Barker College 4 Unit maths trial 1999

Question 1 (Start a new page) **MARKS**

a) Find **3**

i) $\int \frac{4}{x^2+4}\,dx$

ii) $\int \frac{4}{\sqrt{x^2+4}}\,dx$

iii) $\int \frac{4x}{\sqrt{x^2+4}}\,dx$

b) Evaluate **8**

i) $\int_0^{\frac{\pi}{2}} e^x \cos x\,dx$

ii) $\int_0^{\frac{\pi}{2}} \frac{d\theta}{2+\cos\theta}$

c) **4**

i) Find polynomials $p(x), q(x)$ of degrees less than 2, such that $(x+2)p(x)+(x^2+4)q(x)=1$.

ii) Hence evaluate $\int_0^2 \frac{8dx}{(x+2)(x^2+4)}$.

Question 2 (Start a new page) **MARKS**

a) If $z = \dfrac{3+2i}{1-2i}$ then find 3

i) $\bar{z}$

ii) $\arg z$

b) 5

i) Express $\sqrt{6i-8}$ in the form $a+ib$ where a,b are elements of the set of reals.

ii) Hence solve $2z^2 - (3+i)z + 2 = 0$ for z. Express your answer in the form $a+ib$.

c) Neatly sketch each of the following loci on separate Argand Diagrams. 4

i) $\arg \dfrac{z+1}{z-i} = \dfrac{2\pi}{3}$

ii) $z\bar{z} = z + \bar{z}$

d) 3

i) Show on an Argand diagram the locus of z where $|z-4-3i| = 1$.

ii) What are the least values of $|z|$.

Question 3 (Start a new page)

a)

i) Sketch $y = f(x)$, clearly labelling all essential features givei that $f(x) = x^3 - 4x$.

On separate diagrams sketch showing labelling all essentia features

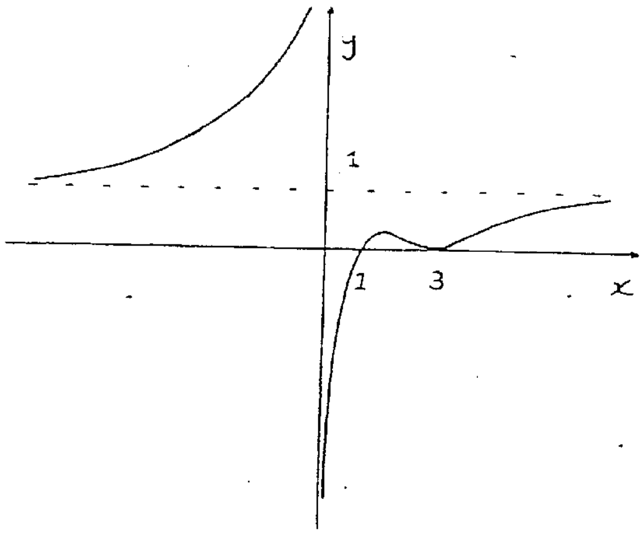
ii) $y^2 = f(x)$

iii) $y = f(\frac{1}{x})$

iv) $y = e^{f(x)}$

v) $|y| = |f(x)|$

b)



The diagram above is of the derivative of $y = f(x)$. i.e. The curve has equation $y = f'(x)$.

i) Sketch the function $y = f''(x)$.

ii) On a separate diagram sketch a possible graph of $y = f(x)$.

iii) Suggest a possible equation for $y = f'(x)$ in terms of x.

Question 4 (Start a new page) **MARKS**

a) 8

Show that the normal to the parabola $x^2 = 4ay$ at the point $P(2ap, ap^2)$ bisects the angle between the lines $x = 2ap$ and SP where S is the focus of the parabola.

b) 7

i) Sketch the hyperbola with equation $\frac{x^2}{4} - \frac{y^2}{2} = 1$, carefully labelling all essential features.

ii) Show that the equation of the tangent to this hyperbola at $P(2\sec\theta, \sqrt{2}\tan\theta)$ is given by $\frac{x\sec\theta}{2} - \frac{y\tan\theta}{\sqrt{2}} = 1$.

iii) Hence prove that the area of the triangle bounded by this tangent and the asymptotes of the hyperbola is independent of the position of P.

Question 5 (Start a new page)

a)

i) Prove that the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ has a magnitude of πab.

ii) Find the volume of a mound with a circular base of equation $x^2+y^2=4$ which has semi-elliptical cross-sections parallel to the y axis, where the ratio of the major axis : minor axis = 2 : 1. The height of each cross-section is the length of the semi-minor axis.

b)

i) Sketch the curve $y=x^2(x^2-1)$ shading the region bounded by the curve and the x-axis.

ii) Find the volume of the solid formed when this shaded area in part i) is rotated about the y-axis.

iii) What is the volume of the solid formed when the area encompassed by the relation $y^2=x^8-2x^6+x^4$ is rotated about the y-axis?

Question 6 (Start a new page) **MARKS**

a) Show that $1+i$ is a root of the polynomial $P(x)=x^3+x^2-4x+6$ and hence completely factorize $P(x)$ over the field of complex numbers. **3**

b) **4**

i) If the polynomial $P(x)=x^4-4x^3+11x^2-14x+10$ has roots of the form $a+ib$ and $a-2ib$ where a, b are real, find the values of a and b.

ii) Find all the zeros of $P(x)$.

iii) Express $P(x)=x^4-4x^3+11x^2-14x+10$ as a product of two quadratic factors with rational coefficients.

c) **5**

i) Prove that if the polynomial $P(x)$ has a root α of multiplicity m then $P'(x)$ has a root α of multiplicity m-1.

ii) Given that the polynomial $P(x)=x^4+x^3-3x^2-5x-2$ has a root of multiplicity 3, find all the roots of $P(x)$.

d) If α,β,γ are the roots of the equation $x^3+qx+r=0$, prove that $(\beta-\gamma)^2+(\alpha-\beta)^2+(\alpha-\gamma)^2=-6q$. **3**

Question 7 (Start a new page)

a) If $I_n = \int_0^1 \frac{x^n}{\sqrt{x+1}}\,dx$ where $n = 0,1,2,3,\ldots\ldots$

i) Show that $x^{n-1}\sqrt{x+1} = \frac{x^n}{\sqrt{x+1}} + \frac{x^{n-1}}{\sqrt{x+1}}$.

ii) Show that $(2n+1)I_n = 2\sqrt{2} - 2nI_{n-1}$ for $n = 1,2,3,\ldots\ldots$

iii) Evaluate $\int_0^1 \frac{x^2}{\sqrt{x+1}}\,dx$.

b)

i) Sketch on an argand diagram the roots of $z^5 - 1 = 0$.

ii) Show that $\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}$.

iii) Hence or otherwise find the exact values of $\cos\frac{2\pi}{5}$ and $\cos\frac{\pi}{5}$.

Question 8 (Start a new page) **MARKS**

a) Prove that if the opposite angles of a quadrilateral are supplementary then the quadrilateral must be cyclic. **4**

b) **5**

i) Show that $\tan^{-1} a + \tan^{-1} b = \tan^{-1} \frac{a+b}{1-ab}$.

ii) Simplify $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c$.

iii) If the equation $x^3 - 2x^2 - 5x + 4 = 0$ has roots α, β, γ show that $\tan^{-1} \alpha + \tan^{-1} \beta + \tan^{-1} \gamma = \frac{\pi}{4}$.

c) **6**

i) Show that $f(x) = \frac{\sec x + \tan x}{2\sec x + 3\tan x}$ is a decreasing function in term of x for the domain $0 < x < \frac{\pi}{2}$.

ii) Deduce that $\frac{\pi}{28} > \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sec x + \tan x}{2\sec x + 3\tan x} dx > \left(\sqrt{2} - 1\right)\frac{\pi}{12}$.

END OF PAPER