Mysterious calculator exercise

Try this on your calculator. Nobody knows why it works.

First we need some definitions.

Suppose \( d_n \) is the divisor function of \( n \), that is, the sum of positive divisors of \( n \)

Hence

\[
\begin{align*}
\text{d}_1 &= 1 \\
\text{d}_2 &= 1+2 = 3 \\
\text{d}_3 &= 1+3 = 4 \\
\text{d}_4 &= 1+2+4 = 7 \\
\text{d}_5 &= 1+5 = 6 \\
\text{d}_6 &= 1+2+3+6 = 12 \\
\end{align*}
\]

etc.

And suppose \( h_n \) is the \( n \)-th harmonic number, \( h_n = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \)

So

\[
\begin{align*}
\text{h}_1 &= 1/1 = 1 \\
\text{h}_2 &= 1/1 + 1/2 = 3/2 \text{ (alternatively \( h_2 = h_1 + 1/2 = 3/2 \))} \\
\text{h}_3 &= 1/1 + 1/2 + 1/3 = 11/6 \text{ (alternatively \( h_3 = h_2 + 1/3 = 11/6 \))} \\
\text{h}_4 &= 1/1 + 1/2 + 1/3 + 1/4 = 25/12 \text{ (alternatively \( h_4 = h_3 + 1/4 = 25/12 \))} \\
\text{h}_5 &= 1/1 + 1/2 + 1/3 + 1/4 + 1/5 = 137/60 \text{ (alternatively \( h_5 = h_4 + 1/5 = 137/60 \))} \\
\text{h}_6 &= 1/1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6 = 49/20 \text{ (alternatively \( h_6 = h_5 + 1/6 = 49/20 \))} \\
\end{align*}
\]

etc.

And define \( f_n \) as Lagarias' number, \( f_n = h_n + e^h \ln h_n \).

Then we have the

Proposition: For all \( n > 1 \), \( f_n > d_n \).

And this is where the calculator comes in.

Check on the calculator for example \( f_2 > d_2 \):

\[
f_2 = 3/2 + e^{3/2} \ln(3/2) = 3.32 > d_2 = 3.
\]

And likewise for others:

\[
\begin{align*}
f_3 &= 11/6 + e^{11/6} \ln(11/6) = 5.62 > d_3 = 4 \\
f_4 &= 25/12 + e^{25/12} \ln(25/12) = 7.98 > d_4 = 7 \\
f_5 &= 137/60 + e^{137/60} \ln(137/60) = 10.38 > d_5 = 6 \\
f_6 &= 49/20 + e^{49/20} \ln(49/20) = 12.63 > d_6 = 12 \\
\end{align*}
\]

etc.

Nobody knows why this works because the proposition is equivalent to the Riemann Hypothesis!