Total marks – 120
Attempt Questions 1–8
ALL questions are of equal value

Barker College Maths Ext. 2
2008 Trial

Answer each question on a SEPARATE sheet of paper

Marks

Question 1 (15 marks) **[START A NEW PAGE]**

(a) If $(\sqrt{3}+i)z = 4\sqrt{3} - 4i$, find

(i) z in the form $a + bi$ **1**

(ii) $|z|$ **1**

(iii) $\arg(z)$ **1**

(iv) z^8 in the form $a + bi$ **1**

(b) Find $\sqrt{-5-12i}$ in the form $a + bi$, and hence solve the equation

$z^2 + (1-2i)z + (\frac{1}{2} + 2i) = 0$ **4**

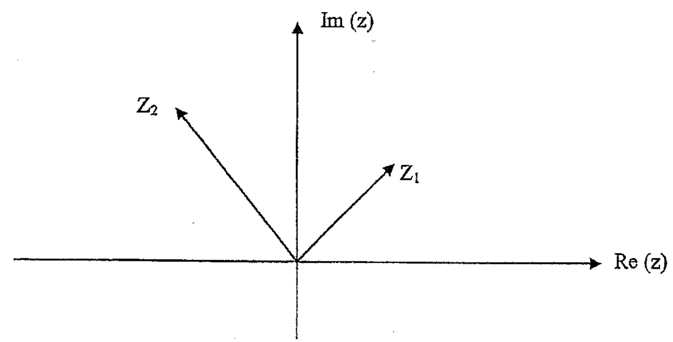
(c) If w is a complex cube root of unity, show that $1 + w + w^2 = 0$, and hence prove that $(1+w)(1+2w)(1+3w)(1+7w) = 31 + 2w$. **3**

Question 1 continues on page 3

Marks

Question 1 (continued)

(d) Let z_1 and z_2 be two given complex numbers as shown on the Argand diagram below.



Let z be a variable complex number.

Sketch and describe the locus of z on an Argand diagram if:

(i) $|z - z_1| = |z - z_2|$ **2**

(ii) $\arg\left(\frac{z - z_1}{z - z_2}\right) = \alpha$, where $0 < \alpha < \pi$ **2**

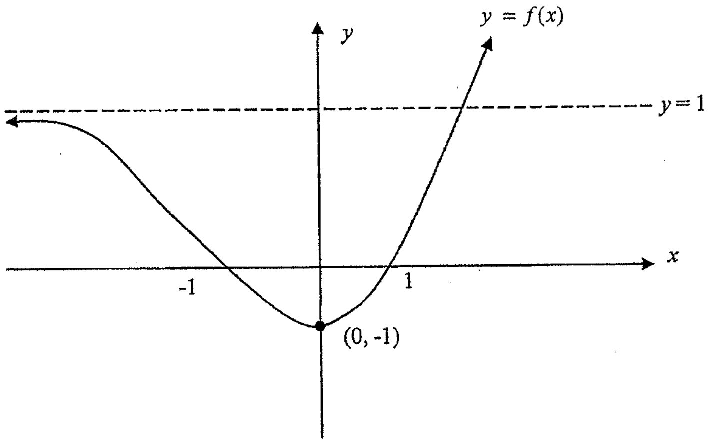
End of Question 1

Marks

Question 2 (15 marks) **[START A NEW PAGE]**

(a) The diagram below shows the graph of $y = f(x)$.

There is a minimum turning point at (0, -1).



On separate diagrams, draw the graph of

(i) $y = f(|x|)$ **2**

(ii) $y^2 = f(x)$ **2**

(iii) $y = \sin^{-1}[f(x)]$ **2**

(iv) $y = ln[f(x)]$ **2**

Question 2 (continued) Marks

b) The equation of a curve is given by $xy^2 + x^2 = 1$

(i) Explain why $x = 0$ is not in the domain of the curve. **1**

(ii) Find the x – intercepts of the curve. **1**

(iii) Re-write the equation of the curve making y the subject, and hence find the domain of the curve. **2**

(iv) The curve has two asymptotes. Write down the equations of both asymptotes. **2**

(v) Hence sketch the curve $xy^2 + x^2 = 1$ **1**

End of Question 2

Marks

Question 3 (15 marks) **[START A NEW PAGE]**

(a) Find

(i) $\int \frac{(x+3)\,dx}{\sqrt[3]{x^2+6x}}$ using the substitution $u = x^2 + 6x$ **2**

(ii) $\int \frac{dy}{y^2+10y+30}$ **2**

(iii) $\int \frac{dx}{2+\cos x}$ using the "t-results" **3**

(iv) $\int x^2 e^{2x}\,dx$ **3**

(b) Factorise $x^3 + x^2 - 6x$ and then find the values of A, B and C such that

$$\frac{x+1}{x^3+x^2-6x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+3}$$

Hence find $\int \frac{(x+1)\,dx}{x^3+x^2-6x}$ **5**

End of Question 3

Marks

Question 4 (15 marks) **[START A NEW PAGE]**

(a) Let $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\,dx$

Show that $I_n + I_{n-2} = \frac{1}{n-1}$

Deduce the value of I_5. **5**

(b) Find the volume of the torus generated by revolving the circle $x^2 + y^2 = 4$ about the line $x = 3$. **4**

(c) (i) Show that

$$\frac{d}{dx}\left\{\frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\sin^{-1}\left(\frac{x}{a}\right)\right\} = \sqrt{a^2-x^2}$$ **3**

(ii) The base of a solid is the circle $x^2 + y^2 = 16x$. Every slice of this solid taken perpendicular to the x axis is a rectangle of height 6 units. Using the result from part (i) above, find the volume of this solid. **3**

End of Question 4

Marks

Question 5 (15 marks) **[START A NEW PAGE]**

(a) Consider the polynomial

$p(x) = ax^4 + bx^3 + cx^2 + d$

where a, b, c and d are integers.

Suppose that α is an integer such that $p(\alpha) = 0$

(i) Prove that d is a multiple of α **2**

(ii) Prove that the polynomial $q(x) = 5x^4 - x^3 + 3x^2 - 3$

does not have an integer root. **2**

(b) Let $P(x) = x^3 - 11x - 14$

Factorise $P(x)$ over the reals and hence find the three roots of $P(x) = 0$ **3**

(c) Find the roots of $q(x) = x^4 - 6x^3 + 12x^2 - 10x + 3$

given that it has a root of multiplicity 3. **2**

(d) Let α, β and γ be the roots of the equation $x^3 - 5x^2 + 5 = 0$

(i) Find the value of $(\alpha-1)(\beta-1)(\gamma-1)$ **2**

(ii) Find the value of $\alpha^3 + \beta^3 + \gamma^3$ **2**

(iii) Find a polynomial equation with integer coefficients whose roots are

α^2, β^2 and γ^2 **2**

End of Question 5

Marks

Question 6 (15 marks) **[START A NEW PAGE]**

(a) For the ellipse $\frac{x^2}{36} + \frac{y^2}{9} = 1$, find

(i) the lengths of the axes **1**

(ii) the eccentricity **1**

(iii) the co-ordinates of the foci **1**

(iv) the equations of the directrices **1**

(b) Let $P(2\sec\theta, \sqrt{5}\tan\theta)$ be a variable point on the hyperbola $5x^2 - 4y^2 = 20$

The tangent at P meets the directrix at T. Show that PT subtends a right angle at the corresponding focus. **6**

(c) (i) If the line $y = mx + b$ is a tangent to the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$,

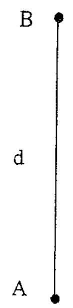
show that $b^2 = 6m^2 + 3$ **2**

(ii) The tangents to this ellipse from a point $P(X, Y)$ meet at right angles. Prove that the locus of P is the circle $x^2 + y^2 = 9$. **3**

End of Question 6

Question 7 (15 marks) **[START A NEW PAGE]**

Marks



A and B are two points d units apart in a vertical line. B is directly above A. Two identical particles are projected from A and B towards each other with the same velocity, u.

The resistance of the medium is kv per unit mass.

(i) Draw a diagram indicating all forces acting on the particles. **1**

(ii) Consider the particle moving upward from A. By writing an expression for $\frac{dv}{dt}$,

(α) show that $t = \frac{1}{k} \ln\left(\frac{g + ku}{g + kv}\right)$ **2**

(β) Hence, find v in terms of t **2**

(γ) Hence, find x in terms of t **2**

(iii) Consider the particle moving downward from B. Given that $\frac{dv}{dt} = g - kv$,

(α) find t in terms of v. **2**

(β) Find v in terms of t **2**

(γ) Find x in terms of t. **2**

(iv) Hence, prove that the particles meet after a time of $\frac{1}{k} \ln\left(\frac{2u}{2u - kd}\right)$ **2**

End of Question 7

Marks

Question 8 (15 marks) **[START A NEW PAGE]**

(a) A particle is moving along the x axis. Its acceleration is given by

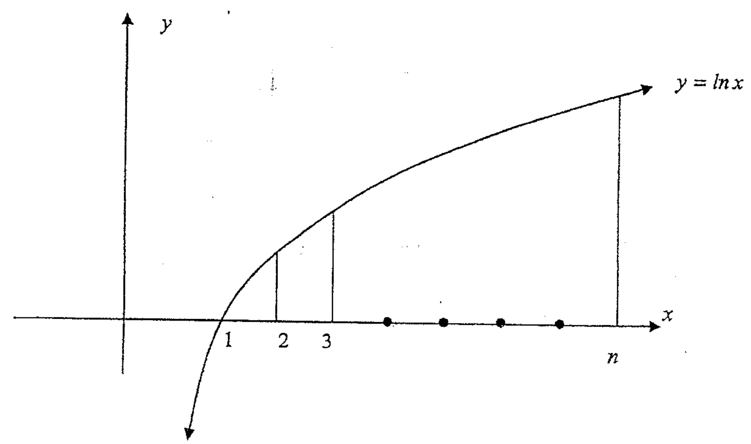
$\frac{d^2x}{dt^2} = \frac{12 - 4x}{x^3}$. The particle starts from rest at the point $x = 6$.

(i) Show that the particle starts moving in the negative x direction. **1**

(ii) Find an expression for velocity, v, in terms of x. **3**

(iii) The path along which the particle moves is bounded. What part of the x axis is the path of the particle? **1**

(b) Consider the area under the curve $y = \ln x$ between $x = 1$ and $x = n$.



(i) Show that this area is exactly equal to $\ln\left(\frac{n^n}{e^{n-1}}\right)$ **2**

Question 8 continues on page 12

Marks

Question 8 (continued)

(ii) Use the Trapezoidal Rule to find an expression which approximates this area. **2**

(iii) Hence show that $n^n > \sqrt{n}\,(n-1)!\,e^{n-1}$ **1**

(c) Given that $\sin^{-1} 2x$, $\cos^{-1} 2x$ and $\sin^{-1}(1-2x)$ are all acute,

(i) Show that $\sin[\cos^{-1} 2x - \sin^{-1} 2x] = 1 - 8x^2$ **3**

(ii) Solve the equation $\cos^{-1} 2x - \sin^{-1} 2x = \sin^{-1}(1-2x)$ **2**

End of Paper