)$ whose derivative is $f(x)$, i.e. $F'(x) = f(x)$.

The process of solving for anti-derivatives is called **anti-differentiation**.

Anti-derivatives are not unique. If $F(x)$ is an anti-derivative of $f(x)$, then so too is the function $F(x) + c$ where c is any number. We write $\int f(x)\, dx = F(x) + c$ to denote the set of all anti-derivatives of $f(x)$. The number c is called the **constant of integration**. For example, since $\frac{d}{dx}(x^3) = 3x^2$, we can write $\int 3x^2\, dx = x^3 + c$

The linearity property of anti-differentiation

The linearity property of anti-differentiation is summarized by the equations:

$\int kf(x)dx = k\int f(x)dx$ for any constant k and

$\int \big(f_1(x) + f_2(x)\big)dx = \int f_1(x)dx + \int f_2(x)\, dx$ for any two functions $f_1(x)$ and $f_2(x)$

Similar equations describe the linearity property of definite integrals:

$\int_a^b kf(x)\,dx = k\int_a^b f(x)dx$ for any constant k and

$\int_a^b \big(f_1(x) + f_2(x)\big)dx = \int_a^b f_1(x)\,dx + \int_a^b f_2(x)dx$ for any two functions $f_1(x)$ and $f_2(x)$

Additivity property of definite integrals

The **additivity property of definite integrals** refers to 'addition of intervals of integration':

$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)dx$ for any numbers a, b and c and any function $f(x)$.

The fundamental theorem of calculus

The **fundamental theorem of calculus** relates differentiation and definite integrals. It has two forms: $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$ and $\int_a^b f'(x)dx = f(b) - f(a)$

Continuous random variables

Probability density function

The **probability density function** of a continuous random variable is a function that describes the relative likelihood that the random variable takes a particular value. Formally, if $p(x)$ is the probability density of the continuous random variable X, then the probability that X takes a value in some interval $[a, b]$ is given by $\int_a^b p(x)\, dx$.

Uniform continuous random variable

A **uniform continuous random variable** X is one whose probability density function $p(x)$ has constant value on the range of possible values of X. If the range of possible values is the interval $[a, b]$ then $p(x) = \frac{1}{b-a}$ if $a \leq x \leq b$ and $p(x) = 0$ otherwise.

Triangular continuous random variable

A **triangular continuous random variable** X is one whose probability density function $p(x)$ has a graph with the shape of a triangle.

Quantile

A **quantile** t_α for a continuous random variable X is defined by $P(X > t_\alpha) = \alpha,$ where $0 < \alpha < 1.$

The **median** m of X is the quantile corresponding to $\alpha = 0.5$: $P(X > m) = 0.5$

Unit 4

Interval estimates for proportions and means

Central limit theorem

There are various forms of the **Central limit theorem,** a result of fundamental importance in statistics. For the purposes of this course, it can be expressed as follows:

"If $\bar{X}$ is the mean of n independent values of random variable X which has a finite mean μ and a finite standard deviation σ, then as $n \to \infty$ the distribution of $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ approaches the standard normal distribution."

In the special case where X is a Bernoulli random variable with parameter p, $\bar{X}$ is the sample proportion $\hat{p}$, $\mu = p$ and $\sigma = \sqrt{p(1-p)}$. In this case the Central limit theorem is a statement that as $n \to \infty$ the distribution of $\frac{\hat{p}-p}{\sqrt{p(1-p)/n}}$ approaches the standard normal distribution.

Margin of error

The **margin of error** of a confidence interval of the form $f - E < p < f + E$ is E, the half-width of the confidence interval. It is the maximum difference between f and p if p is actually in the confidence interval.

Level of confidence

The **level of confidence** associated with a confidence interval for an unknown population parameter is the probability that a random confidence interval will contain the parameter.

Calculus 5

Implicit differentiation

When variables x and y satisfy a single equation, this may define y as a function of x even though there is no explicit formula for y in terms of x. **Implicit differentiation** consists of differentiating each term of the equation as it stands and making use of the chain rule. This can lead to a formula for $\frac{dy}{dx}$. For example,

if $x^2 + xy^3 - 2x + 3y = 0$,

then $2x + x(3y^2)\frac{dy}{dx} + y^3 - 2 + 3\frac{dy}{dx} = 0$,

and so $\frac{dy}{dx} = \frac{2-2x-y^3}{3xy^2+3}$.

Concave up and concave down

A graph of $y = f(x)$ is concave up at a point P if points on the graph near P lie above the tangent at P. The graph is concave down at P if points on the graph near P lie below the tangent at P.

Point of inflection

A point P on the graph of $y = f(x)$ is a point of inflection if the concavity changes at P, i.e. points near P on one side of P lie above the tangent at P and points near P on the other side of P lie below the tangent at P

Second derivative test

According to the second derivative test, if $f'(x) = 0$, then $f(x)$ is a local maximum of f if $f''(x) < 0$ and $f(x)$ is a local minimum if $f''(x) > 0$