

Shore School

4 unit mathematics

Trial HSC Examination 1986

1. (i) (a) Prove that if a polynomial $P(x)$ has a zero of multiplicity m , then $P'(x)$ has a zero of multiplicity $(m - 1)$.
(b) The equation $8x^4 + 12x^3 - 30x^2 + 17x - 3 = 0$ has a root of multiplicity 3. Find all of its roots.
(ii) If in the polynomial $x^3 - a^2x^2 - 2ax - 5$, a is an integer and $x - 5$ is a factor, find the value of a . For this value of 'a' factorise $x^3 - a^2x^2 - 2ax - 5$ over the complex field.
(iii) If α, β, γ are the zero of $2x^3 - 4x^2 - 3x - 1$ find the value of $\alpha^2\beta^2 + \alpha^2\gamma^2 + \beta^2\gamma^2$.
2. (i) Find the equation of the circle with AB as a diameter where A, B have coordinates (x_1, y_1) and (x_2, y_2) respectively.
(ii) At a N.S.W state election 30% of the voters favoured party A. If an interviewer selected 5 voters at random, what is approximately the probability that a majority of those selected favoured party A? Give your answer expressed as a percentage correct to two significant figures.
(ii) Two equal light rods AC, CB are hinged (freely joined) at C , the mass of the hinge being m kilograms. The ends A and B are hinged to two points, A being vertically above B and $AB = a$ metres. If the system revolves with angular velocity ω rad/s about AB , prove that both rods are in tension of $\omega > \sqrt{\frac{2g}{a}}$. If the tensions in the rods are in the ratio 5 : 2, find the period of the motion.
3. (i) If $P(x, y)$ is a general point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and S and S' are the two foci, prove that $PS + PS' = 2a$.
(ii) Show that the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $P(a \sec \theta, b \tan \theta)$ is $\frac{x}{a} \sec \theta - \frac{y}{b} \tan \theta = 1$. If this tangent at P meets the asymptotes of the hyperbola at the two points A and B , show that P is the midpoint of AB .
- 4 (i) If z_1 and z_2 are the roots of the equation $z^2 - 5(1 + i)z + 17i = 0$, show that
(a) the inclination of the line z_1z_2 to the positive direction of the real axis is $\frac{3\pi}{4}$
(b) the area of the triangle Oz_1z_2 is $\frac{15}{2}$ square units.
(ii) If $w = \frac{z-6i}{z+8}$ show that the locus of z is a circle when w is purely imaginary but a straight line when w is real. Find the equations of these loci.
5. (i) Show that $\int_3^6 \frac{dx}{\sqrt{27+6x-x^2}} = \frac{\pi}{6}$
(ii) Evaluate -

- (a) $\int_7^{14} \frac{dx}{(x-2)\sqrt{x+2}}$
 (b) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos 4x \cdot \cos 2x \, dx$
 (iii) Find $\int \sqrt{1+x^2} \, dx$

6. (i) Sketch, using the Calculus, $y = \frac{3x}{(x-1)(x-4)}$ showing all asymptotes and all turning points. (There is no need to find the point of inflexion). Distinguish clearly the stationary points.

(ii) A particle of mass 1 kg is projected from a point O with velocity u m/s along a smooth horizontal table in a medium whose resistance is RV^2 Newtons when the particle has velocity V m/s, R being constant. Find its velocity as a function of t . An equal particle is projected from O simultaneously with the first particle but vertically upwards under gravity with velocity u in the same medium. Show that the velocity V of the first particle when the second is momentarily at rest is given by $\frac{1}{V} = \frac{1}{u} + \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right)$ where $Ra^2 = g$.

7. (i) (a) Show that $\int \frac{dx}{5-4\cos x} = \frac{2}{3} \tan^{-1}\left(3 \tan \frac{x}{2}\right) + C$

(b) Given that $\int_0^\pi \frac{dx}{5-4\cos x} = \frac{\pi}{3}$ show that $\int_0^\pi \frac{\cos x \, dx}{5-4\cos x} = \frac{\pi}{6}$

(ii) If $u_n = \int_0^\pi \frac{\cos nx}{5-4\cos x} \, dx$ show that $u_{n+1} + u_{n-1} - \frac{5}{2}u_n = 0$. Using part (i) find the values of u_2 and u_3 .

(iii) If $p+q = 1$ and $p^2+q^2 = 2$ determine the values of p^3+q^3 and p^4+q^4 without finding the value of p and q .

8. (i) The flat base of a solid is the region bounded by two branches of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and the lines $y = \pm a$. Cross-sections of the solid, perpendicular to the y -axis are equilateral triangles (the ends of their bases are on the hyperbola). Show that the volume of the solid is $\frac{2\sqrt{3}a^3(3b^2+a^2)}{3b^2}$ cubic units.

(ii) Given that $u_n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots 2n}$ for integers $n \geq 1$, prove by Mathematical Induction that $u_n \geq \frac{1}{2\sqrt{n}}$.

(iii) Prove that the expression $\frac{x-a}{x^2-2x+a}$ (for a, x real) can assume all values if $0 < a < 1$.