

Mysterious calculator exercise

Try this on your calculator. Nobody knows why it works.

First we need some definitions.

Suppose d_n =divisor function of n =sum of positive divisors of n

Hence

$$d_1=1$$

$$d_2=1+2=3$$

$$d_3=1+3=4$$

$$d_4=1+2+4=7$$

$$d_5=1+5=6$$

$$d_6=1+2+3+6=12$$

etc.

And suppose h_n = n -th harmonic number= $1/1+1/2+1/3+\dots+1/n$

So

$$h_1=1/1=1$$

$$h_2=1/1+1/2=3/2 \text{ (alternatively } h_2=h_1+1/2=3/2)$$

$$h_3=1/1+1/2+1/3=11/6 \text{ (alternatively } h_3=h_2+1/3=11/6)$$

$$h_4=1/1+1/2+1/3+1/4=25/12 \text{ (alternatively } h_4=h_3+1/4=25/12)$$

$$h_5=1/1+1/2+1/3+1/4+1/5=137/60 \text{ (alternatively } h_5=h_4+1/5=137/60)$$

$$h_6=1/1+1/2+1/3+1/4+1/5+1/6=49/20 \text{ (alternatively } h_6=h_5+1/6=49/20)$$

etc.

And define f_n = n -th Lagarias' number= $h_n+e^{h_n} \ln h_n$.

Then we have the

Proposition For all $n > 1$, $f_n > d_n$.

And this is where the calculator comes in.

Check on the calculator for example $f_2 > d_2$:

$$f_2=3/2+e^{3/2} \ln(3/2)=3.32 > d_2=3.$$

And likewise for others:

$$f_3=11/6+e^{11/6} \ln(11/6)=5.62 > d_3=4$$

$$f_4=25/12+e^{25/12} \ln(25/12)=7.98 > d_4=7$$

$$f_5=137/60+e^{137/60} \ln(137/60)=10.38 > d_5=6$$

$$f_6=49/20+e^{49/20} \ln(49/20)=12.83 > d_6=12$$

etc.

Nobody knows why this works because the proposition is equivalent to the Riemann Hypothesis!

Lagarias proved equivalence in 2001: http://arxiv.org/PS_cache/math/pdf/0008/0008177v2.pdf