



SAINT IGNATIUS' COLLEGE

Trial Higher School Certificate

2007

MATHEMATICS EXTENSION 2

12:30pm – 3:35pm

Tuesday 28th August 2007

Directions to Students

• Reading Time : 5 minutes	• Total Marks 120
• Working Time : 3 hours	
• Write using blue or black pen. (sketches in pencil).	• Attempt Question 1 – 8
• Board approved calculators may be used	• All questions are of equal value
• A table of standard integrals is provided at the back of this paper.	
• All necessary working should be shown in every question.	
• Answer each question in the booklets provided and clearly label your name and teacher's name.	

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Question 1 (15 marks) Use a SEPARATE writing booklet

Marks

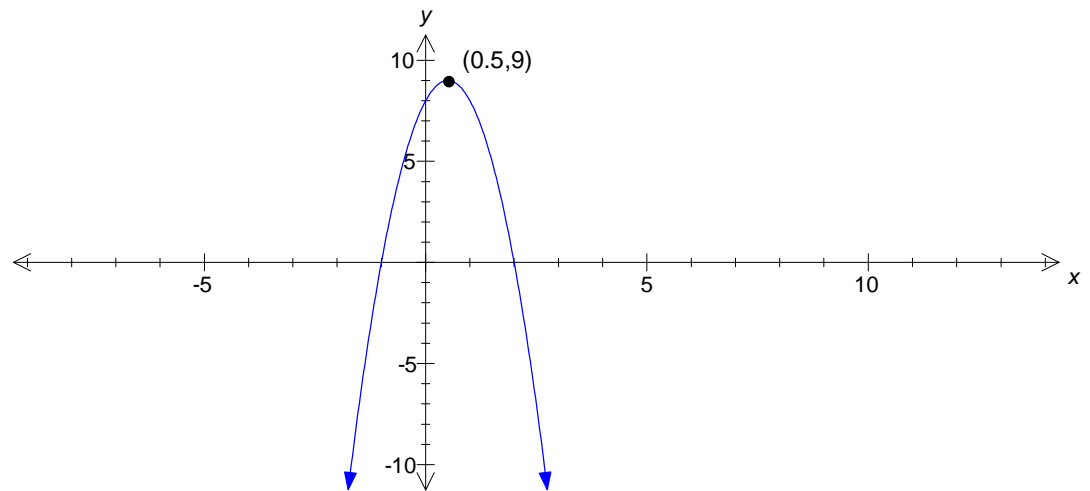
- (a) (i) Show that $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$ 1
- (ii) Hence find the indefinite integral $\int \sin 5x \cos 3x dx$ 2
- (b) Evaluate $\int_0^5 \frac{t dt}{\sqrt{t+4}}$ 2
- (c) Evaluate $\int_{-1}^1 3^x dx$ correct to three significant figures. 2
- (d) Evaluate $\int_0^{\frac{1}{3}} \frac{dx}{\sqrt{1-9x^2}}$ 2
- (e) Find $\int \frac{1}{1+\sin x} dx$, using the substitution $t = \tan \frac{x}{2}$ 3
- (f) Use integration by parts to find $\int \frac{\cos^{-1} x}{\sqrt{1+x}} dx$ 3

Question 2 (15 marks) Use a SEPARATE writing booklet

Marks

(a) Prove that $f(x) = \frac{x^3}{\sin x}$ is an even function. 2

(b) A sketch of $f(x) = -4(x+1)(x-2)$ is shown below.



With the aid of the above diagram, and without the use of calculus, draw a separate half page sketch for each of the following.

(i) $y = |f(x)|$ 1

(ii) $y = f(2x)$ 1

(iii) $y = f(-x)$ 1

(iv) $y = \frac{1}{f(x)}$ 2

(v) $y = \sqrt{f(x)}$ 2

(vi) $y = \log_e f(x)$ 2

(c) Using calculus, show that $e^{-x} + x - 1 \geq 0$ for real x . 4

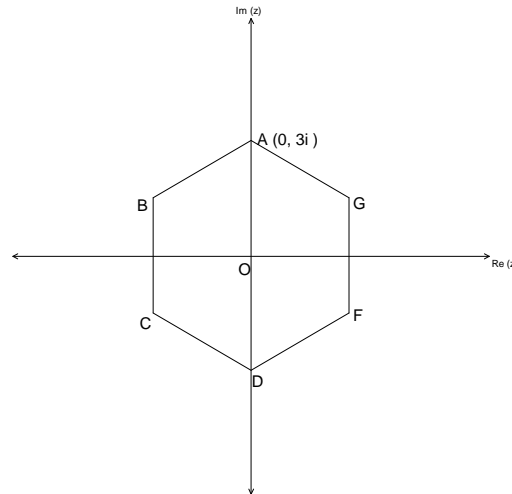
Question 3 (15 marks) Use a SEPARATE writing booklet

Marks

- (a) Express the following in the form $(x + iy)$, where x and y are real: 2

$$\frac{i^2 - 1}{i} + \frac{1}{1+i}$$

- (b)



The Argand diagram above, shows a regular hexagon with vertex A at the point $(0, 3i)$. O is the centre of the hexagon.

- (i) Copy the diagram into your writing booklet.
- (ii) On your diagram show the region within the hexagon in which both the inequalities $|z| \leq 2$ and $-\frac{\pi}{6} \leq \arg z \leq \frac{\pi}{6}$ are satisfied. 2
- (iii) Find in the form $|z - z_1| = R$, the equation of the circle through the points O , B and F . 1
- (iv) Find the complex numbers, in modulus argument form, represented by the points B and C . 2
- (v) The hexagon is rotated anticlockwise about the origin through an angle of $\frac{\pi}{4}$. Express in the form $r(\cos \theta + i \sin \theta)$, where θ is the principal argument, the complex numbers represented by the new positions of B and C . 3

Question 3 continues on page 5

Question 3 (continued)

- (c) If $1 - 2i$ is a root of the equation $z^2 - (3+i)z + k = 0$,
- (i) explain why the conjugate $1 + 2i$ cannot be a root to the equation 1
 - (ii) show that the other root is $2 + 3i$ 1
 - (iii) find the value of k 1
 - (iv) hence, or otherwise, find the two square roots of $-24 + 10i$. 2

Question 4 (15 marks) Use a SEPARATE writing booklet

Marks

- (a) A polynomial $p(x) = x^n + ax^2 - 2$ has a factor of $(x - 1)$ and leaves a remainder of -6 on division by $(x + 2)$.
Find:
- (i) the value of a 1
 - (ii) the value of n 1
 - (iii) the zeros of $p(x)$. 2
- (b) Find the values of a and b so that $p(x) = 2x^3 - (2a + 1)x^2 + (2 + b)x - 1$ has a double root at $x = 1$. 4
- (c) If l, m, n are the roots of the equation $x^3 - 2x + 5 = 0$,
- (i) find the cubic equation whose roots are $2l, 2m, 2n$. 2
 - (ii) find the value of $l^3 + m^3 + n^3$. 2
- (d) Find all the values of k for which the polynomial equation $3x^4 - 4x^3 + k = 0$ has no real roots. 3

Question 5 (15 marks) Use a SEPARATE writing booklet

Marks

- (a) An ellipse has the equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$. O is the centre of the ellipse and S and S' are the foci.
- (i) Find
- (α) the eccentricity 1
 - (β) the co-ordinates of the foci 1
 - (γ) the equations of the directrices. 1
- (ii) Make a third of a page sketch of the ellipse showing the features found in part (i) 2
- (iii) If $P(x_0, y_0)$ is a point on the ellipse show that $(PS + PS')$ is constant. You may mark point P in quadrant one of the above mentioned diagram. 2
- (iv) Show that the equation of the tangent at $P(x_0, y_0)$ is 2
- $$\frac{xx_0}{25} + \frac{yy_0}{16} = 1.$$
- (v) The tangent at P meets the nearer focus at R . If S is the nearer focus to P ,
- (α) write down the co-ordinates of R . 1
 - (β) find expressions for the gradients of PR and SR in terms of x_0 and y_0 . 2
 - (γ) show that the angle PSR is a right angle. 1
- (b) A hyperbola has its centre at the origin and asymptotes $y = \pm \frac{2}{3}x$. Find its equation. 2

Question 6 (15 marks) Use a SEPARATE writing booklet

Marks

- (a) The region bounded by the curve $y = x(2 - x)$ and the x -axis is rotated about the y -axis. Find the volume of the solid of revolution by taking slices perpendicular to the y -axis. 4
- (b) The region bounded by the curve $y = \ln x$, the line $y = 1$ and the co-ordinate axes is rotated about the x -axis.
- (i) By dividing the resulting solid into cylindrical shells, show that each shell has an approximate volume : $\delta v = 2\pi ye^y \delta y$ where δy is the thickness of the shell. 2
- (ii) Hence calculate the volume of the solid. 2
- (c) The base of a particular solid is the circle $x^2 + y^2 = 8$. Find the volume of the solid if every cross section to the x -axis is an isosceles – right angled triangle with the hypotenuse in the base of the solid. 4
- (d) Show that the straight line $lx + my + n = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $a^2l^2 - b^2m^2 = n^2$. 3

Question 7 (15 marks) Use a SEPARATE writing booklet

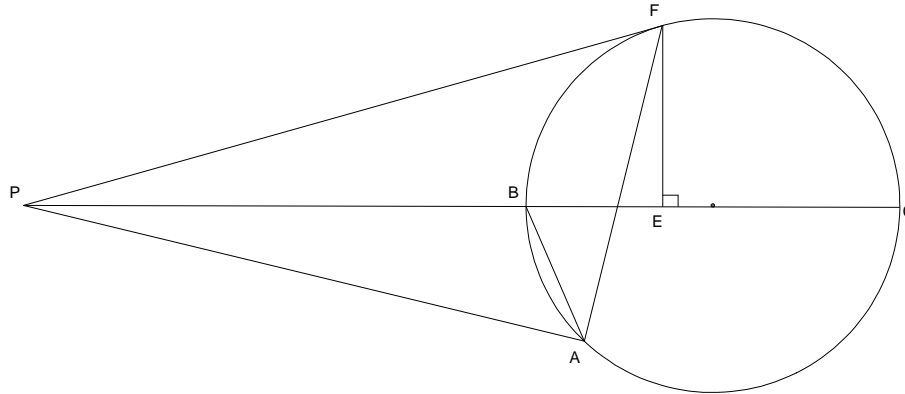
Marks

- (a) Mr Kirkpatrick's mathematics class brought him a ride in a Gondola at Queenstown in New Zealand, during a recent trip. When Mr Kirkpatrick's hands were H metres above the Earth's surface, he dropped overboard his packet of beer nuts of mass m kg. The packet of beer nuts encounters air resistance proportional to its velocity v (which is in metres per second), that is the resistive force is equal to mkv .
Taking Mr Kirkpatrick's hands as the origin and downwards displacement as positive:
- (i) Write down an equation of motion representing the passage of the packet of beer nuts. 2
- (ii) Find the terminal velocity, w , of the packet of beer nuts. 1
- (iii) Show that the equation of motion in part (i) can be written as $\ddot{x} = k(w - v)$. 1
- (iv) Show that the displacement, x metres, of the packet of beer nuts from Mr Kirkpatrick's hands is given by: $x = -\frac{v}{k} - \frac{w}{k} \ln\left(\frac{w-v}{w}\right)$. 4
- (v) If the packet reaches the Earth's surface with a velocity of u metres per second, show that $\ln\left(1 - \frac{u}{w}\right) + \frac{u}{w} + \frac{kH}{w} = 0$. 1
- (vi) Consider the moment when the packet of beer nuts has reached 75% of its terminal velocity.
Find:
- (α) the time, t seconds, for this moment to be reached. 3
- (β) the distance fallen at this moment. 3

Question 8 (15 marks) Use a SEPARATE writing booklet

Marks

- (a) Draw a neat half page sketch of the graph for $y^2 = x^2(4 - x^2)$. 3
- (b)



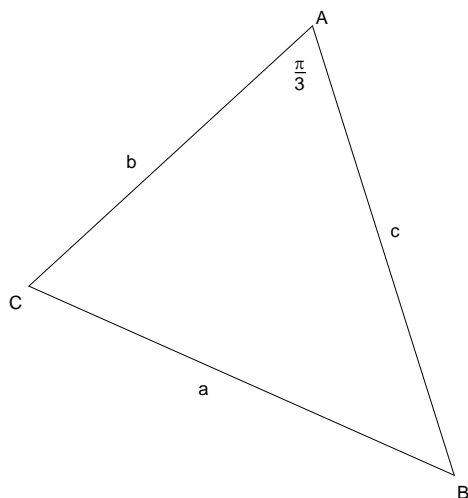
In the above diagram AB and BC are chords of a circle, and F is on the arc ABC such that arc AF is equal to arc FC . E is the foot of the perpendicular from F to the chord BC . CB is extended to P so that $PE = EC$. (Note that B is inside the triangle APF)

- (i) Show that the triangle APF is isosceles. 3
- (ii) Show that $AB + BE = EC$. 4

Question 8 continues on page 11

Question 8 (continued)

(c)



A triangle ABC has sides of varying length a , b and c with a fixed interior angle of $BAC = \frac{\pi}{3}$ as shown in the above diagram.

Use the cosine rule to show that:

- (i) $a^2 \geq bc$, and hence, 3
- (ii) the area of triangle $ABC \leq \frac{a^2\sqrt{3}}{4}$ 2

End of examination