

Name _____

ASCHAM SCHOOL
MATHEMATICS TRIAL EXAMINATION 2013

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen. Black pen is preferred.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11–16.

Total marks – 100

Section I 10 marks

- Attempt Questions 1–10 using the Multiple Choice sheet.
- Allow about 15 minutes for this section.

Section II 90 marks

- Attempt Questions 11–16.
- Allow about 2 hours 45 minutes for this section.
- Do each question in a separate booklet.
- Write your name/number and your teacher's name on each booklet.
- Clearly label the front of each booklet with the number of the question.

Collection

- Start each question of Section II in a new booklet.
- If you use a second booklet for a question, place it inside the first.
Indicate on the outside of the first booklet that you have used two booklets for that question.
- Write your name/number, teacher's name and question number on each booklet.

Section I**10 marks****Attempt Questions 1 – 10****Allow about 15 minutes for this section**Use the multiple-choice answer sheet at the back of this exam paper for Questions 1 – 10

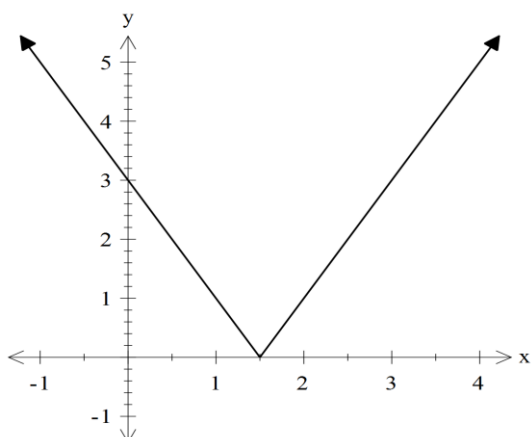
1 Evaluate $\sqrt[3]{3\frac{3}{7}}$ to three significant figures.

- (A) 1.087
- (B) 1.09
- (C) 1.508
- (D) 1.51

2 The first and last terms of an arithmetic series are 10 and 60.
If the sum of the series is 3535, how many terms are there in the series?

- (A) 11
- (B) 101
- (C) 110
- (D) 51

3 What is the equation of the graph drawn below?



- (A) $y = |2x + 3|$
- (B) $y = |2x - 3|$
- (C) $y = |x - 1.5|$
- (D) $y = |x + 1.5|$

4 The perimeter of a sector is 30cm. If the angle at the centre is 3 radians, what is the radius of the circle?

- (A) 10cm
- (B) 20cm
- (C) 6cm
- (D) 3cm

5 Find $\int \frac{1}{x^2} dx$

- (A) $\log(x^2) + c$
- (B) $\frac{-2}{x^3} + c$
- (C) $\frac{-1}{x} + c$
- (D) $2x \log(x^2) + c$

6 What is the equation of a parabola with focus (2, 3) and directrix $y = -5$?

- (A) $(x+1)^2 = 16(y-2)$
- (B) $(x-2)^2 = 16(y+1)$
- (C) $(x-2)^2 = 4(y-1)$
- (D) $(y+1)^2 = 4(x-2)$

7 What is the limiting sum of the series $-\frac{1}{27} + \frac{1}{9} - \frac{1}{3}$?

(A) $\frac{-1}{108}$

(B) $\frac{1}{54}$

(C) Can't be found

(D) $\frac{1}{108}$

8 What is the derivative of $\frac{4}{3x^3}$?

(A) $-\frac{4}{x^4}$

(B) $-\frac{2}{3x^2}$

(C) $-\frac{4}{x^2}$

(D) $-\frac{36}{x^4}$

9 $(2\sqrt{3}-5)^2$ is equal to

(A) $1-20\sqrt{3}$

(B) 37

(C) $37-20\sqrt{3}$

(D) $37-10\sqrt{3}$

- 10** What is the compound interest on \$1000 invested for 5 years at 6% per annum interest, compounded monthly? (to nearest dollar)
- (A) \$1338
 - (B) \$349
 - (C) \$1349
 - (D) \$1025

End of Multiple Choice

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet

- a) Factorise $2x^2 + 3x - 20$. (1)
- b) Find the exact value of $\cos 210^\circ$. (1)
- c) Find $\int \frac{2}{e^{2x}} dx$. (1)
- d) One of the roots of the equation $kx^2 - 2x - 3 = 0$ is -3 .
Find the value of k . (1)
- e) Differentiate $\sin^2 2x$. (2)
- f) Solve $|2x - 3| \leq 5$. (2)
- g) Find $\int \frac{3x}{x^2 - 4} dx$. (2)
- h) Solve for x correct to 2 significant figures:
 $(\log_e 2x)^2 = 16$ (3)
- i) Is $f(x) = (x - x^5)^2$ odd or even? Show all working. (2)

End of question 11

Question 12 (15 marks) Use a new booklet

a) Differentiate with respect to x :

i) $\frac{e^x}{\log_e 2x}$ (3)

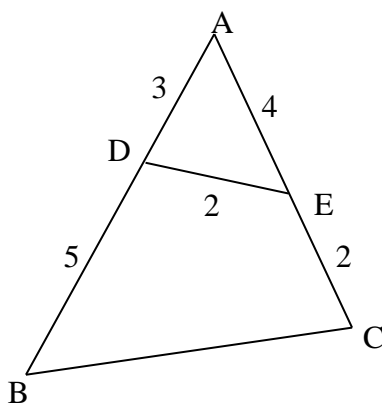
ii) $\log[x^4(x-1)^2]$ (2)

b) Find the primitive of $\cos(\frac{1-x}{5})$. (1)

c) Find, in general form, the equation of the tangent to the curve $y = x \ln x$ at the x intercept. (3)

d) If α and β are the roots of the equation $2x^2 - 3x + 4 = 0$, find the value of $\alpha^2 + \beta^2$. (3)

e) By proving 2 triangles similar, find the length of BC. (3)

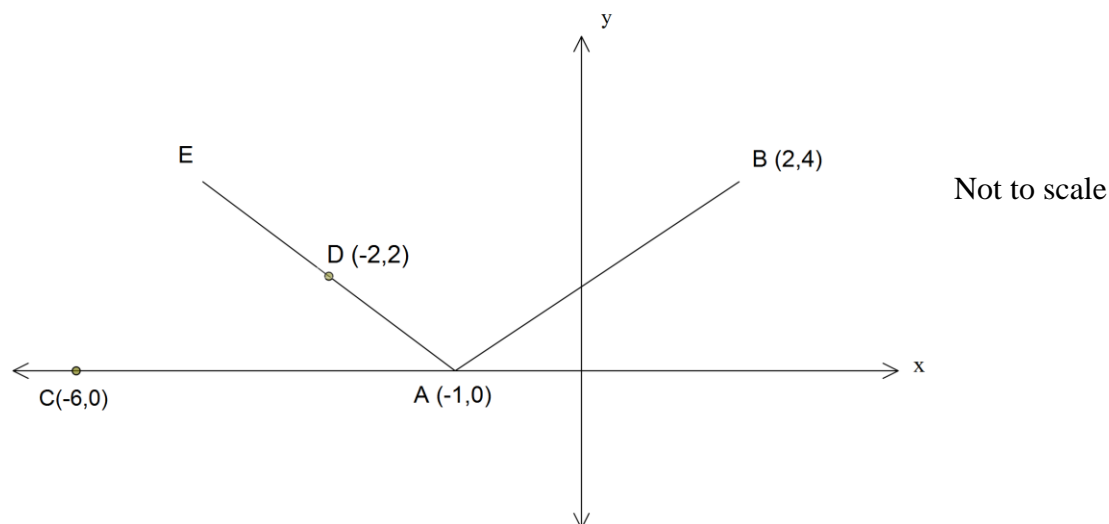


Not to scale

End of question 12

Question 13 (15 marks) Start a new booklet.

- a) A, B and C are the points $(-1,0)$, $(2,4)$ and $(-6,0)$ respectively.
D is the point $(-2,2)$ and is the midpoint of AE.



- i) Find the length of the interval AB (2)
- ii) Find the midpoint of BC (1)
- iii) Find the coordinates of E (2)
- iv) What type of quadrilateral is ABEC?
Give a clear explanation for your answer. (2)
- b) Find the equation of the locus of the point $P(x, y)$ which moves so that it is twice the distance from $R(-3,4)$ as it is from $S(-1,2)$. (4)
- c) The slope at any point on a curve is given by $3\sec^2 2x$.
Find the equation of the curve if it passes through the point $(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$. (4)

End of question 13

Question 14 (15 marks) Start a new booklet.

a) For what values of k will the equation $9x^2 - kx + 1 = 0$ have real and different roots? **(3)**

b) Evaluate $\int_0^n (1 + \sin 2\pi x) dx$. **(3)**

c) i) Sketch the function $y = 4\cos 2x$ for $0 \leq x \leq \pi$. **(2)**

ii) Find the area between the curve $y = 4\cos 2x$ and the x axis from $x = 0$ to $x = \frac{\pi}{2}$. **(3)**

d) A tank contains 50 litres of water. A tap at the base of the tank allows water to flow out at a rate proportional to the quantity of water still in the tank at that time. After 2 minutes, 10 litres have run out.

Use the equation $W = W_0 e^{-kt}$, where W is the amount of water in the tank and t is time in minutes.

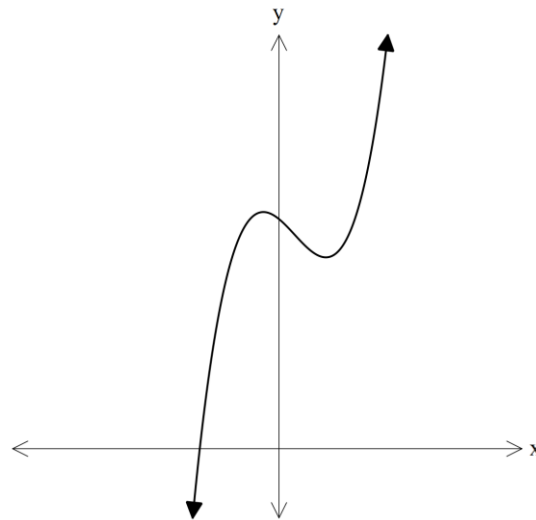
i) Show that $k = -\frac{1}{2} \ln \frac{4}{5}$. **(2)**

ii) How much water has run out after 10 minutes? (To the nearest litre) **(2)**

End of question 14

Question 15 (15 marks) Start a new booklet

- a) The graph of $y = x^3 - x^2 - x + 6$ is sketched below.

**Not to scale**

- i) Find the coordinates of the stationary points. **(3)**
- ii) Find any point(s) of inflexion **(2)**
- iii) For what values of x is the curve decreasing and concave up? **(1)**
- iv) For what values of p has the equation $x^3 - x^2 - x + 6 = p$ exactly two real solutions? **(2)**
- b) Use Simpson's Rule with 5 function values to evaluate, to 2 decimal places,

$$\int_1^3 \log_e x \, dx.$$
 (3)
- c) A particle moves in a straight line such that its position at time t seconds is given by $x = t - \log_e t$.
- i) When is the particle at rest? **(1)**
- ii) Find the exact distance travelled by the particle between $t = \frac{1}{2}$ and $t = 1\frac{1}{2}$. **(3)**

End of Question 15

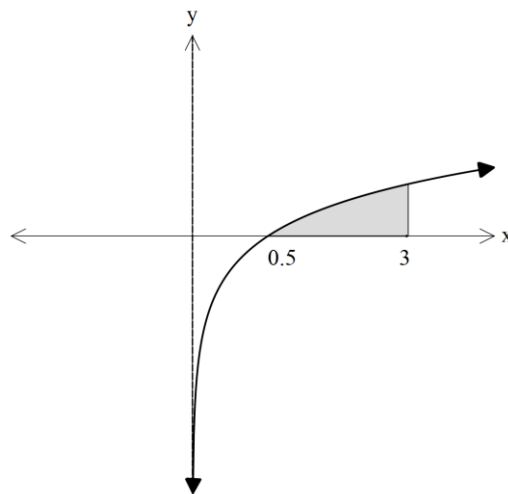
Question 16 (15 marks) Start a new booklet

- a) Diana borrows \$10 000 and arranges to pay it back with interest in 20 equal instalments every three months over 5 years. She is charged 6% per annum interest compounded monthly.

Let A_n be the amount owing after n months and let M be the instalment.

- i) Find the amount owing after the first three months, just after she has made her first payment. **(1)**
- ii) Show that $M = \frac{10000(1.005^{63} - 1.005^{60})}{1.005^{60} - 1}$. **(3)**
- iii) Find the size of the instalment she pays. **(1)**
- iv) How much does Diana pay in interest? **(1)**

- b) The graph of $y = \log_e 2x$ is given below.

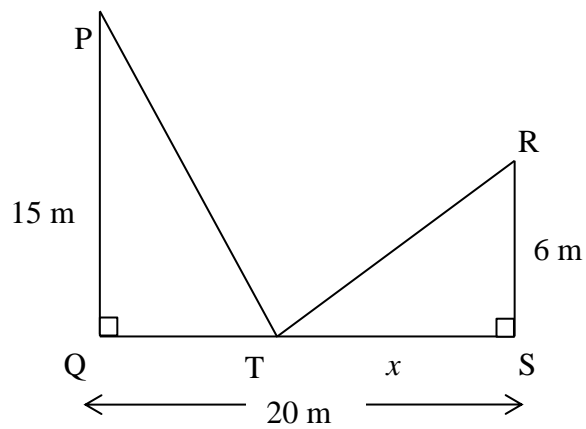


Not to scale

- Find the volume when the shaded region between the curve, the x axis and the line $x = 3$ is rotated about the y axis. **(3)**

Question 16c is on the next page

c)

**Not to scale**

Two poles, PQ and RS are 20 metres apart. PQ is 15 metres high and RS is 6 metres high. A length of wire is attached to the top of each pole and also staked to the ground at T somewhere between the two poles.

Let $TS = x$ metres.

i) Show that the length of wire $L = \sqrt{36 + x^2} + \sqrt{625 - 40x + x^2}$. (2)

ii) Find the shortest length of wire that can be used. (4)

End of exam

Standard Integrals

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

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Student Number: _____

Name: _____

SECTION I Mathematics Multiple Choice Answer Sheet

10 Marks

This sheet must be handed in separately. Detach it from the question paper.

Shade the correct answer:

- | | | | | |
|-----|-------------------------|-------------------------|-------------------------|-------------------------|
| 1. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 2. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 3. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 4. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 5. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 6. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 7. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 8. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 9. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 10. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |