

9 Balfour Street  
Greenwich NSW 2065  
billpender@optusnet.com.au  
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Tom Alegounarias  
Chair, NSW Education Standards Authority  
GPO Box 5300  
Sydney NSW 2001  
brooke.lawson@nesa.nsw.edu.au

Dear Mr Alegounarias,

### Submission on the Board's 2017 Draft Calculus Courses

After reading the rewritten Drafts of the calculus courses (Advanced, Extension 1 and Extension 2), I regret to report that the serious problems I identified in my submission of August 2016 remain. There have been many changes, some considerable, some bringing improvements, and some introducing new problems, but the Drafts remain quite unsatisfactory as a basis for teaching our more able HSC mathematics students and preparing them for university study. As before, they show little respect for the excellent traditions of mathematics teaching in NSW, as expressed in the current excellent 2/3/4 Unit courses, and introducing them would seriously weaken the teaching of mathematics in the State, and compromise our State's standards in mathematics and science.

- The language remains very poor, and often meaningless. One example on page 67 of the Advanced Draft tells the story, '*An asymptote to a curve is a line that the curve begins to imitate at infinity.*' The curve never gets 'to infinity'! If it 'begins' to do something at infinity, what does it do afterwards? The word 'imitate' is not a mathematical word, and makes no sense here — does it refer to gradient, or concavity, or distance, or something else? For example, the curve  $y = \frac{1}{x} \sin x^2$  looks nothing like the line  $y = 0$  that is asymptotic to it.
- There are wrong definitions or missing definitions all through the text. The lack of precision in the text further confuses this, in that it is often unclear whether a statement is intended as a definition, a theorem, or an explanation. These things are fundamental in mathematics, and the subject cannot be taught with such ambiguities, gaps and errors.
- The text seems to have been created initially by cutting-and-pasting of dot-points from other sources. In these resulting Drafts, the coherence of the mathematics has not been thought through, nor has the coherent presentation of the material in the classroom been thought through. The current 2/3/4 Unit syllabuses, however, have been systematically ignored.
- The combination of poor language and mathematical confusion means that the expositions of whole topics are deeply flawed. Introductory calculus, for example, is flawed by confusions and errors with relations, continuity, tangents, turning points, differentiability, inflexions and concavity. Its topics are presented in an unsatisfactory, incoherent teaching order, and its relationship with the geometry of the graphs seems not to have been understood.
- The unity so characteristic of the current 2/3/4 Unit courses as a whole, and of its individual topics, has been badly disrupted. In particular, calculus is disjointed, motion is in scattered fragments, and rates of change has only the odd mention in the Advanced Draft.
- Many results that could be proven are not, making the mathematics arbitrary. Omitting accessible proofs gives students quite the wrong view of mathematics, making it a sequence of disconnected recipes to be learned, rather than a search for the total understanding of a logical universe.

- Euclidean geometry has now been removed entirely. Geometry is the key to the intuitive and to the imaginative understanding of mathematics, and at school to learning proof, besides being essential to topics such as trigonometry, coordinate geometry, and the graphs of functions. Coordinate geometry has similarly been reduced to almost nothing.
- Time is wasted on the relatively trivial topic of descriptive statistics, and on correlation, which involves only pictures and black-box formulae. Both are inappropriate in Advanced.

The Drafts remain completely unsuitable for implementation, and my recommendation remains as before:

*The three calculus Drafts should be withdrawn, then new Drafts structured and written by a properly constituted Syllabus Committee, made up of leading mathematics academics or their direct representatives, together with experienced and knowledgeable teachers. The current 2/3/4 Unit syllabuses should be taken as the basis.*

Considerable effort has been put into these latest Drafts, and some progress has been made. But this very effort has now conclusively proven that having non-experts write mathematics will not work, however diligent and careful they may be.

Mathematics at this level is just too difficult. There are all sorts of tricky technical details that must be finessed. Definitions must be chosen so as to be as close as possible to students' intuitions. Proofs must be refined to their essentials, and hidden logical traps suppressed or bypassed. Subtle and constantly surprising interrelationships must be developed. The teaching order must be logically sound, and effective in the classroom. The relationships within calculus amongst the algebraic manipulations, the graphs of the functions, the underlying geometry, and the properties of the two groups of special functions, are amazingly rich and complex. Convincing and significant applications must be developed systematically to display some of the achievements of the mathematics learnt. Above all, the course must be as true as is possible to the language and structures of the discipline. These are things that only our leading mathematicians can achieve, with advice from expert teachers, and it all takes considerable time and effort — the present Drafts cannot be rescued.

You have in your care some of the most effective school mathematics courses in the world in the current 2/3/4 Unit courses — they satisfy all the criteria of the preceding paragraph wonderfully and imaginatively — and you have a mathematics teaching cohort who respect the courses deeply and teach them very effectively as they prepare their students for university. The courses were developed first by Professor Room, and then refined by generations of very dedicated academics and teachers. Whatever comes next must be an improvement on them, and that certainly cannot be the present text with its incoherencies and muddles.

There is no urgency whatsoever to implement change, because the present courses are universally recognised as excellent.

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Because these second Drafts omit details about projects and assessment, there is no need to repeat my earlier recommendations about such matters. Nevertheless, these are extremely serious issues, each of which is capable of seriously damaging the courses.

The rest of this submission deals only with the Advanced Draft except where otherwise indicated. The three-week time-frame has made a more systematic account impossible, and these remarks are representative only of the poor state of the Drafts. The density of their problems is astonishing.

Systematic criticism of the July Drafts was impossible because of their appalling state, and it is disappointing that time constraints have now made systematic criticism of these latest Drafts impossible. It is also disappointing that we have been asked to review them without any of the 'Support Material' that could help to specify the depth and breadth of treatment.

## General structural issues

What follows is justification and expansion of the remarks made above, but it has been difficult to write coherent criticism of incoherent documents. This section deals with general structural issues, and concerns the Advanced Draft unless otherwise indicated. The final section looks at individual sections of the text.

I do not have the authority to write syllabus text. The textual suggestions made below concern details that are extremely technical and complex, and need academic mathematicians. It seemed constructive, however, this time to attempt replacement text of many items.

### The Syllabus text as exposition

The cramped and artificial dot-point style of the text remains vastly inferior to the expository style of the current 2/3/4 Unit syllabuses. The syllabus text should be conceived as performing many tasks. It should prescribe the limits of what is taught. It should carefully define all words used, and make distinctions as to whether that definition is formally correct or an informal description. It should carefully prescribe which theorems are to be proven with the proof examined, which proofs should be presented but not examined, and which theorems should simply be told to students. And above all, it should expound that material, particularly when it is new, or difficult, or has unexpected relationships with other topics.

I can report that the dot-point style of this text is insanely difficult to read, even after years of teaching experience.

By far the best solution is the structure of the current 2/3 Unit courses, where there are no dot-points, and everything is discursive. A less satisfactory compromise was reached with the current 4 Unit courses, where each set of terse dot-points is followed by a discursive section on '*Applications, Implications and Considerations*'. The Glossary entries must certainly be incorporated into the presentation, because it is so often unclear which section a Glossary item is intended to be relevant to — the Draft Glossaries remain chaotic, as detailed below.

### Extension 1 additions

The current 2/3 Unit syllabus displays both courses in the one document, clearly indicating the material that is for Extension 1 only. This is an excellent approach because so much of the 2 Unit course has to be developed with slightly greater rigour in Extension 1, and the differences are very clear from the document. Teachers would welcome such a structure in a common Advanced/Extension 1 syllabus.

### Definitions

New terms are constantly introduced in any mathematical text, and when this happens, the text must be completely clear whether it is intended as a definition, or an explanation, or a theorem, or a bit of all three — ambiguity here is fatal in all mathematics. Unfortunately, the Drafts are full of such ambiguities, as detailed below.

### Proof

There is an extraordinary resistance in these Drafts to using the word 'proof', which been at the very heart of our discipline at least since Euclid's time. There seems to be an ideology here:

Proof  $\longrightarrow$  Rigidity  $\longrightarrow$  Suppression of creativity  $\longrightarrow$  Unimaginative education

The opposite is the case. Proof, understanding and problem-solving are the single goal of all mathematics. They express the same thing in different ways, and none is possible without the other two. Mathematics taught without proofs of results accessible to proof quickly becomes arbitrary and deeply unsatisfying.

**Clarity about classroom practice:** The drafts often use the ambiguous word ‘understand’, which makes things totally unclear (and looks like a replacement in the earlier vogue phrase ‘choose and use’). From many examples, one dot-point from MA-C1 illustrates the Drafts’ problems:

*‘understand and use the fact that . . . two lines with gradients  $m$  and  $m'$  respectively are perpendicular if and only if  $mm' = -1$ ’.*

What is a teacher to make of the slippery word ‘understand’?

- Recall the result from Years 9–10, or
- See the result proven, but not be examined on it, or
- Learn the proof and be ready to be examined on it or a similar proof, or
- Accept the result without proof?

Even with this simple case, things are unclear, because the usual proof uses congruent triangles, which have been removed from the courses.

The present 2/3/4 Unit syllabuses spell these things out very clearly, often with details of proofs. The Drafts leave everything unclear all through their texts. The intention seems to be to suppress proofs, thus misleading our students completely about the nature of mathematics.

## Euclidean Geometry

Geometry has been the imaginative heart of mathematics from at least Greek times, and remains so in modern research. It enables the abstractions of mathematics to be visualised, so providing motivation, intuition, and a sense of mastery and understanding of the whole.

- ★ Proof was one of the driving forces behind the reintroduction of Euclidean geometry into the senior courses a few decades ago. Students find algebraic proofs very difficult to assimilate, but geometric proofs can be seen as a whole on the diagram.
- ★ Similarity is required for the definitions of the trigonometric functions, because they are ratios of sides. It is required widely elsewhere, as for example at the bottom of page 32, where it is stated that all circles are similar but the implications for the definition of one radian are not drawn out. Even the definition of the gradient of a line requires similarity.
- ★ The congruence tests are needed for the sine and cosine rule:
  - The cosine rule requires the SAS test when the third side is being found, and the SSS test when an angle is being found.
  - The sine rule requires the AAS test when a side is being found, but when an angle is being found, it may not be unique because there is no ‘ASS’ test involving two sides and a non-included angle. The ‘ambiguous case’ referred to in the Glossary is a geometric phenomenon, easily demonstrated with straight-edge and compasses, and the trigonometric phenomenon is a consequence of this.
- ★ Equilateral and isosceles triangles are needed throughout in problems, and in particular for the values of the trigonometric functions of the special angles  $30^\circ$ ,  $45^\circ$  and  $60^\circ$ .
- ★ The new topic of three-dimensional trigonometry will require Euclidean geometry.
- ★ The Cartesian plane is a geometric object, and its properties often rely on Euclidean ideas, as in the theorem above on perpendicular gradients.

A review of Euclidean geometry is required, including similarity. It may be possible to reduce the density of the topic as it is in the present 2 Unit course, but the basic ideas should be there.

**Circle geometry:** The omission of circle geometry is extremely disappointing. Circles are ubiquitous in mathematics, they are only quadratic objects, and the more students know about them, the better. For example, locus problems in complex numbers will be severely restricted without circle geometry, and placing the topic only in Extension 2, as in the July Draft, would make it difficult for Extension 1 students to study mathematics at university. And the radius-and-tangent theorem should be well-known to Advanced students before they embark on calculus.

### Coordinate geometry

Apparently NESAs doesn't like coordinate geometry either. Calculus takes place in the coordinate plane, and its geometry is essential for calculus. The sparse entries in MA-F1 concern linear functions and gradient, but other important areas are missing.

- ★ The midpoint formula is missing. For example, there will no longer be any verification that the point of inflexion of a cubic is the midpoint of a chord through the point of inflexion.
- ★ The perpendicular distance formula, allowing the calculation of areas of triangles, is missing. It is far more useful and important than the irrelevant Heron's formula.
- ★ Two lines with gradients  $m$  and  $m'$  being perpendicular if and only if  $mm' = -1$  belongs in MA-F1, because it is part of coordinate geometry. It is at the start of calculus in MA-C1.1 (and there is no problem reviewing it there). Also, the gradients should be  $m_1$  and  $m_2$  because  $m'$  in calculus means the derivative — the notation was not edited.

Section MA-F1.3 suggests that direct variation, rather than geometry, is being understood as the only motivation for linear functions. NSW has always had a different and most effective approach.

It is inappropriate for Extension 1 students to be proving theorems using vectors when they are not proving things using Euclidean or coordinate geometry.

### Digital technology should not be prescribed

Digital technology should not be prescribed. For example, MA-C1 page 37 prescribes:

*Use digital technology to plot the gradient functions of  $f(x) = x^2$ ,  $f(x) = x^3$ .*

- ★ There is no reason whatsoever to use digital technology to sketch the derivatives of  $x^2$  and  $x^3$ . In fact, a graph-paper sketch with some tangents drawn and their gradients measured tells a student far more about differentiation than does a picture on a screen.
- ★ In large numbers of schools, the technology is not available, or does not work properly.

Here, and everywhere else, it should say, 'using digital technology or otherwise'. Teachers teach best when they follow their own intuitions and enthusiasms. The only reasonable exceptions in these courses are large data sets.

### The structure of the calculus course

It is good that the chain, product and quotient rules have been moved back to the start of calculus so that effective drill can take place. But the development of calculus, and then of its applications, remains most unsatisfactory.

**The second derivative (Section MA-C3)** should be introduced as soon as differentiation begins. It starts as drill work that continues the practice with differentiation of algebraic functions. But it soon demonstrates effortlessly some extremely significant facts about the functions in this course:

- ★ Polynomials vanish on repeated differentiation.
- ★ Exponential functions are their own derivatives (apart from constants factors).
- ★ The sine and cosine functions cycle with period 4 when differentiated (apart from constants factors).

Section MA-C3 in Year 12 is far too late, and it appears that the presentation of these key properties of the most common functions has not been thought through.

**Maximisation and minimisation (Section MA-C3)** is always one of our best ways of selling calculus to students. It is now also removed to Section MA-C3, the second-last topic of calculus, which is far too late.

**Systematic curve-sketching (Section MA-C3)** should also begin in Year 11, or very early in Year 12, when only the algebraic functions are accessible to calculus, because this application within mathematics is the first application of calculus, and particularly of the second derivative, that students can understand and master. Again, Section MA-C3 in Year 12 is far too late.

A SYSTEMATIC SUMMARY OF CURVE-SKETCHING techniques should be given once points of inflexion have been taught. Section MA-C3 omits such a summary entirely:

1. *Find the domain, and any vertical or horizontal asymptotes.*
2. *Test whether the function is odd or even or neither.*
3. *Find the  $x$ -intercepts, and use test points with  $f(x)$  to examine the sign of the function.*
4. *Find the stationary points, and use test points with  $f'(x)$  to find any turning points and horizontal points of inflexion.*
5. *Find the zeroes of  $f''(x)$ , and use test points with  $f''(x)$  to find any points of inflexion.*

(Of course, not all steps are appropriate or even possible, depending on the function.)

This systematic approach should then be applied as the set of functions expands to cover first exponential and logarithmic functions, and then trigonometric functions.

THE INTERACTION OF ALL THESE TECHNIQUES, and the way in which they will be presented in the classroom, have not been thought through:

- ★ Section MA-C1 does not mention  $x$ -intercepts, evenness or oddness, or asymptotes.
- ★ Section MA-E1 on exponential and logarithmic functions does not mention asymptotes.
- ★ Section MA-F2 on functions is so concerned with shifting and stretching that it forgets to mention evenness and oddness.
- ★ Section MA-C2 seems to have forgotten about sketching unknown functions.
- ★ Section MA-S5 wants sketching of normal distribution curves using digital technology, even though the course has developed all the methods necessary to sketch these curves, from evenness and asymptotes to finding points of inflexion.

**Integration (Section MA-C4)** comes at the very end of calculus, with no applications of integration afterwards, either within the topic or in succeeding topics. This means that integration will not be assimilated. Yet integration is the most important part of calculus in undergraduate science.

THE CURRENT SYLLABUSES have the correct solution. Integration of algebraic functions should come immediately after curve-sketching and maximisation, using only algebraic functions, so that this reversing of differentiation to find areas can be presented as an essential part of the story. After this, the two groups of special functions are developed, and curve-sketching, maximisation, and integration applied immediately to each. In this way, each method of calculus is revised again as the special functions are introduced, and the whole thing is revised as a unity when it is later applied to motion and rates of changes.

THE ARBITRARY CONSTANT: One of the worst aspects of Section MA-C4 is that it does not deal adequately with problems involving the evaluation of the arbitrary constant of integration using boundary values. This is ubiquitous in science, and must be taught in any course on introductory integration. See the comments below on motion.

THE PRIMITIVE OF  $1/x$  is one of our two motivations for the calculus of the exponential and logarithmic functions. This aspect of the logarithmic functions goes unnoticed in the Drafts.

**Solids of revolution:** This is a simple application of the definite integral, and should have its usual place in the Advanced course.

- ★ It is one of the few places where three-dimensional ideas occur.
- ★ There are no logical or computational problems, provided reasonable functions are chosen.
- ★ The development of the definite integral by slicing is difficult to understand, but the development of the formula for solids of revolution is exactly the intuition that is needed. Science also constantly develops integrals by such slicing methods.
- ★ We promised students in Years 9–10 that we would eventually prove the formulae for the volumes of spheres and cones. As always, not doing so when it is so simple makes mathematics arbitrary, and distorts its methods.

This omission is another example of the Drafts not caring about the straightforward applications of calculus, while at the same time talking endlessly about real-world applications.

### **Motion as an application of calculus versus the structure of the strands**

In the present 2/3/4 Unit courses, motion is an impressively coherent unit of work that follows curve-sketching, maximisation, integration, and the two groups of special functions. In the drafts, it is split up into many incoherent fragments.

Motion is by far the most satisfactory practical application of calculus at school, because the first derivative is modelled by velocity, which we can see, and the second derivative by acceleration, which we can feel. Yet these insights, and even simple ideas such that a ball is motionless at the top of a vertical flight, take sustained teaching and exercises for students to assimilate.

Motion is also an excellent application of calculus because one integrates acceleration to get velocity, and then integrates velocity to get displacement. Yet none of this is mentioned because no calculus follows Section C4.2 on integration. In particular, there is no mention of any specific motion problems requiring the evaluation of the arbitrary constant of integration using boundary values — again, this is a ubiquitous process in science.

**Simple harmonic motion**, in all its various forms, is the reason why the trigonometric functions are so important in science. Everything in Extension 1 has been prepared for it, including  $r \cos(nt + \alpha)$ , and also now phase, and it certainly should be included. The present 3 Unit course makes the mistake of introducing it through the differential equation  $\frac{d^2x}{dt^2} = -n^2x$ , which can't be solved in our courses, and thus causes endless teaching problems and confusion. But there is no problem whatsoever defining simple harmonic motion as motion whose displacement–time equation is a sine or cosine function, and proceeding from there in exactly the way that other functions are used to model motion.

Even the Advanced course should be dealing with such motion as one example of motion, leaving out the complications of compound angles and phase, and perhaps also the name.

### **Rates of change**

With all the talk about rates of change, there are none in the Advanced Draft. One expects to see:

- ★ Given  $Q$  as a function of time, find  $\frac{dQ}{dt}$ , and solve a problem.
- ★ Given  $\frac{dQ}{dt}$  and a boundary condition, find  $Q$  as a function of time, and solve a problem.
- ★ Given that  $Q = Ae^{kt}$ , show that  $\frac{dQ}{dt} = kQ$ , and solve a problem.
- ★ Given that  $\frac{dQ}{dt} = kQ$ , show that  $Q = Ae^{kt}$  is a solution, and hence solve a problem given boundary conditions.

Such things are in the Extension 1 course, but why has such important material been omitted from the Advanced Drafts?

### The structure of the mathematics versus the structure of the strands

The writers have had to battle with the straight-jacket of strands, despite the fact that once calculus begins, everything combines with everything else. The result is incoherence and inconsistency. Besides the matters mentioned above:

- ★ Placing trigonometry before calculus, and exponentials after calculus, means that the derivative of  $e^x$  has been placed in the logarithms and exponentials section. In contrast, the derivative of the trigonometric functions is in a calculus section.
- ★ This may be the cause of radian measure being introduced far too early, near the start of Year 11.

The structure of Strands is quite inappropriate for senior calculus because it is in conflict with the structure of the mathematics.

Of course the material in any Year can be taught in any order, but this limited flexibility doesn't help a teacher when curve-sketching is placed in Year 12 instead of Year 11, and radian measure is introduced in Year 11 instead of Year 12.

The teaching order of the current 2/3 Unit syllabuses was disturbed by artificial rules about how much of the Year 11 course can be examined in the HSC. If the Board could only eliminate these restrictions, then the artificial order of the Drafts could be eliminated.

### Cutting, pasting and editing

It is clear that larger amounts of these Drafts have been obtained by cutting and pasting from elsewhere. It is also clear that the editing process has gone astray with a great deal of this material. The result, as detailed in what follows, is inappropriate notation, inappropriate language, wrong definitions, and incoherence on every scale from the very local to the global. The presentations of every topic are badly affected by this, and by the subsequent failure to think through the implications of what is being written for the coherence of the mathematics, and for its presentation by teachers in the classroom.

It is as if the writing has been done in complete ignorance of the excellent NSW traditions of mathematics teaching, and particularly of its insistence on the geometric representation of just about every phenomenon in the courses. This problem is well displayed in calculus, where the relationship of the algebra to the graph is insufficiently emphasised, and the curve-sketching implications of calculus are delayed until the very end of calculus instead of being presented as near to the start as possible.

One example not mentioned below — the Advanced Glossary item for 'function' gives the set-theoretic idea of a mapping from a map  $S$  to a set  $T$ , which is irrelevant because these courses need functions of real numbers. By itself, that causes no harm, but then the Extension 1 Glossary states that the inverse sine function is defined (where 'defined' is the wrong word here) by

$$\sin^{-1} : [-1, 1] \rightarrow \Re.$$

Such set-theoretic notation has no place in school (and neither does the Fraktur font). Where was it cut-and-pasted from?

## Details of individual sections

I will now turn to the fine details of individual sections. As before, these remarks concern the Advanced Draft unless otherwise indicated.

### Functions MA-F1: Working with Functions

Functions correctly begin the syllabus, because they are the basis on which calculus is built, and students' attention must be turned to them at the very start, in sharp contrast to Year 10.

**F1.1 Algebraic techniques:** As remarked earlier, a review of coordinate geometry needs pride of place here, because all these functions will live in the Cartesian plane, where they will display their characteristic geometric features.

**Numbers are required** for the functions in this course, and there should be a short review of the types of numbers significant in the course:

- *Review four important types of numbers:*
  - *The whole numbers are the numbers 0, 1, 2, 3, ... used to count finite sets.*
  - *The integers ..., -2, -1, 0, 1, 2, ... include the opposites of all the whole numbers.*
  - *The rational numbers are the numbers that can be written as fractions  $\frac{a}{b}$ , where  $a$  and  $b$  are integers and  $b \neq 0$ .*
  - *The real numbers are the points on the number line.*

The first definition avoids axiomatics by begging the question as to what finite sets are, and the last definition must make an appeal to geometry to avoid very sophisticated work with limits.

**Surds are required** throughout for quadratics and trigonometry, and their revision needs to be far more explicit:

- *Review the arithmetic of quadratic surds — simplification, rationalising the denominator, addition, subtraction, multiplication and division of surdic expressions.*

Add the following item to the Glossary:

*Surd: An expression  $\sqrt[n]{a}$ , where  $a$  is a rational number and  $n \geq 2$  is an integer, is called a surd if it not itself rational.*

**Index laws** is the term, not *indicial laws* as cut-and-pasted from elsewhere — changing well-accepted language achieves nothing. They may be mentioned here, but the main place where index laws must be revisited, and are not, is Section MA-E1 on exponential and logarithmic functions.

**F1.2 Introduction to functions:** This section is badly flawed by the description of a relation as a 'mapping'. The Glossary entry is different but still wrong: '*Graphically, a relation between two real variables  $x$  and  $y$  is the set of ordered pairs  $(x, y)$  that can be plotted on the Cartesian number plane.*'

- ★ Was 'the set' intended to be 'a set'? As it stands, there would only be a single relation — the whole Cartesian plane.
- ★ What is 'Graphically' doing? Is this a definition or a description?
- ★ The term 'number plane' is inappropriate unless we are dealing with complex numbers.

There are many other problems of language and of the order with which things are presented:

- ★ The very first dot-point asks students to '*understand what is meant by a variable*', with no function mentioned. I can't answer this, and there is no Glossary entry for 'variable'.
- ★ A function may determine a variable, but it doesn't 'define' it.
- ★ One can define odd and even functions, but one can't 'define' their properties.

- ★ The relationship between a function and its graph is not made clear. In particular, tables of values and the resulting ordered pairs are not mentioned.

Three approaches to a function are required — a rule, a graph, and a set of ordered pairs — together with the more general idea of a relation. Only functions whose domains and ranges are subsets of  $\mathbf{R}$  should be considered at this stage. Writing this text absolutely requires mathematicians, and it requires a discursive explanation, but keeping to NESAs's rigid and unmathematical syntax:

- *Understand a function informally as a rule that, given one quantity (the independent variable), determines another quantity (the dependent variable).*
- *Use function notation, as in  $f(x) = x^2 + 3$ , and variable notation, as in  $y = x^2 + 3$ .*
- *Use a table of values to create a representative set of ordered pairs  $(x, y)$  satisfying  $y = f(x)$ , and hence sketch the graph of the function on the Cartesian plane.*
- *Understand the formal definition of a function as a set of ordered pairs  $(x, y)$  of real numbers such that no two ordered pairs have the same first component (or  $x$ -component).*
- *More generally, define a relation as any set of ordered pairs  $(x, y)$  of real numbers.*
- *Understand that a relation is a function if and only if it passes the vertical line test, that is, no vertical line intersects the graph more than once.*

**Tables of values** are missing almost entirely from the three Drafts (the single exception being tables of values of the gradients of secants on page 37 when introducing the derivative as a limit.) Tables of values have been the life-blood of functions in NSW for decades, yet this Draft ignores them. How else, before one knows any rules, does one graph a function? What does it mean algebraically for a point to lie on a curve?

**Simultaneous equations**, both linear and non-linear, and the relationship with their graphs, should be covered. This looks like an oversight, because it's been mentioned in F1.3.

**Continuity** should not be mentioned here. It is a complicated idea that requires limits, and it belongs in introductory calculus. See the section below labelled **Calculus**.

**The horizontal line test** should not be mentioned here, and the term 'one-to-one' should never be used. General inverse functions are not a concern of the Advanced course, and this is a distraction — it should mention inverse functions in the section on logarithms, which is the only place where the Advanced course requires them, and this should be brief, and directed specifically towards the understanding of exponential and logarithmic functions and their graphs.

**One-to-one, one-to-many, . . . , and the horizontal line test:** The terms 'one-to-one', 'one-to-many', 'many-to-one' and 'many-to-many' are unnecessary and conceptually difficult distractions at school. The last three are never used again in the Drafts, and in Extension 1, the first is easily and much better avoided by passing straight from the horizontal line test to the question whether the inverse relation is a function (this is not even a concern of the Advanced course).

There are also logical problems here. Injective, left injective and right injective are tricky ideas, and are easily confused or conflated with their surjective analogies.

This material has been cut-and-pasted from elsewhere, with the implications for the Advanced and Extension 1 syllabuses not thought through.

### F1.3 Polynomial function up to degree 3

These objects should be called 'linear functions' and 'quadratic functions', not 'linear relationships' and 'quadratic relationships', which are meaningless terms.

Surds, by the way, are required to '*understand the role of the discriminant*'.

**Maximum or minimum value of a quadratic function:** The vertex is mentioned, but the maximum or minimum value of the quadratic is not mentioned, despite the central role that this is about to play in calculus (or would play if the order were satisfactory). The structure of the material has not been thought through.

**Inflexions and concavity** should not be mentioned here, except for the simple idea, already introduced in Year 10, that graphs of quadratics are ‘concave up’ or ‘concave down’. Beyond that, calculus is required, and students have as yet no way of determining at what points a factored cubic is concave up or concave down or has an inflexion.

#### F1.4 Introduction to functions

- ★ The first dot-point on polynomials belongs in the previous section F1.3 on polynomials.
- ★ Replace  $ax \pm b$  by  $ax + b$  twice in the third dot-point. The constant  $b$  can be negative.
- ★ Given the graph of  $y = f(x)$ , for any function  $f(x)$ , it is trivial to draw  $y = |f(x)|$ . The third dot-point should not be restricted to linear functions.

**Reflections in the  $x$ - and  $y$ -axes** are required in F1.2 for odd and even functions (as in the newly inserted Glossary entries). This has also been omitted from the later section MA-F2, and F1.4 should have:

- *Given the graph of  $y = f(x)$ , sketch  $y = -f(x)$  and  $y = f(-x)$  and  $y = -f(-x)$ , using reflections in the  $x$ - and  $y$ -axes.*
- *Describe the symmetries of the graphs of odd and even functions.*

**Shifting and stretching horizontally and vertically** has been delayed until Year 12 (MA-F2), which is appropriate in Advanced given the formality of the identities (but perhaps not in Extension 1, where they are needed earlier). Yet this section is exactly where the phenomenon is noticed, and once noticed, it can be identified and explained so easily using the graphs of parabolas, cubics, hyperbolas and circles. Thus a dot-point should be added:

- *Identify informally how graphs are shifted and stretched horizontally and vertically as the constants in the function change.*

#### Functions MA-F2: Graphing techniques

It is appropriate in Advanced that the formal statements of shifting and stretching are only introduced here. But informal observation should have been done in MA-F1, as indicated above.

**Reflections** should be stated formally here if they are only introduced informally in MA-F1.

**Shifting and stretching:** The various laws are in an incoherent state that students will never understand, and teachers will not be able to teach systematically:

1. The initial indigestible dot-point should be replaced by a statement that students are able to deal with compositions of transformations. No one can learn the formula given here.
2. The roles of subtraction, and of division by constants, have not been clarified.
3. TECHNOLOGY should not be prescribed in the content — the qualification ‘or otherwise’ should always be added.
4. TABLES OF VALUES are the obvious way in which these things are observed and understood. It is quite inappropriate to mention technology, but not mention tables of values.

Here is a suggested rewriting. (My own opinion, however, is that horizontal stretching is a little too much for Advanced students, and that it should be treated only when it arises in trigonometric functions, which are the only situation in which it occurs seriously in Advanced.)

- *Examine how vertical and horizontal shifting and stretching of graphs arises from the varying of the constants in any function within the scope of this syllabus (including Extension 1 functions for Extension 1 students), using tables of values, digital technology, or other methods.*
  - *Examine the graphs of  $y = f(x)$ ,  $y = f(x - a)$  and  $y - b = f(x)$  (which can be rewritten as  $y = f(x) + b$ ). Hence establish the rules for shifting right by  $a$  units and shifting up by  $b$  units:*
    - *To shift right by  $a$  units, replace  $x$  by  $x - a$ .*
    - *To shift up by  $b$  units, replace  $y$  by  $y - b$ .*
  - *Examine the graphs of  $y = f(x)$ ,  $y = f(x/k)$  and  $y/c = f(x)$  (which can be rewritten as  $y = cf(x)$ ). Hence establish the rules for stretching horizontally by a factor of  $k$  and vertically by a factor of  $c$ :*
    - *To stretch horizontally by a factor of  $k$ , replace  $x$  by  $x/k$ .*
    - *To stretch vertically by a factor of  $c$ , replace  $y$  by  $y/c$ .*
  - *Examine the composition of these transformations, and the role of the constants in the associated function, and recognise that the order in which transformations are applied is important in the construction of the resulting function or graph.*

This formulation of the rules shows the perfect symmetry between shifting and stretching, and between horizontal and vertical.

**Asymptotes** have been appallingly defined, as remarked on page 1. The glossary entry should be:

*Asymptotes: A horizontal line  $y = b$  is an asymptote to a curve  $y = f(x)$  on the right or left if respectively :*

$$\lim_{x \rightarrow \infty} f(x) = b \quad \text{or} \quad \lim_{x \rightarrow -\infty} f(x) = b .$$

*A vertical line  $x = c$  is an asymptote to  $y = f(x)$  if*

$$f(x) \rightarrow \infty \quad \text{or} \quad f(x) \rightarrow -\infty \quad \text{as } x \rightarrow c+ \quad \text{or as } x \rightarrow c- .$$

It's hard to avoid limits from above and below with vertical asymptotes.

Oblique asymptotes may or may not be considered appropriate for the Glossary. If so:

*A line  $y = mx + b$  is an asymptote to a curve  $y = f(x)$  on the right or left if respectively :*

$$\lim_{x \rightarrow \infty} (mx + b - f(x)) = 0 \quad \text{or} \quad \lim_{x \rightarrow -\infty} (mx + b - f(x)) = 0 .$$

**Linear and non-linear inequalities** need to be solved by finding the points of intersection and examining the graphs. This is standard and important work in calculus. Compare the remarks about non-linear simultaneous equations in the discussion of MA-F1.

## **Trigonometric Functions — MA-T1 and MA-T2**

The early transition from trigonometry to trigonometric functions is good, but there are many problems.

**Radian Measure** is motivated by calculus, because the derivative  $\frac{d}{dx} \sin x = \cos x$  requires radian measure, and the introduction of radian measure should thus be delayed until  $\sin x$  is about to be differentiated. Unmotivated ideas are destructive of good mathematics.

There is already much material here that Advanced students find very challenging — the definition of the trigonometric functions for any angle, the sketching of their graphs, trigonometric identities, and even the use of trigonometry in non-right-angled triangles. In particular, the new topic three-dimensional trigonometry would be difficult at this level.

**Heron's formula and proofs:** Herons' formula is never mentioned again in the Drafts, and should be omitted in favour of the perpendicular distance formula. But the issue of proof that arises here is a problem throughout the Drafts. Are the proofs of the sine rule, the cosine rule and Heron's formula intended to be given and provable, given but not provable, assumed from Year 10, or just told to the students as arbitrary results? This is a central problem that any senior mathematics syllabus must address, and it has been largely ignored.

**Notation in identities:** In Topic MA-T2, identities are written using 'identically equals':

$$\cos^2 x + \sin^2 x \equiv 1 .$$

Why are trigonometric identities different from algebraic or exponential identities? This will cause endless confusion, because we are not writing

$$x^2 - y^2 \equiv (x + y)(x - y) \quad \text{or} \quad e^{x+y} \equiv e^x e^y .$$

Is it  $\sin^2 x - \sin^2 y \equiv (\sin x + \sin y)(\sin x - \sin y)$  or  $\sin^2 x - \sin^2 y = (\sin x + \sin y)(\sin x - \sin y)$ ? Of course student should know what an identity is, but this notation must be dropped. Once again, cutting-and-pasting and poor editing are at work.

**Defining the trigonometric functions:** Topic MA-T2 *defines*  $\tan x$  as  $\frac{\sin x}{\cos x}$  (with the  $\equiv$  sign now used for a definition, not an identity). This should be an identity, not a definition, and being the formal text of a senior syllabus, the qualification 'provided that  $\cos x \neq 0$ ' must be added. Similar qualifications should be added to the definitions of  $\sec x$  and  $\operatorname{cosec} x$ .

The definition of  $\cot x$  is tricky because  $\cot 90^\circ$  is defined and  $\tan 90^\circ$  is not. The definitions of the three functions  $\sec x$ ,  $\operatorname{cosec} x$  and  $\cot x$  should therefore be given explicitly.

It would be far preferable, however, to define the other four functions  $\tan x$ ,  $\sec x$ ,  $\operatorname{cosec} x$  and  $\cot x$  as ratios in the same way as  $\sin x$  and  $\cos x$ , and then extend them to all angles in the same manner as  $\sin x$  and  $\cos x$  were extended. Each of these six functions has a life of its own in triangle trigonometry before being extended to all angles, and preferencing  $\sin x$  and  $\cos x$  suppresses this.

**All Stations To Central and the related angle:** Despite the obvious copying from other States, there is no reason to drop the loved and effective ASTC mnemonic, which students rely on all the time in this work. Similarly, the term 'related angle' has been omitted, here and in the Glossary. All further evidence of contempt for, or ignorance of, the NSW traditions of mathematics teaching.

**Amplitude and periodicity:** Page 33 requires students to '*understand . . . the resulting periodicity of these functions . . .*', but does not define period or state whether a value is required here or a verbal description. The Glossary entry for 'period' is flawed, in particular because it begs the question by using the word 'cycle', which is otherwise undefined.

The easier concept 'amplitude' is not mentioned, although there is an entry in the Glossary, badly flawed by its use of the word 'centre' to refer to a horizontal line, and by its assumption that motion is being considered.

**Domain, range, odd and even:** The far easier concepts of domain and range, already introduced and drilled, are also omitted. The point here is to move attention to the trigonometric functions, and these ideas are an essential part of that task.

The stand-out omission here, however, is the absence of oddness and evenness, and of the resulting symmetries of the graph. The fact that the sine function is odd and the cosine function is even underlies all their applications — indeed this is the main reason why these two symmetries are introduced into the courses. This shows once again how little care has been given to the graphical implications of this material, and to the coherence of the courses.

**Inverse functions and inverse trig functions in Extension 1 (ME-F1.3 and ME-T1)**

The presentation of inverse functions in Section ME-F1.3 is flawed in a number of ways:

- ★ There is no definition of inverse relation. Indeed the term is never used.
- ★ The algebraic definition offered of inverse function is a theorem in our courses. It is also wrongly stated, because the domain of  $x$  is usually different for the two expressions.
- ★ The text seems to assume that every function has an inverse function, because it seems to assume, quite falsely, that it remains the same function when its domain has been restricted.
- ★ The horizontal line test, introduced at the start of the Advanced draft, is not mentioned. Neither is the relationship between the domains and ranges of a function and its inverse function established when the inverse relation is a function.
- ★ The theorem that the inverse of a composite is the composite of the inverses is a complicated algebraic irrelevance in our courses. The principal reason for introducing inverse functions is the calculus of the inverse trigonometric functions and the extra primitives that they allow.

Such algebraic approaches, not grounded in geometry, are quite foreign to the NSW tradition. Here is an attempt to rescue the material, assuming that the inverse of a relation is never needed:

- *Define the inverse relation of a function  $y = f(x)$  to be the relation obtained by reversing all the ordered pairs of the function. Apply this to familiar tables of values and graph the results.*
  - *Understand why the graph of the inverse relation is therefore obtained by reflecting the graph of the function in the line  $y = x$ . (The reflection reverses each ordered pair.)*
  - *Using the fact that this reflection exchanges horizontal and vertical lines, develop the horizontal line test to test whether the inverse relation of the function is again a function.*
  - *Write the rule or rules for the inverse relation by exchanging  $x$  and  $y$  in all their occurrences in the function rules, including any restrictions, and solve for  $y$ , if possible.*
- *Use the notation  $f^{-1}(x)$  to write the inverse relation when it is a function, and identify the relationships between the domains and ranges of  $f(x)$  and  $f^{-1}(x)$ .*
  - *Develop the identities:*

$$f^{-1}(f(x)) = x, \text{ for all } x \text{ in the domain of } f(x), \text{ and}$$

$$f(f^{-1}(x)) = x, \text{ for all } x \text{ in the domain of } f^{-1}(x).$$
- *When the inverse relation is not a function, create new functions  $g(x)$  whose inverse relations are functions by restricting the domain, and compare the effectiveness of different restrictions.*
  - *In particular, understand that  $y = \sqrt{x}$  is not the inverse function of  $y = x^2$ , but is the inverse function of the restricted function  $y = x^2$ , where  $x \geq 0$ .*

**Inverse trigonometric functions** suffer similar problems of language in stating, for example, that  $y = \arcsin x$  is the inverse function of  $y = \sin x$ . This is false — it is the inverse function of a new function, namely the *restriction of  $y = \sin x$  to the closed interval  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$* .

Also, the inverse trigonometric functions are motivated in this course by the need for primitives of two further algebraic functions  $\frac{1}{\sqrt{1-x^2}}$  and  $\frac{1}{1+x^2}$ . What then is the topic doing in Year 11, when it achieves no purpose there except to introduce more confusing terminology and notation? It should follow integration, as it does in the present 3 Unit course, but of course the Strands seem to have dictated that integration is the end of calculus. It's a mess.

**Calculus – MA-C1**

It is good that the latest Draft uses tangents to introduce differentiation rather than rates of change. But little else has changed, and as remarked above, the exposition is very badly compromised by mistakes and confusions in almost all the significant words used, and in many structural features.

**Tangents:** The Glossary item makes the fatal mistake of saying that a tangent does not cross the curve (consider  $y = x^3$  at the origin), although it is unclear what the text actually means by the phrase ‘does not intersect the curve at that point’. An informal and a formal definition are needed:

Informal definition: *Suppose that a graph is continuous at a point  $P$ , and that there is a line  $t$  that approximates the graph near  $P$  on both sides of  $P$ . Then the line  $t$  is called the tangent at  $P$ .*

Formal definition: *Suppose that the derivative of a function  $f(x)$  exists at  $x = a$ , that is,*

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \text{ exists.}$$

*Then the line through  $P(a, f(a))$  with gradient  $f'(a)$  is called the tangent to  $y = f(x)$  at  $P$ .*

And tangents are not ‘tangent lines’ as copied-and-pasted from elsewhere. Neither are intervals ‘line segments’.

**Vertical tangents and cusps in Extension 1:** It would be wise to define vertical tangents and cusps in the Extension 1 course, because its systematic treatment of inverse functions will routinely throw up functions such as  $y = x^{\frac{1}{3}}$  and  $y = x^{\frac{2}{3}}$ .

**Continuity:** The Drafts make the grave mistake of defining the term ‘continuous function’. This will cause endless confusion, because a *continuous function* is a function that is continuous at every point in its domain, so that  $y = \frac{1}{x}$  is a continuous function. Continuous functions are not the sort of thing to teach at school, and serve no purpose in the three courses.

The Glossary entries for ‘continuous function’ and ‘discontinuous function’ then make the situation impossible by confusing ‘continuous’ with ‘connected’:

*Continuous function: A function is continuous when ... its graph is a single unbroken curve.*

*Discontinuous function: A discontinuous function has at least one break or gap in the graph of the function.*

I would not use the term ‘continuous function’ at all (even though the current syllabus does use it, but with warnings). The term ‘discontinuous function’ can surely only mean, ‘a function that is not continuous’, and should not ever be used.

**Continuity at a point:** The concept that is needed in school calculus is ‘continuity at a point’. This is the idea highlighted in the current 2/3 Unit syllabus, but is completely missing from the Drafts. The glossary item should be (following page 70 of the current syllabus):

*Continuity at a point: A function  $f(x)$  is continuous at  $x = a$  if:*

1.  $f(x)$  is defined at  $x = a$ .
2. The limit  $\lim_{x \rightarrow a} f(x)$  of the function, as  $x$  approaches  $a$ , exists.
3.  $f(a)$  is equal to that limit.

I personally find that students understand these things better if one takes separately the limits from above and below:

$$\lim_{x \rightarrow a^+} f(x) \quad \text{and} \quad \lim_{x \rightarrow a^-} f(x).$$

This also makes it much easier to talk about curves such as  $f(x) = \frac{x}{|x|}$  that arise when differentiating functions such as  $y = |x|$ . It helps also with integrating a semi-circle  $f(x) = \sqrt{1-x^2}$ , which has to be ‘continuous on the closed interval  $[-1, 1]$ ’, which is defined by

1.  $f(x)$  is continuous for each value of  $x$  in the open interval  $-1 < x < 1$ ,
2.  $f(1) = \lim_{x \rightarrow 1^-} f(x)$  and  $f(-1) = \lim_{x \rightarrow 1^+} f(x)$ .

**Proving the formula for the derivative of  $x^n$ :** Again there is the slippery ‘understand and use’. When  $n$  is a positive integer, there are two standard approaches to the proof:

Using the sum of GP:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} \\ &= \lim_{t \rightarrow x} \frac{t^n - x^n}{t - x}, \text{ where } t = x + h \\ &= \lim_{t \rightarrow x} \frac{(t-x)(t^{n-1} + t^{n-2}x + \dots + x^{n-1})}{t-x} \\ &= \lim_{t \rightarrow x} (t^{n-1} + t^{n-2}x + \dots + x^{n-1}) \\ &= nx^{n-1}. \end{aligned}$$

Using the product rule, and assuming that we already have the result for  $x^{n-1}$ ,

$$\begin{aligned} f'(x) &= \frac{d}{dx}(x x^{n-1}) \\ &= 1 \times x^{n-1} + x \times (n-1)x^{n-2} \\ &= 1 \times x^{n-1} + (n-1)x^{n-1} \\ &= nx^{n-1}. \end{aligned}$$

The Drafts have now placed both GPs and mathematical induction in Year 12, blocking both approaches. But the induction here is easily understood without the formality normally associated with mathematical induction, and the following dot-point should be placed after the chain-product-quotient dot-points:

- *Understand (not examinable) how assuming the result  $\frac{d}{dx}x^{n-1} = (n-1)x^{n-2}$ , where  $n \geq 2$  is an integer, allows one to prove, using the product rule, that  $\frac{d}{dx}x^n = nx^{n-1}$ .*
- *Understand (not examinable) how the derivatives of rational powers of  $x$  can be established using the derivatives of positive integer powers of  $x$  and the chain, product and quotient rules.*

Similar non-examinable proofs are needed for the chain, product and quotient rules. Without these proofs, introductory calculus becomes a set of arbitrary formulae with little understanding.

**Discontinuities** are a problem. Is  $x = -5$  a discontinuity of  $y = \log_e x$ ? The word is probably best used only colloquially, and I quote the wisdom of the current 2/3 Unit syllabus:

*The behaviour of  $1/x$  and  $|x|/x$  near the origin should be demonstrated, but discontinuities should not be further stressed.*

## Calculus in Year 12 — Definitions

Many of the associated definitions in the Glossary are quite wrong (asymptotes and continuity have already been mentioned).

**Differentiable (Glossary):** The word is *differentiable*, with *smooth* only added in brackets if desired. As discussed above with ‘continuous’, the idea should only be introduced at school as a pointwise concept — as things stand,  $y = \frac{1}{x}$  would not be a differentiable function because it would not be a continuous function according to the Glossary’s definitions. We should use:

*Differentiability at a point: A function is called differentiable (or smooth) at a point  $P(a, f(a))$  on its graph if there is a tangent at  $P$ . That is, if the derivative exists at  $x = a$ ,*

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \text{ exists.}$$

*It is easily shown that if  $f(x)$  is differentiable at  $x = a$ , then it is continuous at  $x = a$ .*

**Local and global maximum and minimum (Glossary):** The definitions are correct. But the diagram need to be changed so that two things are clear. First, not all extrema are turning points, as for example in  $y = |x|$ , and extrema may occur at endpoints, as for example in the semi-circle  $y = \sqrt{1 - x^2}$ . Secondly, a global extremum is also a local extremum, so whatever replaces the point to the right should be labelled ‘Global (and local) maximum’.

**Turning point (Glossary):** Another fatal mistake — the curve must be differentiable at the point. (The origin is not a turning point of  $y = |x|$ , only a minimum.)

*Turning point: A turning point of a curve  $y = f(x)$  is a stationary point that is also a local maximum or minimum.*

*To test whether a stationary point is a turning point or a horizontal point of inflexion, test whether the gradients of tangents change sign around the point. One ‘sufficiently close’ point on each side should be enough.*

The diagram labelling is irrelevant. Two points should be labelled ‘maximum turning point’, and the other points ‘minimum turning point’. There should also be a horizontal point of inflexion.

**Concavity (Glossary)** is the sign of the second derivative, which is positive, negative or zero. It does not have a value. It is certainly not ‘the rate of turning of the tangent’, which could only mean some value  $\frac{d\theta}{dt}$  corresponding perhaps to curvature.

The curve  $y = x^4$  is neither concave up nor concave down at  $x = 0$ , and is an important counter-example to use when discussing concavity, points of inflexion and turning points. The Glossary ‘definition’ is no help here — where did its extraordinary definition come from?

Concavity must be defined in terms of the second derivative. Also, concavity should only be given as a pointwise concept, like continuity and differentiability:

*Concavity at a point: If a function  $f(x)$  is doubly differentiable at  $x = a$ , then the concavity of the curve  $y = f(x)$  at  $x = a$  is the sign of  $f''(x)$  at  $x = a$ , which may be positive, negative or zero.*

*If  $f''(x)$  is positive, the curve is called concave up at  $x = a$ , and the gradients of the tangents are increasing near  $x = a$ .*

*If  $f''(x)$  is negative, the curve is called concave down at  $x = a$ , and the gradients of the tangents are decreasing near  $x = a$ .*

**Point of inflexion (Glossary):** The definition is wrong because it omits the condition that  $f(x)$  be differentiable at the point (piecewise-defined functions cause problems here). There are several approaches, but the most straightforward is probably:

*Point of inflexion: A point of inflexion is a point on the curve where the tangent exists and crosses the curve.*

*Within these courses, points of inflexion can be found by taking the zeroes of the second derivative, and checking whether the concavity changes around the point.*

*(One should avoid functions such as  $y = x^{5/3}$ , which is not doubly differentiable at  $x = 0$ , but which has a tangent at the origin that crosses the curve.)*

## Logarithms and Exponentials — MA-E1

Students find this topic far more difficult than one would expect, and it needs more careful structuring.

**Calculus and logarithmic functions:** The straight-jacket of the Strands is very apparent here. The calculus of the exponential and logarithmic functions base  $e$  is part of calculus, but it can't put into calculus because of doctrines about Strands. The result is incoherence. Even with the present structure, the material on  $e^x$  could have been separated off into a second content subsection:

MA-E1.1 Logarithms and exponential functions to any base

MA-E1.2 Logarithms and exponential functions to base  $e$

**Indices** need explicit revision here. The three index laws (they are not 'indicial laws') and the special indices 0 and 1 need revision. Fractional indices and negative indices need particular attention, because few Year 11 students have any confidence with them. Surely any teacher would have included these things?

**Logarithms** need more careful introduction. First, the base  $a$  is restricted in two ways — it must be positive, and it cannot equal 1, and these two restriction must each be explained. Page 39 does not give either restriction, and the Glossary entry omits the restriction  $a \neq 1$ . The Glossary item also used the wrong symbol  $\leftrightarrow$  for 'if and only if'. It should be  $\iff$ , but words are better for logic.

Secondly, pages 39–40 do not mention that you can only take the log of a positive number — the variable  $b$  in the first dot-point has not been restricted.

The ideas of domain, range and asymptotes, developed earlier as tools for dealing with functions, are ignored on pages 39–40 just when they become extremely useful and important and make things clear.

**Inverse functions:** A general definition of inverse functions is probably inappropriate for Advanced students, but the idea is extremely important in this topic, and should be developed specifically for it within the dot-points. The word 'inverse' is used on page 39, in the unfortunate phrase 'inverse relationship', but is not explained, nor is the word 'inverse' in the Glossary.

A Glossary entry is probably not advisable, so the word needs to be explained on page 39:

- *Define logarithms as indices:  $y = a^x$  is equivalent to  $x = \log_a y$ , and explain why this definition only makes sense when  $a > 0$  and  $a \neq 1$ .*
- *Use the term 'inverse function' in describing how each function  $y = a^x$  and  $y = \log_a x$  is the inverse function of the other.*

— *Understand and use the fact that the function  $y = \log_a x$  defined in this way satisfies the two laws*

$$\log_a a^x = x, \text{ for all real } x \quad \text{and} \quad a^{\log_a x} = x, \text{ for all } x > 0.$$

— *Draw up tables of values for say  $y = 2^x$  and  $y = \log_2 x$ , and note how the rows of one are exchanged to give the rows of the other.*

— *Sketch the two graphs, and note how each is the reflection of the other in the diagonal line  $y = x$ .*

— *Identify the domain and range, the  $x$ - and  $y$ -intercepts, and the asymptotes of each curve, and explain how each property can be obtained from the corresponding property of the other function.*

- *Derive the logarithmic laws from the index laws,*

$$\log_a mn = \log_a m + \log_a n \quad \log_a \frac{m}{n} = \log_a m - \log_a n, \quad \log_a m^n = n \log_a m$$

*and establish the additional identities,*

$$\log_a 1 = 0, \quad \log_a a = 1, \quad \log_a \frac{1}{a} = -1, \quad \log_a \frac{1}{x} = -\log_a x$$

*and hence simplify logarithmic expressions.*

**Logarithmic and exponential functions base  $e$  — MA-E1 and MA-C2 and MA-C4:**

As remarked above, this should be separated out, at least to a different subtopic.

**Incoherence:** The fact that this is spread over three sections already tells a story. This is difficult material for Advanced students to assimilate, as we all know, and they need a coherent account of it all, not have things separated out all over the place.

**The definition and calculation of  $e$ :**

The Glossary defines  $e$  as  $e \equiv \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ . This definition is irrelevant to this course, because questions of ‘continuous compounding’ never arise. The definition is also notoriously difficult to apply or to approximate from.

And why has  $\equiv$  rather than  $=$  been used? This is a typical cut-and-paste item inserted from somewhere else with no consideration about the coherence of the courses.

Page 39 takes the most straightforward approach, but it has not been developed. Here is what is needed (and I have had to ignore the dot-point straight-jacket here):

- A. *After convincing oneself first by sketching exponential functions and their gradient functions, it is easy to show informally that the derivative of  $f(x) = a^x$  is a multiple of  $a^x$ :*

$$f'(x) = \lim_{h \rightarrow 0} \frac{a^{x+h} - a^x}{h} = a^x \lim_{h \rightarrow 0} \frac{a^h - 1}{h}.$$

Hence  $f'(0) = \lim_{h \rightarrow 0} \frac{a^h - 1}{h}$  is the gradient of the tangent at the  $y$ -intercept, and we can reasonably believe from the sketch that this tangent, and hence this limit, exists. Thus

$$\frac{d}{dx}(a^x) = m a^x, \text{ where } m \text{ is the gradient at the } y\text{-intercept.}$$

- B. *The curve  $y = 2^x$  passes through  $(1, 2)$ , so its gradient at the  $y$ -intercept is less than 1. The curve  $y = 4^x$  passes through  $(-\frac{1}{2}, \frac{1}{2})$ , so its gradient at the  $y$ -intercept is greater than 1. One is then convinced that there is a base  $e$  somewhere between 2 and 4 so that  $y = e^x$  has gradient exactly 1 at the  $y$ -intercept, and the resulting function  $e^x$  is thus its own derivative,*

$$\frac{d}{dx}(e^x) = e^x.$$

This is not a good way to approximate  $e$ . Calculators or digital technology are useful here, but they are still black boxes. Once integration has been done, however, we reach the simple definite integral

$$\int_1^e \frac{1}{x} dx = 1.$$

Graph paper and counting squares can now be used to approximate  $e$ , and one can even redefine  $e$  in terms of this identity. Thus the Glossary item should be

*Euler's number: This number is  $e \doteq 2.71828$ .*

*Informal definition: Euler's number  $e$  is the base of the exponential function  $e^x$  such that  $\frac{d}{dx}(e^x) = e^x$ .*

*Formal definition: Euler's number  $e$  is defined by  $\int_1^e \frac{1}{x} dx = 1$ .*

**Digital technology is not required:** As mentioned above, there is no need for digital technology here (although some teachers may prefer it), and the text should always say, ‘or otherwise’.

**Some later dot-points seem repeats:** Some of the later dot-points on page 40 seem repeats of earlier dot-points. The problem seems related to the fact that  $e^x$  and its derivative is introduced in the middle of the content dot-points, after which logarithms to other bases are resumed. Cutting-and-pasting, and poor editing, seems the cause.

**The primitive of  $1/x$ :** As mentioned above, the establishing of a primitive of  $1/x$  is one of our two motivations for this group of special functions (the other being the finding of a function that is its own derivative). This major achievement is not even mentioned in the Section MA-C4 on integration, which means that little thought has gone into producing a coherent account of this difficult material. The solution is to introduce integration much earlier using only algebraic functions, as indicated above.

The Drafts introduce a further problem in their assertion that the primitive of  $1/x$  is  $\log|x|$ . The current syllabuses make the very wise decision to restrict  $x$  to positive real numbers:

$$\int \frac{1}{x} dx = \ln x, \quad x > 0 \quad (\text{current examination papers, including Extension 2}),$$

but the Drafts state the primitive as

$$\int \frac{1}{x} dx = \ln|x| + c, \quad \text{for } x \neq 0 \quad (\text{Advanced, page 56}).$$

1. This unnecessary complication will be a stopper for most Advanced and Extension 1 students.
2. In any definite integral,  $x$  cannot cross the origin because  $x = 0$  is not in the domain.
3. The definite integral  $\int_{-2}^{-1} \frac{1}{x} dx$  is, by symmetry,  $-\int_1^2 \frac{1}{x} dx$ .
4. The assertion in the Drafts is actually wrong. There is no connection between the constants of integration in the two disconnected regions, so the correct form should be:

$$\int \frac{1}{x} dx = \begin{cases} \ln x + c_1, & \text{for } x > 0, \\ \ln(-x) + c_2, & \text{for } x < 0, \end{cases} \quad \text{where } c_1 \text{ and } c_2 \text{ are constants.}$$

We surely don't want this sort of thing in the courses.

### Differential equations in the Extension 1 course — ME-C3

I believe that differential equations are far too sophisticated for school, where instead systematic integration should be pushed as far as is reasonable. The only exceptions are exponential growth and decay, and the demonstration that  $\sin nx$  and  $\cos nx$  satisfy  $\frac{d^2y}{dx^2} = -n^2y$ .

The Draft looks reasonable in introducing only one method — separation of variables. The problem is that by omitting integrating factors, the solutions of even the simplest proper DE  $\frac{dy}{dx} = y$  becomes unbearably complicated. In fact, students are being taught quite the wrong approach to it:

Using separation of variables,

$$\frac{dy}{dx} = y$$

$$dx = \frac{dy}{y}$$

$$x = \ln|y| + c, \quad \text{for some constant } c$$

$$x = \begin{cases} \ln y + c, & \text{for } y > 0 \\ \ln(-y) + c, & \text{for } y < 0 \end{cases}$$

$$y = e^{x-c} \text{ or } y = -e^{x-c}$$

$$y = e^{-c}e^x \text{ or } y = -e^{-c}e^x$$

so  $y = Ae^x$ , for some constant  $A \neq 0$ .

We note also that  $A = 0$  yields a solution,

so  $y = Ae^x$ , for some constant  $A$ .

Using the integrating factor  $e^{-x}$ ,

$$\frac{dy}{dx} = y$$

$$-e^{-x}y + e^{-x}\frac{dy}{dx} = 0$$

$$\frac{d}{dx}(e^{-x}y) = 0$$

$$e^{-x}y = A, \quad \text{for some constant } A$$

$$y = Ae^x.$$

The left-hand column uses the form given in the Drafts of the primitive of  $1/x$ , and ignores other logical problems, which I know by experience that more able Extension 2 students will bring up and argue endlessly over.

DEs should be omitted from school, and this difficult material left to the universities. Was the  $\log|x|$  primitive of  $1/x$  added so that separation of variables could be carried out?

**DEs in ME-C3 and rates of change in ME-C1:** What is the relationship between this section ME-C3 and the earlier Extension 1 section ME-C1 on rates of change? Are the methods used in ME-C3 to solve a DE meant to supersede the methods used in the earlier section? There is no comment on the matter, so one assumes that it has not been considered.

### Systematic differentiation and integration in sections MA-C2, MA-C3 and MA-C4

This is all a terrible muddle, first because trigonometric functions and mixed up with exponential and logarithmic functions, and secondly because the required standard forms are not given.

**Subsection C2.1:** This contains three things:

- ★ The first steps in differentiating the trigonometric functions.
- ★ One further very trivial step in differentiating exponential functions, namely  $\frac{d}{dx}(ke^x)$ .
- ★ The first step in differentiating  $\ln x$ .

This is the first time students have used calculus with the trigonometric functions. A sustained treatment is needed, not just getting the formula and moving on to exponential functions. And what is the Draft doing presenting this very new material without relating it to the graphs of a multitude of trigonometric functions in all the ways so far established?

**Subsection C2.2:** This is only the Advanced course. What are needed are the standard forms for differentiating  $\sin(ax + b)$ ,  $e^{ax+b}$  and so forth. Students should be turning to the chain rule setting-out, not to a formula, to differentiate objects such as  $\sin f(x)$ .

And again, where are the graphs that illustrate all these things? The presentation of the material has just not been thought through.

**Subsection C4.1:** These integrals are completely unsystematic in their presentation. We have  $ax + b$  forms for the trigonometric and exponential functions, but the full reverse chain rule for power and logarithmic functions. There is good reason for doing this eventually with logarithmic functions, but not at the start, and Advanced students can't handle the reverse chain rule for power functions. And if the  $ax + b$  forms are used for the trigonometric functions, why were their differential versions not in the differentiation section?

**Extension 1 rates of change in Section ME-C1:** How can Extension 1 deal with exponential functions in the Year 11 rates of change section when the differentiation of  $e^{kx}$  is only in Year 12 of the Advanced course?

### Arithmetic and geometric sequences — MA-M1

First, this topic is not part of financial mathematics — financial mathematics is an application of it. Sequences must be separated out of M1.2 and given its own topic. It would be far preferable to have it in Year 11 before starting calculus, because:

1. A series is a discrete integral.
2. An AP with constant difference is a discrete analogy of a linear function whose derivative is a constant, and a GP with constant ratio is a discrete analogy of an exponential function whose derivative is a multiple of itself. These things have only the briefest mention in the Drafts.
3. The difference-of- $n$ th-powers identity is the most straightforward way to establish the derivative of  $x^n$ .

**Series:** The Glossary definition is confused:

*'A series is the sum of the terms of a particular sequence. The  $n$ th term of a series is the sum of the first  $n$  terms of the related sequence.'*

The second sentence is correct, but the first sentence looks as if one could take the sum of all the terms of an infinite sequence. The language of partial sums makes things straightforward:

*Series: The  $n$ th partial sum of a sequence is the sum of the first  $n$  terms.*

*The resulting sequence of partial sums is called the series associated with the sequence.*

**Arithmetic and geometric means:** These are important ideas and need attention here, to show how they create an AP or GP with three terms (the insertion of more than one mean, as in the current syllabuses, has proven unrewarding).

The important connection here is with the exponential and logarithmic functions, which convert between an arithmetic mean to a geometric mean and vice versa. It would not make sense to lose that connection. In particular, financial mathematics uses both means — for example, if an item costs \$1000 and \$100 000 at two dates due to constant inflation, then its cost midway between these two dates is \$10 000, not \$50 500.

### Financial mathematics — MA-M1

As indicated above, this should be a far broader topic, involving first general rates of change, and secondly, exponential rates of change. The exponential form section should develop far more clearly the relationship between exponential functions and GPs. For example, assuming a constant inflation rate of 4% per annum, a GP can find the future cost of a car in 5 years time, but an exponential function (ignoring fancy interpolation) is required to find its value in  $5\frac{1}{4}$  years time.

Superannuation and housing loans have proven extremely difficult for Advanced and Extension 1 students. They certainly can't jump into them without earlier more straightforward rates of change.

### The normal distribution — MA-S5

This section is now teachable, although it will be very difficult in Advanced. First, the idea of a continuous random variable is introduced and developed beforehand in S5.1. Secondly, the formula for the normal distribution is given, so the topic can be properly linked with functions and calculus. There are still a number of problems, however.

#### Probability density function:

Without a clear definition of a probability density function, nothing will ever be clear. Nowhere here, or in the Glossary, is it stated that a probability density function is a function  $f(x)$  with the two properties:

$$f(x) \geq 0, \text{ for all real } x \quad \text{and} \quad \int_{-\infty}^{\infty} f(x) dx = 1 .$$

Page 64 seems to be dealing with PDFs defined only on a closed interval  $[a, b]$ . One can do this, of course, but because the normal distribution is the target, surely it is better only to consider functions defined for all real numbers. In any case, MA-S5 does not make any transition from closed intervals to the whole real line, so everyone will be confused.

It's no good being squeamish about integrals from  $-\infty$  to  $\infty$ , because they are fundamental to the normal distribution, and if they are not explained, then everything is unclear.

**Digital technology** should not be specified for sketching  $y = e^{-\frac{1}{2}x^2}$ . There is no problem differentiating it twice and applying the systematic curve-sketching methods to find its asymptote, its symmetry, and its inflexions. Once again, the Drafts develop methods, and then fail to use them.

**The definite integral of  $e^{-\frac{1}{2}x^2}$ :** The fact that the definite integral of  $e^{-\frac{1}{2}x^2}$  over  $(-\infty, \infty)$  is  $\sqrt{2\pi}$  cannot be proven (except as a last-question-in-the-4-Unit-paper-type question for the most able Extension 2 students). But the fact must be clearly stated in the text, which it is not at the moment. Without this fact, the formula for the normal distribution is just a ‘black box’.

**Mean:** The mean has not been defined here or in the Glossary. The Glossary further complicates matter by referring to ‘expected value’, and then fails to give any definition there either. There is no problem with:

*Mean or expected value: The mean or expected value of a continuous random variable  $X$  with probability density function  $f(x)$  is*

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx .$$

For the normal distribution  $y = e^{-\frac{1}{2}x^2} / \sqrt{2\pi}$ , this integral is immediately zero by symmetry.

**Variance:** Similarly, variance is also not defined here or in the Glossary. Most annoyingly, it is defined as the square of the standard deviation, which in turn is defined as the square of the variance (if these statements are intended as definitions, which is unclear, as so often in the Glossary).

The definition should be given, and its analogy explained to variance of discrete distributions, defined already in MA-S3 (although that section in its present form should go — see below). Using  $\mu$  for the mean:

$$\text{Var}(X) = E((X - \mu)^2) = E(X^2) - \mu^2 = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2 .$$

This can be calculated for simple probability density functions. Students will need to be told that this is 1 for the normal distribution, although Extension 2 students will later have the machinery of integration by parts to prove it. The Drafts need to state this assumption clearly.

**$z$ -scores and the definition of standard deviation:** How can they find the  $z$ -score of a sample if they have no definition of standard deviation?

Whether students use the formula or technology for standard deviation, the question will immediately arise as to whether they divide by  $n$  or by  $n - 1$ . Calculators usually have both buttons, so students will be asking this question. It may or may not be reasonable to divide only by  $n$  at school, but the issue must be mentioned, and reasonable teaching instructions given and justified. The matter seems not to have been considered.

## Probability — MA-S2

Why the introduction of conditional probability? My concern here is for the Advanced student who needs probability at university. It will not be conditional probability that will hold such a student up, but the lack of knowledge of  ${}^n C_r$ . I would prefer to see a moderate course in binomial probability, with counting involving the multiplication principle, and reaching  ${}^n C_r$  but no further. Thankfully, the quite unsuitable material that was in the July Drafts is now gone from Advanced.

The probability and conditional probability that is presented here, however, is incoherent. The principal reasons for this are that there are no definitions of any of the significant ideas, but only formulae that are obtained from goodness knows where.

**Probability is undefined:** There is no definition of probability here. Perhaps it is being defined on page 44 as the limit of the relative frequency as the number of trials increases, which would not be not a satisfactory way to proceed, but that seems unlikely because relative frequency is being distinguished from a concept called ‘theoretical probability’, not that any other sort of probability seems to be discussed here. The Glossary is no help because the word ‘probability’ is missing.

The term ‘sample space’ occurs over the page, so perhaps sample space and event space is what is intended? If so, some set theory is needed as follows, if the ludicrous rules of outcomes would ever allow it to be written into the text:

- *Suppose that the results of an experiment can be divided into a finite number of equally likely possible outcomes, meaning that we have no reason to believe that any one of these outcomes is more likely than any other. Then the set  $S$  of equally likely possible outcomes is called the sample space of the experiment.*
- *In such a situation, any subset  $A$  of those outcomes is called an event, and the probability of that event is defined to be*

$$P(A) = \frac{|A|}{|S|}.$$

In the Glossary items ‘sample space’ and ‘event space’:

- ★ The word ‘set’ is probably intended to mean ‘finite set’.
- ★ The word ‘possible outcomes’ is probably intended to mean ‘equally likely possible outcomes’, which is a sharply different idea. This is a very serious problem.
- ★ The word ‘random experiment’ is undefined, and causes more problems.

**Some very basic finite set theory** is needed to support this. Specifically Venn diagrams, union, intersection, disjoint sets and complement are needed, and the usual formulae need to be proven for subsets  $A$  and  $B$  of a sample space  $S$ :

$$|A \cup B| = |A| + |B| - |A \cap B| \quad \text{and} \quad |\bar{A}| = |S| - |A|,$$

and the fact that  $A$  and  $B$  are disjoint if and only if  $|A \cup B| = |A| + |B|$ .

**Combinations of events:** Now the notation can be extended to events (although I would far prefer at school that the words ‘and’ and ‘or’ were used instead of the symbols  $\cap$  and  $\cup$ ). They should now be able to ‘*prove and use the rules*’ rather than just ‘*review and use the rules*’:

- *Prove and use the rules for combinations of events  $A$  and  $B$ :*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \text{and} \quad P(\bar{A}) = 1 - P(A).$$

- *Prove and use the fact that two events  $A$  and  $B$  are disjoint if and only if*

$$P(A \cup B) = P(A) + P(B).$$

This will all need quite a lot of subsequent drill to establish the ideas in students’ minds.

**Conditional probability is undefined:** No definition of conditional probability is given, but the intention seems to be to imply it by the phrase, ‘reduced sample space’. What is needed is thus:

- *Define the probability of an event  $A$ , conditional upon an event  $B$  occurring, to be the probability obtained by removing all outcomes of the complementary event  $\bar{B}$  from the sample space. That is,  $P(A|B) = \frac{|A \cap B|}{|B|}$ .*

- *Hence establish and use the formula  $P(A|B) = \frac{P(A \cap B)}{P(B)}$ .*

**Independent events are undefined:** There is a definition, and there is a theorem:

- *Define two events  $A$  and  $B$  with the same sample space as independent if  $P(A|B) = P(A)$ .*

- Hence prove and use the fact that the events  $A$  and  $B$  are independent if and only if  $B$  and  $A$  are independent, and if and only if

$$P(A \cap B) = P(A) \times P(B) \text{ (the multiplication law).}$$

**Tree diagrams and independence:** Finally we have the problem of what ‘independent events’ means when they are successive events on a tree diagram. This problem is almost sufficient to ditch conditional probability entirely at this level. It has not been addressed at all in the last dot-point on page 45 — in fact, the crucial word ‘independent’ is not even used, thus undermining the whole theory of tree diagrams. Typical of these drafts, there is no coherence even within one subsection, and little thought given as to how things hang together.

**Better in Year 12:** This all looks too sophisticated for Year 11.

### The binomial distribution in Extension 1 — ME-A1 and ME-S1

The binomial distribution is a very satisfactory topic in Extension 1, but there are some problems in both the binomial distribution section and the earlier preparatory section on combinatorics.

**Proofs:** There are at least two easily accessible combinatorial proofs of the formula for  ${}^n C_r$ , and the notes should indicate these:

- Count the ordered selections of  $r$  objects, then divide by the number of orderings of  $r$  objects.
- Count the number of ordering of  $r$  Ys (Yesses) and  $n - r$  Ns (Noes).

A minor point: The object on the fourth last line of 38 is not a formula.

**Arrangements in a circle:** This must exclude objects that are not all distinct, as the guardians of the present 2/3/4 Unit course realised a few years ago.

**The Pascal triangle** in the Glossary should not be printed as a disembodied equilateral triangle, but should be given in a form of a table, with rows indexed by  $n$  and columns by  $r$ . Students otherwise have great difficulty identifying the role of the two variables  $n$  and  $r$ .

It would be far more straightforward to describe it as a table of values of  ${}^n C_r$  (defined combinatorially). The subsequent proof that  ${}^n C_r$  is the appropriate coefficient in the binomial expansion is combinatorial. After this, Pascal’s triangle becomes also a table of the coefficients of the binomial expansion.

**Better all in Year 12:** This is difficult work, and combinatorics and the binomial distribution are best studied together, and in Year 12, when students are more mature. One suspects that some doctrine of Strands is insisting that there be a probability or statistics topic in Year 11.

**Simpler probabilities first:** Within or straight after all the counting in ME-S1, simpler probabilities should be calculated — this should work up to and include probability questions involving  ${}^n C_r$ , as is done so effectively in the current 3 Unit course, but not yet talk about the probability distribution. Jumping straight into the binomial distribution without initial work on probabilities would mean that students approach the binomial distribution without any previous intuitions about the sorts of individual probabilities that they are talking about.

**The mean and variance** of the binomial distribution must be proven — they are not hard at this level. But the identities required have been omitted from the treatment of the binomial theorem. In the present 3 Unit course, those identities are there (obtained by differentiating the binomial expansion once and then twice), but are unfortunately unmotivated. Now that they are motivated — correctly — by the distribution, why have they been removed from the syllabus just as they are needed?

**Notation:** Why, at the bottom on page 54, has the symbol  $\bar{x}$  been used for a theoretical mean?

**Approximating the binomial by the normal** should wait for the expertise of proper statisticians at university. It is not appropriate at school.

### **Descriptive statistics — MA-S1**

This review section should be dropped entirely. Drawing all these pictures would waste students' time, blunt their enthusiasm for mathematics, and create contempt for the subject from teachers and students. It would also teach them nothing. In particular, there is still no formula for standard deviation here or in the Glossary, even though it is a quadratic object entirely accessible to methods that students have been studying since Year 8. The section serves no purpose in this course.

### **Bivariate analysis — MA-S4**

This section should also be dropped entirely. Some of it is drawing pictures, and some of it is total 'black-box' mathematics. It has to be black-box, because the formulae, let alone the proofs, are far too difficult for school. Again, this section would waste the time of teachers and students and would bring the subject into disrepute.

### **The examination tail wagging the dog**

There is great suspicion that the very questionable examination arrangements published in July, wanting common questions between Advanced and Standard, are the reason for including these two sections of trivial and black-box material in the courses. If so, this is outrageous.

### **Discrete Probability Distributions — MA-S3**

I really have no idea what is intended here, and what material would actually be taught in the classroom. I note the word 'binomial' in the *Subtopic Focus*, but its absence from the content.

Is it all some kind of typo? Did page 46 get left in the Draft by mistake?

### **General conclusion**

To repeat my remark on page 2 — these Drafts cannot be rescued. A properly constituted Syllabus Committee will need to start again.

Yours sincerely,



Dr Bill Pender

Retired Subject Master in Mathematics, Sydney Grammar School

Cc:

Anna Wethereld

Inspector, Mathematics and Numeracy

NSW Education Standard Authority