



*2014  
HIGHER SCHOOL CERTIFICATE  
TRIAL EXAMINATION*

# Mathematics

## General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators and templates may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

## Total marks – 100

### Section 1 Pages 2- 4

- **10 marks**
- Attempt Questions 1-10
- Allow about 15 minutes for this section

### Section 11 Pages 5-10

- **90 marks**
- Attempt Questions 11 -16
- Allow about 2 hours 45 minutes for this section

**Section 1 : Objective Response Questions.**

Answer on the answer sheet provided.

- 1 What are the coordinates of the *focus* of the parabola  $(x - 4)^2 = 8(y + 3)$ ?  
(A)  $(4, -1)$   
(B)  $(-4, 1)$   
(C)  $(-3, -1)$   
(D)  $(-3, -4)$
  
- 2 What is the equation of the tangent to the curve  $y = x^2 - 5x$  at the point  $(1, -4)$ ?  
(A)  $y = -3x - 1$   
(B)  $y = -3x - 7$   
(C)  $y = 3x + 7$   
(D)  $y = 3x - 7$
  
- 3 For what values of  $k$  does the equation  $x^2 - 6x - 3k = 0$  have real roots  
(A)  $k \geq -3$   
(B)  $k \leq -34$   
(C)  $k \geq 3$   
(D)  $k \leq 3$
  
- 4 The fourth term of an arithmetic series is 27 and the seventh term is 12. What is the common difference?  
(A)  $-5$   
(B)  $5$   
(C)  $13$   
(D)  $42$

5 What is the area enclosed between the curves

$$y = x^2 + 1 \text{ and } y = 3x + 1 \quad ?$$

- (A)  $\frac{3}{2}$  square units
- (B)  $\frac{9}{2}$  square units
- (C)  $\frac{27}{2}$  square units
- (D)  $\frac{45}{2}$  square units

6 The graph  $y = f(x)$  passes through the point  $(1, 4)$  and  $f'(x) = 3x^2 - 2$ .

Which of the following expressions is  $f(x)$ ?

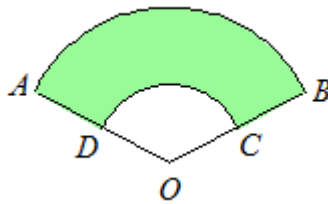
- (A)  $x^3 - 2x$
- (B)  $2x - 1$
- (C)  $x^3 - 2x + 3$
- (D)  $x^3 - 2x + 5$

7 What is the value of  $\sum_{r=1}^{10} (5x + 2)$ ?

- (A) 59
- (B) 295
- (C) 590
- (D) 795

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- 8 A car windscreen wiper traces out the area  $ABCD$  where  $AB$  and  $CD$  are arcs of circles with a centre  $O$  and radii 40 cm and 20 cm respectively. Angle  $AOB$  measures  $120^\circ$ .

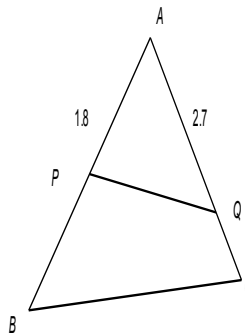


Not to scale

What is the area of  $ABCD$ ?

- (A)  $419 \text{ cm}^2$   
(B)  $1257 \text{ cm}^2$   
(C)  $1676 \text{ cm}^2$   
(D)  $2095 \text{ cm}^2$
- 9 If  $AB = 6$ ,  $AC = 4$  and  $\triangle APQ \parallel \triangle ACB$ , find the length of  $PQ$  if  $BC = 5$ .

- (A) 2.25  
(B) 3.375  
(C) 1.44  
(D) 2.4



- 10 On the Gill family holiday at Perisher snow fields 4 cm of snow falls on the first day. In each following days the snowfalls increase by 1.5 cm, so on the second day there is 5.5 cm, on the third day there is 7 cm. How much snow falls on the 10th day?
- (A) 15 cm  
(B) 17.5 cm  
(C) 19 cm  
(D) 107.5 cm

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**Marks**

**Part B**

**Question 11:**

**Start a new Booklet**

**Marks**  
**(15)**

- (a) Evaluate  $\left(\frac{1}{e^{2.5}} - 1\right)^2$  correct to 3 significant figures. (2)
- (b) Factorise fully  $36x^2 - 16y^2$  (2)
- (c) Find integers  $a$  and  $b$  such that  $(\sqrt{2} + 1)(5\sqrt{2} - 3) = a\sqrt{2} + b$  (2)
- (d) Given that the vertices of triangle ABC are A (2, 8), B (-3, -7) and C (5, -1)
- (i) Find the equation of the line through B and C (1)
- (ii) Determine the length of the altitude from A to BC. (2)
- (iii) Find the length of the side BC (1)
- (e) Express  $0.4\dot{7}$  as a fraction in its simplest form using an algebraic method. (2)
- (f) Solve for  $x$ :  $|4x + 1| < 5$  and graph the solution set. (3)

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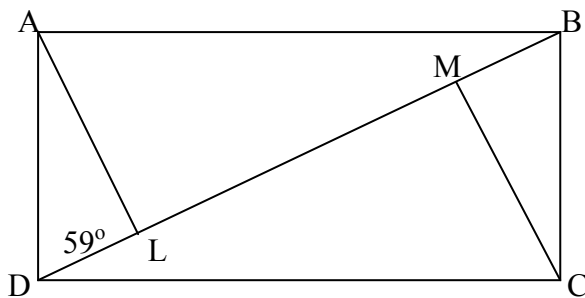
- | <b>Question 12:</b> | <b>Start a new Booklet</b>  | <b>Marks<br/>(15)</b> |
|---------------------|---|-----------------------|
| (a)                 | Find $\int \frac{4}{x^2} dx$  | (2)                   |
| (b)                 | Show that the value of the definite integral $\int_0^2 \frac{3x^2}{x^3+1} dx$ is $2 \ln 3$  | (2)                   |
| (c)                 | Find the first derivative of:   |                       |
| (i)                 | $y = \frac{x^3 + 3}{x}$   | (2)                   |
| (ii)                | $y = e^x \ln 2x$  | (2)                   |
| (d)                 | A series is $\log(x^{-1}) + \log x + \log(x^3) + \log(x^5) + \dots$<br>Is this series arithmetic or geometric? Fully justify your answer. | (2)                   |
| (e)                 | If $f(x) = \log_e 2x$ evaluate $f'(2) + f''(2)$   | (2)                   |
| (f)                 | The first two terms of a geometric series are 15 and 12.  |                       |
| (i)                 | Calculate the next term in the series.  | (2)                   |
| (ii)                | Determine the limiting sum.   | (1)                   |

**Question 13:**

**Start a new Booklet**

- (a) For the parabola with equation  $x^2 = -8y$ .
- (i). Find the coordinates of the focus (S) of the parabola. (1)
  - (ii). Find the equation of the directrix of the parabola. (1)
  - (iii). Show that the point A(-8, -8) lies on the parabola. (1)
  - (iv). Find the equation of the focal chord of the parabola which passes through A. (2)
  - (v). Find the equation of the tangent to the parabola at A. (1)

- (b) Given ABCD is a rectangle,  $\angle ADL$  is  $59^\circ$ .  $AL \perp BD$  and  $CM \perp BD$ .



- (i) Find the size of  $\angle MBC$ , give reason. (1)
  - (ii) Prove that  $\triangle ADL$  is congruent to  $\triangle CBM$ , give reasons. (2)
  - (iii) Hence show that  $AL = MC$ , give reasons. (1)
- (c) If  $\alpha$  and  $\beta$  are solutions to the equation  $4x^2 + 5x - 1 = 0$ , without solving the equation, find the value of:
- (i)  $\alpha + \beta$  (1)
  - (ii)  $\alpha\beta$  (1)
  - (iii)  $\frac{1}{\alpha} + \frac{1}{\beta}$  (1)
- (d) Find  $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1}$ . (2)

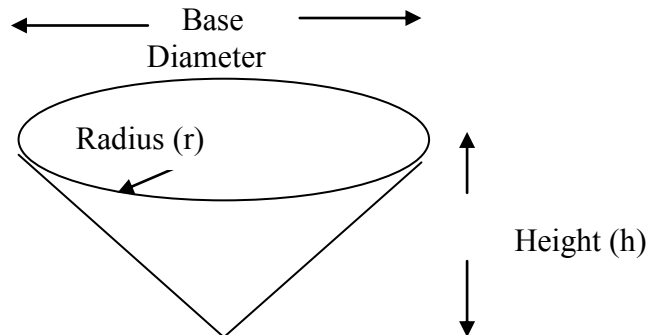
**Question 14:**                      **Start a new Booklet**                      **(15)**

- (a)      A woman walks 120 metres on a bearing of  $312^\circ$ , then turns and walks for a further 96 metres on a bearing of  $056^\circ$ .
- (i)      Draw a diagram in your answer booklet, labeling all given information. (2)
- (ii)     Determine how far the woman is from her starting point to the nearest kilometre? (2)
- (iii)    Hence find the bearing of the woman from her starting point? (2)
- 
- (b)      For the function  $y = 2x^3 - \frac{x^4}{2}$  find:
- (i)      the first and second derivative. (2)
- (ii)     the two stationary points and determine their nature. (3)
- (iii)    Sketch the function showing all intercepts on the axes, over the domain  $-5 \leq x \leq 5$ . (2)
- 
- (c)      Prove that  $\frac{\sin^2 x}{1 - \cos x} + \frac{\sin^2 x}{1 + \cos x} = 2$  (2)



**Question 15:**                      **Start a new Booklet**

- (a) The diagram below represents a conical water container. The sum of its base diameter and its height is 60 metres.



- (i) Write an expression for the height in terms of the radius ( $r$ ). (2)
- (ii) Show that the volume is given by :  $V = 20\pi r^2 - \frac{2}{3}\pi r^3$  (2)
- (iii) Find the radius which makes the volume a maximum. (3)
- 
- (b) (i) Sketch the graph of  $y = e^{-x}$  showing the  $y$  – intercept. (1)
- (ii) Find the exact area of the region bounded by the curve  $y = e^{-x}$ , the  $x$ -axis and the lines  $x = 1$  and  $x = -\ln 3$ . (3)
- 
- (c) Given  $\log_m p = 1.75$  and  $\log_m q = 2.25$ . Find
- (i)  $\log_m pq$  (1)
- (ii)  $\log_m \frac{q}{p}$  (1)
- (iii)  $\sqrt[5]{pq^2}$  in terms of  $m$  (2)

**Question 16:**

**Start a new Booklet**

**(15)**

(a) Use the table

$x$	3	3.25	3.5	3.75	4	4.25	4.5
$f(x)$	1.0	0.8	0.65	0.55	0.5	0.48	0.45

to find an approximation to the value of the definite integral

$$\int_3^{4.5} f(x)dx,$$

using Simpson's Rule. Give your answer correct to 3 significant figures. (4)

(b) Con and Angela want to buy an investment property on the Gold Coast. They decide to borrow \$250 000 to buy an apartment. Interest is calculated monthly on the balance still owing, at a rate of 6% per annum. The loan is to be repaid in full at the end of 15 years with equal monthly repayments of \$ $M$ .

Let \$ $A_n$  be the amount owing after the  $n$ th repayment.

- (i) Derive an expression for  $A_3$ . (1)
- (ii) Find the value of  $M$ . (2)
- (iii) Hence, calculate the amount still owing after 5 years of payments at this rate. (2)
- (iv) At the end of 5 years, the interest rate is increased to 7.2% per annum and Con and Angela decide to increase their repayments to \$2400 per month. How many more months are required to pay off the remainder of the loan? (2)

(c) The area bounded by the curve  $y = \frac{1}{x}$ , the x-axis and the ordinates  $x = a$  and  $x = 4$  is rotated about the x-axis. If the volume generated is  $\frac{\pi}{2}$  unit<sup>3</sup>, where  $0 < a < 4$ , find the value of  $a$ . (4)

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left( x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left( x + \sqrt{x^2 + a^2} \right)$$

NOTE :  $\ln x = \log_e x, \quad x > 0$