



2008
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

Extension 1

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Use a separate answer booklet for each question

Total marks (84)

- Attempt Questions 1-7
- All questions are of equal value

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Question 1 (12 marks)

Marks

(a) Evaluate $\int_0^{2\sqrt{3}} \frac{3}{4+x^2} dx$

2

(b) Evaluate $\lim_{x \rightarrow 0} \frac{\sin(2x)}{3x}$

2

(c) The point $P(x, y)$ divides the interval joining $A(5, 3)$ to $B(-1, 0)$ externally in the ratio $2 : 5$.

2

Find the coordinates of the point P

(d) Solve $\frac{5}{2x-1} \leq 1$

3

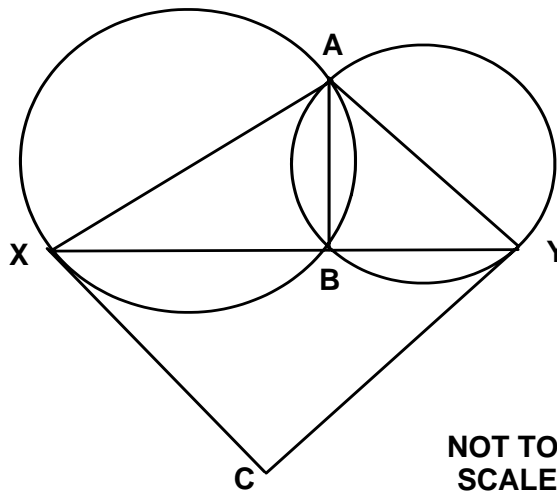
(e) Use the substitution $u = 2 - x$ to evaluate $\int_0^1 \frac{x}{2-x} dx$

3

Question 2 (12 marks) – Use a SEPARATE writing booklet

Marks

- (a) State the domain and range of $y = 2\cos^{-1}\left(\frac{x}{3}\right)$ 2
- (b) Find the size of the acute angle between the lines whose equations are $y = 2x - 3$ and $y = 4 - 3x$ 2
- (c) Find the coefficient of the term in x^3 in the expansion of $\left(\frac{1}{x^2} - x\right)^9$ 2
- (d) (i) Find the $\frac{d}{dx}(x^2 \ln x)$ 1
- (ii) Hence (or otherwise) find $\int x \ln x dx$ 2
- (e) In the diagram below, AB is a common chord of the two circles.
A straight line through B intersects the circles at X and Y as shown.
The tangents to the circles at X and Y intersect at C .
Copy or trace the diagram onto your own paper.
Let $\angle BYC = \alpha$ and $\angle CXY = \beta$



- (i) Explain why $\angle BYC = \angle BAY$ 1
- (ii) Hence, prove that $AXCY$ is a cyclic quadrilateral 2

Question 3 (12 marks) Use a SEPARATE writing booklet

- | | Marks |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|
| (a) Find $\int \cos^2\left(\frac{x}{2}\right)dx$ | 2 |
| (b) Find the derivative of $\sin^4 3x$ | 2 |
| (c) The function $f(x) = x - \cos x$ has a zero near $x = 0.7$
Taking $x = 0.7$ as a first approximation, use one application of Newton's method to find a second approximation to the zero.
Give your answer correct to three decimal places. | 2 |
| (d) Let $P(x) = x^3 + 3x^2 + Ax + B$.
$(x + 2)$ is a factor of $P(x)$.
When $P(x)$ is divided by $(x - 1)$, the remainder is 9.
Find the values of A and B. | 3 |
| (e) Use mathematical induction to prove that $9^n - 4^n$ is divisible by 5 for all integers $n \geq 1$ | 3 |

Question 4 (12 marks) Use a SEPARATE writing booklet

- | | |
|-------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------|
| (a) (i) Express $\sin \theta + \sqrt{3} \cos \theta$ in the form $R \sin(\theta + \alpha)$ where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$ | Marks
2 |
| (ii) Hence, or otherwise, solve $\sin \theta + \sqrt{3} \cos \theta = 1$ for $0 \leq \theta \leq 2\pi$ | 2 |

(b) Consider the function $y = \frac{1}{x^2 - 4}$.

You are given that this is an even function.

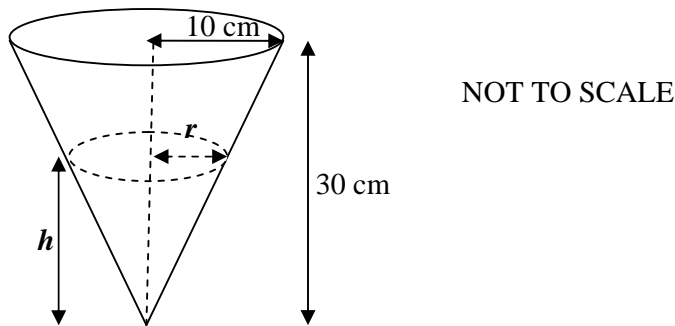
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|------------------------------------------------------------------------------------------------------------------------------------------------------|----------|
| (i) Make a neat sketch of the graph of $y = \frac{1}{x^2 - 4}$, clearly showing any asymptotes and points of intersection with the coordinate axes. | 3 |
|------------------------------------------------------------------------------------------------------------------------------------------------------|----------|

- (ii) Hence, or otherwise, determine the values of k so that the equation

$\frac{1}{x^2 - 4} = k$	1
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has solutions that are real and different.

(c)



The sketch shows a conical container whose height is 30 cm and radius 10 cm.

The container is initially full of water.

The water now drains out of the container at a constant rate of $2\pi \text{ cm}^3$ per second.

After t seconds, the height of the water in the container is h cm and the radius of the surface of the water is r cm, as shown.

- | | |
|------------------------------------------------------------|----------|
| (i) Using similar triangles, show that $r = \frac{h}{3}$ | |
| (Note: you do NOT need to prove the triangles are similar) | 1 |

- | | |
|----------------------------------------------------------------------------------------------------------------------|----------|
| (ii) Find the rate at which the height of water in the container is decreasing when the height of the water is 3 cm. | 3 |
|----------------------------------------------------------------------------------------------------------------------|----------|

(Volume of a cone = $\frac{1}{3}\pi r^2 h$)

Question 5 (12 marks) Use a SEPARATE writing booklet

Marks

- (a) Let α , β and γ be the roots of $x^3 + 4x^2 + kx - 36 = 0$
- (i) Find the value of $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$ 2
- (ii) Given that two of the roots are equal in magnitude but opposite in sign, find the third root and hence find the value of k 2

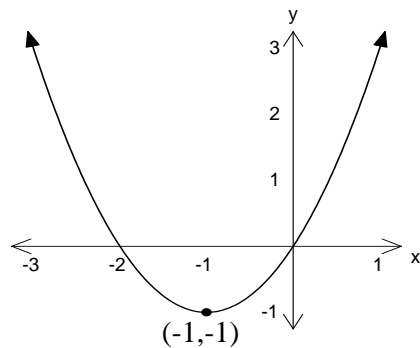
- (b) A cup full of hot water is cooling in a room where the temperature is a constant 20°C .

At time t minutes, its temperature is decreasing according to the equation

$$\frac{dT}{dt} = -k(T - 20), \text{ where } k \text{ is a positive constant.}$$

- (i) Show that $T = 20 + Ae^{-kt}$ satisfies the above equation. 1
- (ii) The initial temperature of the cup of water is 80°C . 3
 After 10 minutes its temperature has decreased to 50°C .
 Find the temperature of the cup of water after 20 minutes. Give your answer correct to the nearest degree.

(c)



The sketch shows the graph of $f(x) = x^2 + 2x$

- (i) Explain why $f(x)$ does not have an inverse function. 1
- (ii) Let $g(x) = x^2 + 2x$ where $x \geq -1$ 1
 State the domain of $g^{-1}(x)$, the inverse function of $g(x)$
- (iii) Find an expression for $y = g^{-1}(x)$ in terms of x 2

Question 6 (12 marks) Use a SEPARATE writing booklet

Marks

(a) A body is moving with simple harmonic motion along the x -axis.

Its velocity, $v \text{ ms}^{-1}$, is given by $v^2 = 8 - 2x - x^2$, where x is in metres.

(i) Find the endpoints of the motion

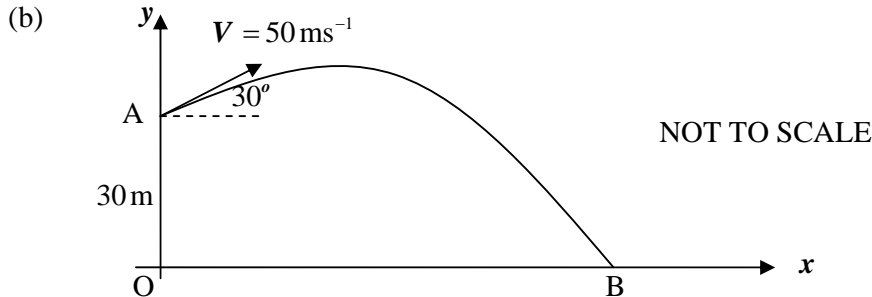
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(ii) Find the maximum speed of the body.

1

(iii) Find an expression for the acceleration of the body in terms of x .

2



A body is projected from a point 30 metres above level ground with velocity 50 ms^{-1} and inclined at an angle of 30° to the horizontal as shown in the sketch. The body lands on the ground at the point B.

The equations of motion of the body are $\ddot{y} = -10$ and $\ddot{x} = 0$

Hence the vertical and horizontal displacements of the body are given by

$$y = 30 + 25t - 5t^2 \quad \text{and} \quad x = 25\sqrt{3}t$$

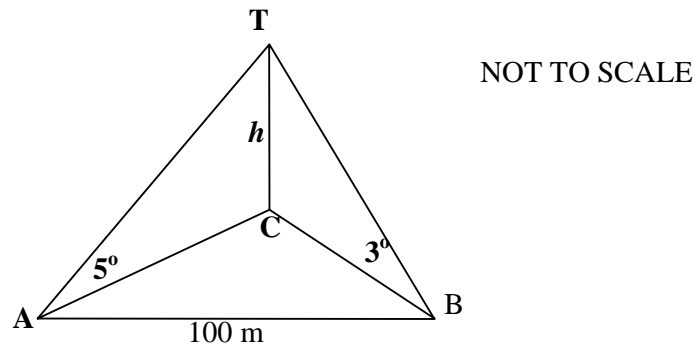
(i) Find the maximum height above ground level that is reached by the body.

2

(ii) Find the range of the projectile (ie the distance from O to B)

2

(c)



In the diagram, TC represents a vertical tower that is due North of the point A.

The point B is 100 m due East of A.

From A the angle of elevation of the top of the tower is 5° , and from B the angle of elevation of the top of the tower is 3° .

Let the height of the tower be h metres.

(i) Show that $AC = \frac{h}{\tan 5^\circ}$

1

(ii) Using a similar expression for BC , find the height of the tower, correct to the nearest metre.

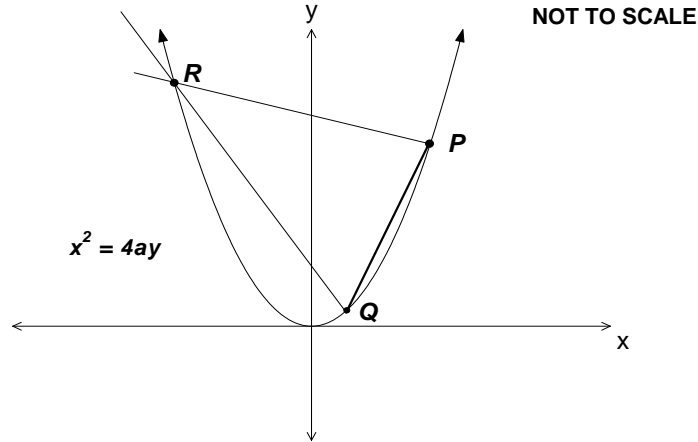
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Question 7 (12 marks) – Use a SEPARATE writing booklet

Marks

- (a) (i) Show that in the binomial expansion of $\left(x - \frac{1}{x}\right)^{2n}$, the term that is independent of x is $(-1)^n {}^{2n}C_n$ **1**
- (ii) Show that $(1+x)^{2n} \left(1 - \frac{1}{x}\right)^{2n} = \left(x - \frac{1}{x}\right)^{2n}$ **1**
- (iii) Deduce that $({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - \dots + ({}^{2n}C_{2n})^2 = (-1)^n {}^{2n}C_n$ **2**

(b)



$P(2ap, ap^2)$, $Q(2aq, aq^2)$ and $R(2ar, ar^2)$ are points on the parabola $x^2 = 4ay$.

The normals at the points P and Q intersect at R .

PQ is a chord of the parabola.

- (i) Show that the equation of the normal at P is $x + py = 2ap + ap^3$ **2**
- (ii) Hence show that $r = -p - \frac{2}{p}$ **2**
- (iii) Show that $pq = 2$ **2**
- (iv) Hence find the equation of the locus of the midpoint of the chord PQ **2**

END OF EXAMINATION

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STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE : $\ln x = \log_e x, x > 0$