

Shore - Sydney Church of England Grammar School

4 unit mathematics

Trial HSC Examination 1993

1. Evaluate the following integrals. Give your answers correct to 3 significant figures.

(a) $\int_3^4 \frac{4}{x^2-3x+2} dx$ (b) $\int_1^2 2^x dx$ (c) $\int_0^1 \sin^{-1} x dx$ (d) $\int_0^{\frac{\pi}{2}} \frac{dx}{2+\sin x}$ (e) $\int_0^{1.5} \sqrt{9-x^2} dx$

2. (a) For the hyperbola $\frac{x^2}{144} - \frac{y^2}{25} = 1$ find:

(i) the eccentricity

(ii) the coordinates of the foci

(iii) the equations of the asymptotes

(iv) the equations of the directrices

Sketch the graph showing the above information.

(b) $P(a \cos \theta, b \sin \theta)$ is any point on the ellipse whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and S is the focus of the ellipse. Prove that the line through S perpendicular to the tangent at P , and the line OP produced, meet on the directrix corresponding to the focus S .

3. (a) Solve for z : $z + \frac{2+8i}{z} = 4 + i$

(b) Express $w = \frac{(-1+i\sqrt{3})(1+i)}{\sqrt{3}-i}$ in modulus-argument form.

(c) Given that $(2-i)$ is a zero of $2x^3 + mx^2 + nx + 15$, determine m and n , where m and n are real. Hence factorise $2x^3 + mx^2 + nx + 15$ in the real field.

4. (a) Determine all the roots of $8x^4 - 25x^3 + 27x^2 - 11x + 1 = 0$ given that it has a root of multiplicity 3.

(b) α, β, γ are the roots of $x^3 + 2x^2 - 3x + 4 = 0$

(i) Evaluate $\alpha^2 + \beta^2 + \gamma^2$

(ii) Evaluate $\alpha^3 + \beta^3 + \gamma^3$

(iii) Find the equation whose roots are $\frac{\alpha\beta}{\gamma}, \frac{\alpha\gamma}{\beta}, \frac{\beta\gamma}{\alpha}$.

5. (a) Let $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x dx$, where n is an integer. Show that $I_n = \frac{1}{n-1} - I_{n-2}$ and hence evaluate I_7 .

(b) (i) Write down the general solution of $\tan 4\theta = 1$.

(ii) Use De Moivre's theorem to express $\cos 4\theta$ and $\sin 4\theta$ in terms of $\cos \theta$ and $\sin \theta$.

Hence show that $\tan 4\theta = \frac{4 \tan \theta - 4 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta}$.

(iii) Find the roots of the equation $x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$ in trigonometric form. Hence show that $\tan^2 \frac{\pi}{16} + \tan^2 \frac{3\pi}{16} + \tan^2 \frac{5\pi}{16} + \tan^2 \frac{7\pi}{16} = 28$.

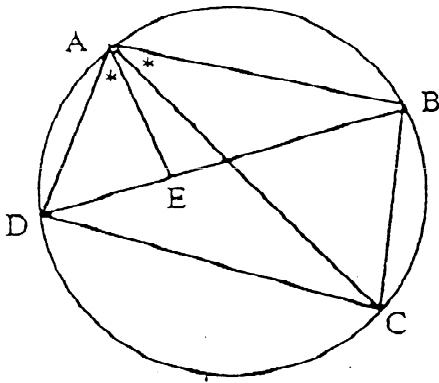
6. (a) A solid has a base in the form of a circle with centre the origin and radius 6 units. If every section perpendicular to the x -axis is an equilateral triangle, show that the volume of the solid is $288\sqrt{3}$ cubic units.

(b) The region bounded by the curve $y = x^3$, the line $y = 1$ and the y -axis is rotated about the line $y = -1$. By noticing that rectangular strips taken parallel to the axis of rotation give rise to cylindrical shells, find the volume of the solid of revolution.

7. (a) A quiz consists of twenty True-False questions. Find the chance that someone who knows the correct answers to ten of the questions, but answers the remaining ones by tossing a coin, will obtain a score of at least 85% on the quiz.

(b) By using mathematical induction prove that: $\sin(n\pi + x) = (-1)^n \sin x$ for all positive integral values of n .

(c)



$ABCD$ is a cyclic quadrilateral. E is a point on diagonal BD , such that $\angle DAE = \angle BAC$. Prove that:

(i) $AB \cdot CD = AC \cdot BE$ (ii) $BC \cdot DA = AC \cdot DE$ (iii) $AB \cdot CD + BC \cdot DA = AC \cdot BD$

8. (a) Prove that $\frac{a^2+b^2}{2} > (\frac{a+b}{2})^2$, where a and b are positive, real and unequal.

(b) Sketch $y = 1 + x^2$ and hence sketch on separate diagrams: (do not use calculus)

(i) $y = \frac{1}{x^2+1}$ (ii) $y = \frac{x}{x^2+1}$ (iii) $y = |\frac{x}{x^2+1}|$ (iv) $y = \pm \sqrt{\frac{x}{x^2+1}}$

(c) For the rational function $F(x) = \frac{x^4}{x^2-1}$

(i) Find if $F(x)$ is odd or even or neither.

(ii) Show algebraically that the range of $F(x)$ is: $y \leq 0$ or $y \geq 4$. Hence calculate the coordinates of its three turning points.

(iii) Considering large values of $|x|$ and any discontinuities, sketch the graph of $y = F(x)$. Show also the curved asymptotes by dotted lines.