Teacher	
Student Number	

2008
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**84**)

- Attempt Questions 1-7
- All questions are of equal value

Q1 (12)	Q2 (12)	Q3 (12)	Q4 (12)	Q5 (12)	Q6 (12)	Q7 (12)	(84)	(100)

Question 1 (12 Marks) Start a fresh sheet of paper. **Marks**

(a) Solve the inequality $\dfrac{4-2x}{x+5} \leq 2$ **3**

(b) Find $\lim\limits_{x \to 0} \dfrac{\sin\left(\frac{\pi}{4}x\right)}{2x}$ **2**

(c) Find the acute angle between the lines: $x - 2y + 1 = 0$ **2**

$y = 5x - 4$

(d) Differentiate $ln\,(\sin^{-1} 2x)$ **2**

(e) Find the Cartesian equation of the parabola given $x = t - 2$ and $y = 3t^2 - 1$. **1**

(f) Find the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sec\,x\,tan\,x\; dx$, giving the answer in the simplest

exact form **2**

End of Question 1

Question 2 (12 Marks) Start a fresh sheet of paper. **Marks**

(a) i. Prove that $\sin\theta\sec\theta = \tan\theta$ **1**

ii. Hence solve $\sin\theta\sec\theta = \sqrt{3}$. $(0 \leq \theta \leq 2\pi)$ **1**

(b) Use the process of mathematical induction to show that: **4**

$$1 + 3 + 9 + \ldots\ldots\ldots\ldots\ldots + 3^{n-1} = \frac{1}{2}\left(3^n - 1\right)$$

(c) i) Express $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ **2**

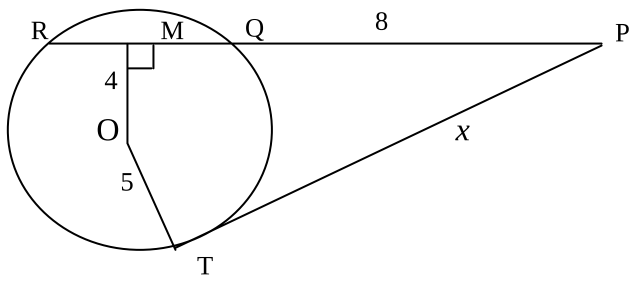
ii) Hence solve the equation $\cos x - \sqrt{3}\sin x = -2$ for $0 \leq x \leq 2\pi$ **1**

(d) From the top of a cliff an observer spots two ships out at sea. One is north east with an angle of depression of 6° while the other is south east with an angle of depression of 4°. If the two ships are 200 metres apart, find the height of the cliff, to the nearest metre. **3**

End of Question 2

Question 3 (12 Marks) Start a fresh sheet of paper. **Marks**

(a) **2**



PT is a tangent to the circle, centre O. OM is perpendicular to the secant RQ.

Find the value of *x*.

(b) If $\alpha = \sin^{-1}\left(\frac{2}{3}\right)$ and $\beta = \sin^{-1}\left(\frac{3}{5}\right)$, find the value of $\sin(\alpha + \beta)$ **3**

(c) Evaluate $\int \cos^2 4x \, dx$ **2**

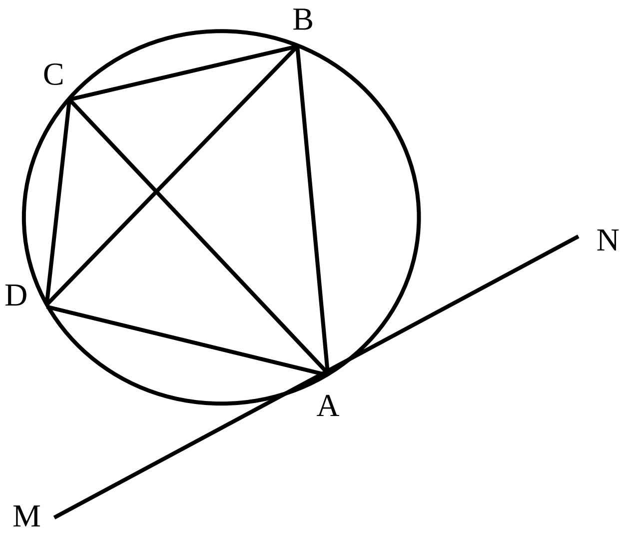
(d) Using the substitution $u = x - 2$, evaluate $\int_3^4 \frac{x^2}{(x-2)^2} \, dx$ **3**

(e) Find the value of $1 + \cos^2 x + \cos^4 x + \cos^6 x + \ldots\ldots\ldots\ldots$ $(0 < x < \frac{\pi}{2})$ **2**

End of Question 3

Question 4 (12 Marks) Start a fresh sheet of paper. **Marks**

(a)



ABCD is a cyclic quadrilateral. *MAN* is the tangent at *A* to the circle through *A*, *B*, *C* and *D*. *CA* bisects $\angle BCD$.

Copy the diagram.

Show that *MAN* is parallel to *DB*. **4**

(b) A first approximation to the solution of the equation $x^3 - 3x^2 + 1 = 0$ is 0·5. Use one application of Newton's method to find a better approximation. Give your answer correct to two decimal places. **2**

(c) Let P($2ap$, $2ap^2$) and Q($2aq$, $2aq^2$) be points on the parabola $y = \dfrac{x^2}{2a}$. **6**

i. Find the equation of the chord PQ.

ii. If PQ is a focal chord, find the relationship between *p* and *q*.

iii. Show that the locus of the midpoint of PQ is a parabola.

End of Question 4

Question 5 (12 Marks) Start a fresh sheet of paper. **Marks**

(a) The polynomial $x^3 - 4x^2 + 5x - 1 = 0$ has 3 roots, namely α, β and γ.

i. Find the value of $\alpha + \beta + \gamma$ **1**

ii. Find the value of $\alpha\beta\gamma$ **1**

iii. Find the equation of the polynomial with roots 2α, 2β and 2γ. **2**

(b) Consider the function $f(x) = x + \frac{1}{x}$ for $x \geq 1$

(i) Show that the function f (x) is increasing and the curve $y = f(x)$ is concave up for all values of $x > 1$ **2**

(ii) On the same diagram, sketch the graphs of $y = f(x)$ and the inverse function $y = f^{-1}(x)$, showing the coordinates of the end points and the equation of the asymptote. **2**

(iii) Find the equation of the inverse function $y = f^{-1}(x)$, in its simplest form **2**

(c) Given that $v = \frac{dx}{dt}$, prove that $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d^2x}{dt^2}$ **2**

End of Question 5

Question 6 (12 Marks) Start a fresh sheet of paper. **Marks**

(a) Consider the polynomial $P(x) = x^3 - kx^2 + kx - 1$, where k is a real constant.

(i) Show that 1 is a root of the equation $P(x) = 0$ **1**

(ii) Given that α, $\alpha \neq 1$, is a second root of $P(x) = 0$, show that $\frac{1}{\alpha}$ is also a root of the equation. **1**

(iii) Show that $\alpha^2 + \frac{1}{\alpha^2} = k^2 - 2k - 1$ **2**

(b) Given P = 2000 + Ae^{kt},

i. Prove that it satisfies the equation $\frac{dP}{dt} = k(P - 2000)$ **1**

ii. Initially, P = 3000, and when t = 5, P = 8000. Use this information to find the values of 'A' and '*k*'. **2**

iii. How long does it take the value of 'P' to double and what is the rate of change of 'P' at this time. **3**

(c) The two equal sides of an isosceles triangle are of length 6cm. If the angle between them is increasing at the rate of 0·05 radians per second, find the rate at which the area of the triangle is increasing when the angle between the equal sides is $\frac{\pi}{6}$ radians. **2**

End of Question 6

Question 7 (12 Marks) Start a fresh sheet of paper. **Marks**

a) A bomb is dropped from a helicopter hovering at a height of 800 metres. At the same time a projectile is fired from a gun located on the ground 800 metres to the west of the point directly beneath the helicopter. The intent is for the projectile to intercept the bomb at a height of exactly 400m. (Use acceleration due to gravity = 10 m / s^2) **8**

i. Show that the time taken for the bomb to fall to a height of 400m is $4\sqrt{5}$ seconds.

ii. Derive the formula for the horizontal and vertical components of the displacement of the projectile.

iii. Find the initial velocity and angle of inclination of the projectile if it is to successfully intercept the bomb as intended.

b) A particle moves so that its distance x centimetres from a fixed point O at time t seconds is $x = 8\sin 3t$. **4**

i. Show that the particle is moving in simple harmonic motion.

ii. What is the period of the motion?

iii. Find the velocity of the particle when it first reaches 4 centimetres to the right of the origin.

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Solutions Question 1 2008	Marks/Comments
a. $\dfrac{4-2x}{x+5} \le 2$ x ? - 5 $\therefore 4 - 2x = 2x + 10$ $-6 = 4x$ $x = -1.5$ solution is $x < -5,\ x \ge -1\frac{1}{2}$	**1 for x ? - 5** **1 for other limit** **1 for correct inequality**
b. $\lim\limits_{x\to 0} \dfrac{\sin\left(\frac{\pi}{4}x\right)}{2x} = \lim\limits_{x\to 0} \dfrac{1}{2}.\dfrac{\pi}{4}.\dfrac{\sin\left(\frac{\pi}{4}x\right)}{\frac{\pi}{4}x}$ $= \dfrac{\pi}{8}$	**1 – working** **1- correct answer**
c. $x - 2y + 1 = 0$? $y = \frac{1}{2}x + \frac{1}{2}$? $m_1 = \frac{1}{2}$ $y = 5x - 4$ $\quad m_2 = 5$ $\tan\theta = \left\|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right\| = \left\|\dfrac{\frac{1}{2} - 5}{1 + 5\left(\frac{1}{2}\right)}\right\| = \left\|\dfrac{-4\frac{1}{2}}{3\frac{1}{2}}\right\| = \dfrac{9}{7}$? = 52°8'	**1 – substituting gradients into correct formula** **1- correct answer**
d. $\dfrac{d}{dx}[ln(\sin^{-1} 2x)] = \dfrac{1}{\sin^{-1} 2x} \cdot \dfrac{2}{\sqrt{1-4x^2}}$	**1 – deriving $\sin^{-1}x$** **1 – fully correct**
e. $x = t - 2$ $y = 3t^2 - 1$ From (1) $t = x + 2$ sub in (2) $y = 3(x+2)^2 - 1$ $= 3x^2 + 12x + 12 - 1$ $y = 3x^2 + 12x + 11$	**1 – Cartesian equation**

f.

	Criteria
• writes primitive and substitutes for x	1
• evaluates in simplest surd form	1

Answer

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec x \tan x\, dx = \left[\sec x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \sec\tfrac{\pi}{3} - \sec\tfrac{\pi}{4} = 2 - \sqrt{2}$$

Solutions Question 2 2008	Marks/Comments
a. i. $\sin\theta\sec\theta = \sin\theta.\dfrac{1}{\cos\theta}$ = tan ? ii. $\sin\theta\sec\theta = \sqrt{3}$ i..e. $\tan\theta = \sqrt{3}$ $\therefore$? $= \dfrac{\pi}{3}, \dfrac{4\pi}{3}$	**1 mark** **1 mark for both answers**
b. $1 + 3 + 9 + \ldots\ldots + 3^{n-1} = \frac{1}{2}\left(3^n - 1\right)$ Test n = 1. $3^{1-1} = \frac{1}{2}\left(3^1 - 1\right) = 1$ $\therefore$ true for $n = 1$ $n = k$ *i.e.* $1 + 3 + 9 + \ldots\ldots + 3^{k-1} = \frac{1}{2}\left(3^k - 1\right)$ *Prove true for n=k + 1* i.e. $1 + 3 + 9 + \ldots\ldots + 3^{k-1} + 3^k = \frac{1}{2}\left(3^k - 1\right) + 3^{(k+1)-1}$ $= \frac{1}{2}\left(3^k - 1\right) + 3^k$ $= \frac{1}{2}\left(3^k - 1 + 2.3^k\right)$ $= \frac{1}{2}\left(3.3^k - 1\right)$ $= \frac{1}{2}\left(3^{k+1} - 1\right)$ Which is in the form $\frac{1}{2}\left(3^n - 1\right)$ where $n = k + 1$ $\therefore$ True for $n = k + 1$ when true for $n = k$, But it is true for $n = 1$ $\therefore$ True for $n = 1 + 1 = 2$ And $2 + 1 = 3$ Etc. Hence by Mathematical Induction $1 + 3 + 9 + \ldots\ldots + 3^{n-1} = \frac{1}{2}\left(3^n - 1\right)$	**1 mark for test $n = 1$** **1 mark** **1 mark** **1 mark**

c)

Solutions Question 2 2008 (Cont'd)	
i • finds value of R	1
• finds value of α	1
ii • solves equation for x	1

Answer

i. $\cos x-\sqrt{3}\sin x = 2\left(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x\right)$

$= 2\left(\cos\frac{\pi}{3}\cos x-\sin\frac{\pi}{3}\sin x\right)$

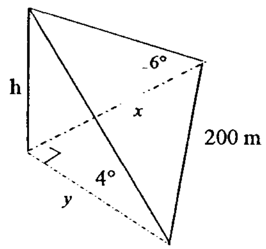
$= 2\cos\left(x+\frac{\pi}{3}\right)$

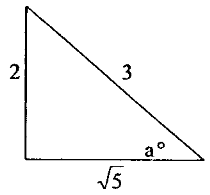
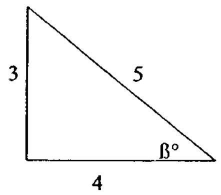
ii. $\cos x-\sqrt{3}\sin x=-2, \quad 0\le x\le 2\pi$

$\cos\left(x+\frac{\pi}{3}\right)=-1, \quad \frac{\pi}{3}\le x+\frac{\pi}{3}\le 2\pi+\frac{\pi}{3}$

$x+\frac{\pi}{3}=\pi$

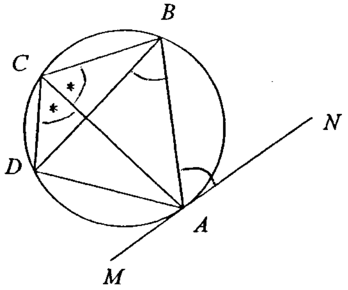
$x=\frac{2\pi}{3}$

d.  $\tan 6° = \frac{h}{x}$, $x=\frac{h}{\tan 6°}$; $\tan 4° = \frac{h}{y}$, $y=\frac{h}{\tan 4°}$	1 mark for the expressions for x and y
Now, $(200)^2=\left(\frac{h}{\tan 6°}\right)^2+\left(\frac{h}{\tan 4°}\right)^2$ $=h^2\left(\frac{1}{\tan^2 6°}+\frac{1}{\tan^2 4°}\right)$	1 mark for the use of Pythagoras
$h=\sqrt{(200)^2\div\left(\frac{1}{\tan^2 6°}+\frac{1}{\tan^2 4°}\right)}$ $=11{\cdot}64381501$ $=12$ metres (nearest metre)	1 Mark for correct answer
e. x^3-2x^2-4x+7 divisible by $(x+3)$ $\therefore f(-3)=$ remainder $\therefore$ Remainder $=(-3)^3-2(-3)^2-4(-3)+7$ $=-27-18+12+7$ $=-26$	1 mark

Solutions Question 3 2008	Marks/Comments
a. By Pythagoras RM $=\sqrt{5^2-4^2}=3$	**1 for length RM**
Since the line through the centre of a circle perpendicular to a chord bisect the chord, RQ = 6, so PR = 14. Now $(PT)^2 = PQ\,.\,PR$; $x^2=(8)(14)$; $x^2=112$; $x=\sqrt{112}=10{\cdot}583\ldots\ldots$ $=10.6$ units (1dp)	**1 – correct answer**
b. $\alpha=\sin^{-1}\left(\frac{2}{3}\right)$ $\beta=\sin^{-1}\left(\frac{3}{5}\right)$  	
$\sin\alpha=\frac{2}{3}$, $\cos\alpha=\frac{\sqrt{5}}{3}$; $\sin\beta=\frac{3}{5}$, $\cos\beta=\frac{4}{5}$	**1 for values of cos a and cos ß**
Sin (a + ß) = sin a cos ß + sin ß cos a $=\frac{2}{3}\cdot\frac{4}{5}+\frac{3}{5}\cdot\frac{\sqrt{5}}{3}$	**1 use of result**
$=\frac{8+3\sqrt{5}}{15}$	**1 answer**
c. $\int\cos^2 4x\,dx$ — $\cos 2x=2\cos^2 x-1$; $2\cos^2 x=\cos 2x+1$; $\cos^2 x=½(\cos 2x+1)$; $\therefore\cos^2 4x=½(\cos 8x+1)$	
$\int\cos^2 4x\,dx=\frac{1}{2}\int(\cos 8x+1)dx$	1
$=\frac{1}{2}\left[\frac{1}{8}(\sin 8x)+x\right]+c$ $=\frac{\sin 8x}{16}+\frac{x}{2}+c$	1
Solutions Question 3 2008 (Cont'd)	

d. $\int_3^4 \frac{x^2}{(x-2)^2}$ $u = x - 2 \quad du = dx$ $x = u + 2$ $x = 3,\ u = 1$ $x = 4,\ u = 2$ $= \int_3^4 \frac{x^2}{(x-2)^2}$ $= \int_1^2 \frac{(u^2 + 4u + 4)du}{u^2}$	1 integral using change of variable
$= \int_1^2 \left(1 + \frac{4}{u} + 4u^{-2}\right) du$ $= \left[u + 4\ln u - 4u^{-1}\right]_1^2$	1
$= 2 + 4\ln 2 - \frac{4}{2} - (1 + 4\ln 1 - \frac{4}{1})$ $= 4\ln 2 - (1 + 0 - 4)$ $= 4\ln 2 - (-3)$ $= 4\ln 2 + 3$	1
e. $1 + \cos^2 x + \cos^4 x + \cos^6 x + \ldots\ldots\ldots\ldots$ $a = 1, r = \cos^2 x \qquad S_\infty = \frac{a}{1-r} = \frac{1}{1-\cos^2 x}$	1
$= \frac{1}{\sin^2 x}$ $= \operatorname{cosec}^2 x$	1

→ QUESTION (4):



1 $\angle BAN = \angle BCA$ (angle between tangent and chord drawn to point of contact is equal to angle subtended by the chord in the alternate segment)

1 $\angle BCA = \angle DCA$ (given that AC bisects $\angle BCD$)

1 $\angle DCA = \angle DBA$ (angles subtended at the circumference by the same arc DA are equal)

1 $\therefore MAN \parallel DB$ (equal alternate angles on transversal BA since $\angle BAN = \angle DBA$)

b. $f(x) = x^3 - 3x^2 + 1 \quad f(0\cdot5) = 0\cdot5^3 - 3(0\cdot5)^2 + 1 = 0\cdot375$ $f'(x) = 3x^2 - 6x \quad f'(0\cdot5) = 3(0\cdot5)^2 - 6(0\cdot5) = -2\cdot25$	1 mark sub into formula
$a_1 = a_0 - \frac{f(a_0)}{f'(a_0)} = 0\cdot5 - \frac{0\cdot375}{-2\cdot25} = 0\cdot67$ (2 dp)	1 for answer
c. P($2ap$, $2ap^2$) and Q($2aq$, $2aq^2$) i. $m = \frac{2ap^2 - 2aq^2}{2ap - 2aq} = \frac{2a(p+q)(p-q)}{2a(p-q)} = p + q$	1
$y - 2ap^2 = (p + q)(x - 2ap)$ $y - 2ap^2 = px + qx - 2ap^2 - 2apq$ $y = px + qx - 2apq$	1
ii. $y = \frac{x^2}{2a}$? $x^2 = 2ay \qquad \therefore 4A = 2a$ $A = \frac{2a}{4} = \frac{a}{2}$ $\therefore$ focus is $S\left(0, \frac{a}{2}\right)$	1
$y = px + qx - 2apq$ is a focal chord if passes through S. i.e. $\frac{a}{2} = p(0) + q(0) - 2apq$ $\frac{a}{2} = -2apq$ $a = -4apq$ $pq = -\frac{1}{4}$	1

Solutions Question 4 2008 (Cont'd)	
iii.	
Midpoint PQ $= \left(\dfrac{2ap+2aq}{2}, \dfrac{2ap^2+2aq^2}{2}\right)$	
$= \left(\dfrac{2a(p+q)}{2}, \dfrac{2a(p^2+q^2)}{2}\right)$	
i.e. $x = a(p+q)$ $y = a(p^2+q^2)$	
$p+q = \dfrac{x}{a}$ $= a[(p+q)^2 - 2pq]$	
$= a\left[\left(\dfrac{x}{a}\right)^2 - 2\left(-\dfrac{1}{4}\right)\right]$	**1 achieving this step**
$y = \dfrac{x^2}{a} + \dfrac{a}{2}$	
$ay = x^2 + \dfrac{a^2}{2}$	
$ay - \dfrac{a^2}{2} = x^2$	
$x^2 = a\left(y - \dfrac{a}{2}\right)$	**1 for equation**
Which is a parabola with vertex $\left(0, \dfrac{a}{2}\right)$, *focal length* $\dfrac{a}{4}$	

QUESTION 5

Solutions Question 5 2008	Marks/Comments
a. $x^3 - 4x^2 + 5x - 1 = 0$	
i. $\alpha + \beta + \gamma = -\dfrac{b}{a} = \dfrac{-(-4)}{1} = 4$	1
ii. $\alpha\beta\gamma = \dfrac{-d}{a} = \dfrac{-(-1)}{1} = 1$	1
iii. The equation with roots 2a, 2ß and 2? takes the form	
$x^3 - (2\alpha + 2\beta + 2\gamma)x^2 + (2\alpha 2\beta + 2\alpha 2\gamma + 2\beta 2\gamma)x - 2\alpha 2\beta 2\gamma$	
i.e. $x^3 - 2(\alpha+\beta+\gamma)x^2 + 4(\alpha\beta + \alpha\gamma + \beta\gamma)x - 8\alpha\beta\gamma = 0$	
Now,	1
$\alpha + \beta + \gamma = 4$	
$\alpha\beta + \alpha\gamma + \beta\gamma = \dfrac{c}{a} = 5$	1
$\alpha\beta\gamma = 1$	
$\therefore$ Equation is $x^3 - 2(4)x^2 + 4(5)x - 8(1) = 0$	
i.e. $x^3 - 8x^2 + 20x - 8 = 0$	

i. $f(x)=x+\dfrac{1}{x}$ for $x\ge 1$

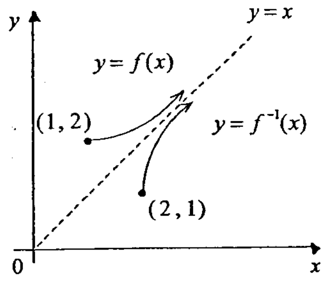
$f'(x)=1-\dfrac{1}{x^2}>0$ for $x>1$

$\therefore f(x)$ is increasing for $x>1$

$f''(x)=\dfrac{2}{x^3}>0$ for $x>1$

$\therefore y=f(x)$ is concave up for $x>1$

ii.



iii. $y=x+\dfrac{1}{x}$, $x\ge 1$ and $y\ge 2$

Rearrangement gives $x^2-xy+1=0$, $x\ge 1$ and $y\ge 2$

Considering this quadratic in x: $x=\dfrac{y\pm\sqrt{y^2-4}}{2}$, $x\ge 1$ and $y\ge 2$

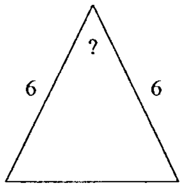
Clearly the branch $x=\dfrac{y-\sqrt{y^2-4}}{2}$ contains points for which $x<1$.

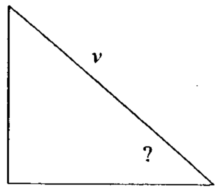
Hence expressing x as the subject of $y=f(x)$, $x=\dfrac{y+\sqrt{y^2-4}}{2}$, $y\ge 2$.

Interchanging x and y, the inverse function is $f^{-1}(x)=\dfrac{x+\sqrt{x^2-4}}{2}$, $x\ge 2$

Solutions Question 5 2008 (Cont'd)	Marks/Comments
c. $\dfrac{d^2x}{dt^2}=\dfrac{d}{dt}\left(\dfrac{dx}{dt}\right)$	
$=\dfrac{dv}{dt}$	1
$=\dfrac{dv}{dx}\times\dfrac{dx}{dt}$	
$=\dfrac{dv}{dx}\times v$	1
$=\dfrac{dv}{dx}\times\dfrac{d}{dv}\left(\dfrac{1}{2}v^2\right)$	
$=\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$	
$\therefore \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)=\dfrac{d^2x}{dt^2}$	

Solutions Question 6 2008	Marks/Comments
a.	
i. $P(x)=x^3-kx^2+kx-1$ $P(1)=1-k+k-1=0$ ii. Product of the roots is 1. Hence if the roots are $1,\alpha,\beta$, then $\alpha\beta=1$. $\therefore\frac{1}{\alpha}$ is the 3rd root. iii. $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$ $\therefore \alpha^2+\frac{1}{\alpha^2}+1^2=k^2-2k$ $\therefore \alpha^2+\frac{1}{\alpha^2}=k^2-2k-1$	
b.	
i. $P=2000+Ae^{kt}$ $\dfrac{dP}{dt}=kAe^{kt}$ $=k(2000+Ae^{kt}-2000)$ $=k(P-2000)$	1 mark
ii. $P=2000+Ae^{kt}$ *when t = 0, P = 3000* $3000=2000+Ae^{0}$ $\therefore A=1000.$	1 for 'A'
$P=2000+1000e^{kt}$ *when t = 5, P = 8000* $8000=2000+1000e^{5k}$ $6000=1000e^{5k}$ $6=e^{5k}$ $\ln 6=5k$ $k=\ln 6\div 5$ $=0{\cdot}358351893$	1 for 'k'
iii. $P=2000+1000e^{0\cdot358351893\cdot t}$ $6000=2000+1000e^{0\cdot358351893\cdot t}$ $4000=1000\,e^{0\cdot358351893\cdot t}$	1 correct substitution
$4=e^{0\cdot358351893\cdot t}$ $\ln 4=0{\cdot}358351893t$ $t=\ln 4\div 0{\cdot}358351893$ $t=3{\cdot}868528072$ i.e. 3·9 years	1 for number of years
$\dfrac{dP}{dt}=k(P-2000)$ $\dfrac{dP}{dt}=0{\cdot}358351893\,(6000-2000)$ $=1433{\cdot}407572$	1 for rate

Solutions Question 6 2008 (Cont'd)	Marks/Comments
(c)  $\frac{d\theta}{dt} = 0\cdot05$	
$A = ½\, ab \sin C$ $= ½\,(6)(6) \sin ?$ $= 18 \sin ?$ $\frac{dA}{d\theta} = 18 \cos\theta$	
$\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$ $\frac{dA}{dt} = 18 \cos\theta \times 0\cdot05$	1
When $? = \frac{\pi}{6}$ $\frac{dA}{dt} = 18 \cos\frac{\pi}{6} \times 0\cdot05$ $= 0\cdot779422863$ $= 0\cdot78\ cm^2/s$ (2dp)	1

Solutions Question 7 2008	Marks/Comments
a. i. $\ddot{y} = -10$ $\dot{y} = -10t + C$ When $t = 0$, $\dot{y} = 0$, so $C = 0$ $\therefore \dot{y} = -10t$ $y = -5t^2 + C$	**1 for equation for y**
When $t = 0$, $y = 800$, so $C = 800$. $\therefore y = -5t^2 + 800$ When $t = 400$ $400 = -5t^2 + 800$ $-400 = -5t^2$ $t^2 = 80$ $t = \sqrt{80} = 4\sqrt{5}$	**1 for time**
ii.  $\dot{x}_h = v \cos ?$ $\dot{y}_v = v \sin ?$	
$\ddot{x} = 0$ $\ddot{y} = -10$ $\dot{x} = c$ $\dot{y} = -10t + c$ When $t = 0$, $\dot{x}_h = v \cos ?$ When $t = 0$, $\dot{y}_v = v \sin ?$ $\therefore \dot{x}_v = v \cos ?$ $\therefore \dot{y}_v = -10t + v \sin ?$	**1 for working**
$x = vt \cos ?$ $y = -5t^2 + vt \sin ?$ When $t = 0$, $x = 0$, $\therefore c = 0$ When $t = 0$, $y = 0$, $\therefore c = 0$ $\therefore x = vt \cos ?$ $y = -5t^2 + vt \sin ?$	**1 mark for each equation (2)**

Solutions Question 7 2008 (Cont'd)	Marks/Comments
iii. Successfully intercepts the bomb if when $t = 4\sqrt{5}$, $x = 800$ *and* $y = 400$. *i.e.* $800 = 4\sqrt{5}\, v \cos ?$ ---------(1) $400 = -5(4\sqrt{5})^2 + 4\sqrt{5}\, v \sin ?$ ---------(2) *From (1)* $v = \frac{800}{4\sqrt{5}\cos\theta} = \frac{200}{\sqrt{5}\cos\theta}$ ---- (3) *Sub (3) in (2)* $400 = -400 + 4\sqrt{5}\left(\frac{200}{\sqrt{5}\cos\theta}\right)\sin ?$ $800 = 800 \tan ?$ $\tan ? = 1$ $? = 45°$ $v = \frac{200}{\sqrt{5}\cos 45°} = 126{\cdot}49$ m/s i.e. 126*m/s*	**1 for working** **1 for angle** **1 for velocity**
b. i. $x = 8\sin 3t.$ $\dot{x} = 24\cos 3t$ $\ddot{x} = -72\sin 3t$ $= -72x$ *Which is in the form* $-n^2x$ ii. Period of motion is $\frac{2\pi}{n}$ i.e. $\frac{2\pi}{3}$ iii. iii. $x = 8\sin 3t.$ *When* $x = 4$ $4 = 8\sin 3t$ $\frac{1}{2} = \sin 3t$ $3t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \ldots\ldots\ldots$ $t = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \ldots\ldots\ldots$	**1 mark** **1 mark** **1 for value of *t***

Now, $\dot{x} = 24\cos 3t$ *When* $t = \frac{\pi}{18}$, $\dot{x} = 24\cos 3\left(\frac{\pi}{18}\right)$ $= 24\cos\frac{\pi}{6}$ $= 12\sqrt{3}$ *cm/s*	**1 for velocity**