

EPPING BOYS' HIGH SCHOOL



YEAR 12 MATHEMATICS (2U) TRIAL HSC EXAMINATION 2008

Time allowed: 3 hours (Plus 5 minutes reading time)

Student Number: _____

Class: 12M **Teacher:** _____

DIRECTIONS TO CANDIDATES:

- ◆ There are 10 questions.
- ◆ **ALL** questions must be attempted.
- ◆ **ALL** questions are of equal value.
- ◆ **Write using black or blue pen.**
- ◆ All necessary working should be shown in every question.
- ◆ Full marks may not be given unless you show working.
- ◆ Full marks may not be awarded for careless or badly arranged work.
- ◆ **Start each question in a new booklet.**
- ◆ Approved calculators may be used.
- ◆ **Do not** use white out on your answer paper.
- ◆ Cross out neatly leaving deleted work able to read.
- ◆ Fill in the information at the top of this page.
- ◆ Borrowing is not allowed.
- ◆ A table of standard integrals is provided at the back of this paper.

1	2	3	4	5	6	7	8	9	10	Total	%
12	12	12	12	12	12	12	12	12	12	/120	100

Total Marks – 120**Attempt Questions 1 – 10****All questions are of equal value****Marks**

Begin each question on a SEPARATE sheet of paper. Extra paper is available.

Question 1 (12 marks) Use a SEPARATE sheet of paper or booklet.

- a) Evaluate $e^{2.4} - 1$ correct to 3 significant figures. 2
- b) Solve $|2x - 4| \leq 2$ 2
- c) If $\frac{4}{2 - \sqrt{3}} = a + b\sqrt{3}$ find the values of a and b . 2
- d) Find the sum of the first ten terms of the series $4\frac{1}{2} + 3 + 1\frac{1}{2} + \dots$ 2
- e) Factorise $2z^2 + 6zy + xz + 3xy$ 2
- f) Find the perpendicular distance from the point $(1, 3)$ to the line $6x - 8y + 5 = 0$ 2

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet.**Marks**a) Differentiate with respect to x

(i) $2x^3 + x^{-3}$ **2**

(ii) $\frac{1}{e^{2x}} - \sin x$ **2**

b) (i) Find $\int \sec^2 x - e^{4x} dx$ **2**

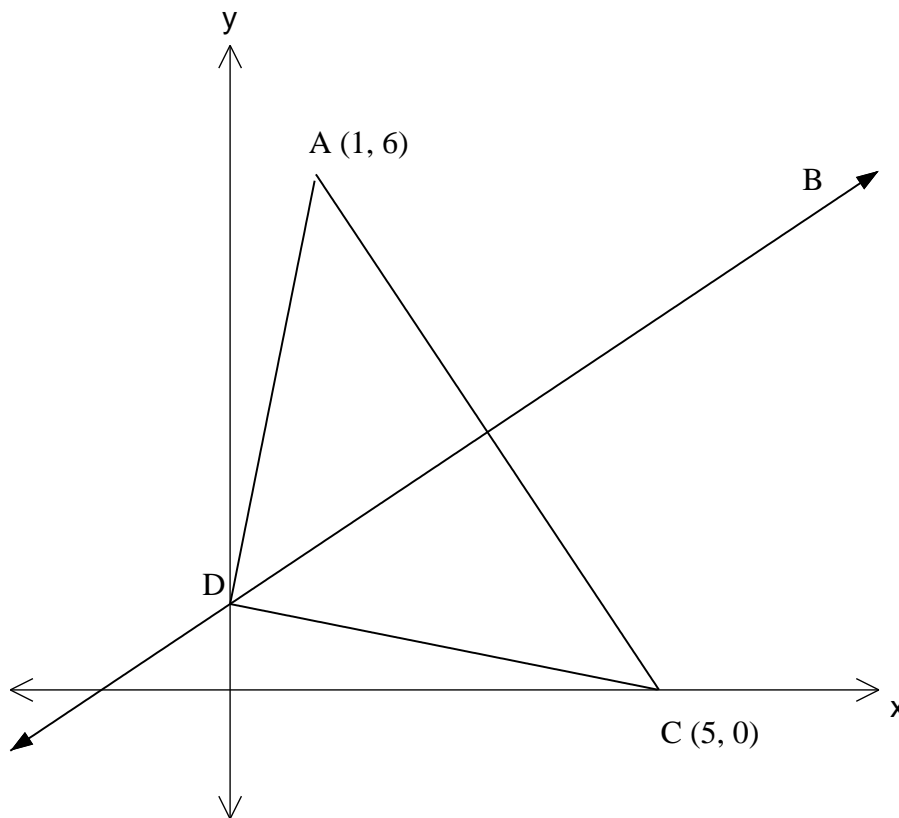
(ii) Evaluate $\int_1^e x^2 + \frac{2}{x} dx$ **3**

c) Find the area enclosed by the curve $y = \cos x$, the line $x = \frac{\pi}{3}$ and the x and y axes. **3****End of Question 2**

Question 3 (12 marks) Use a SEPARATE writing booklet.

Marks

a)



The points A and C have coordinates (1, 6) and (5, 0) respectively.
The line BD has an equation of $2x - 3y + 3 = 0$ and meets the y axis in D.

- | | | |
|------|---|----------|
| i) | The point M is the midpoint of AC. Show that M has coordinates (3, 3). | 1 |
| ii) | Show that M lies on BD. | 1 |
| iii) | Find the gradient of the line AC. | 1 |
| iv) | Show that BD is perpendicular to AC. | 2 |
| v) | Find the distance AC. | 1 |
| vi) | Explain why the quadrilateral ABCD is a kite regardless of the position of B. | 1 |

Question 3 continues on page 5

Question 3 continued**Marks**

- (b) i Find the sum of the first 200 positive integers. **2**
 $1 + 2 + 3 + 4 + \dots + 200$
- ii. The series $1 + 5 + 7 + 11 + \dots + 199$ is formed by omitting from the first 200 positive integers all those which are multiples of 2 or 3. **3**
Find the sum of the series.

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet.

Marks

a) Show that:

2

$$\sqrt{\frac{\operatorname{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = \tan x$$

b) Express 2.12^c (radians) as an angle in degrees correct to the nearest minute

1

c) What is the domain and range for $y = \sqrt{9 - x^2}$

2

d) Ally and Bella are standing on level ground on opposite sides of a tower which is 142 metres high. Ally is due west and measures the angle of elevation of the top of the tower as 16° . Bella is due east and measures the angle of elevation of the top of the tower as 20° .

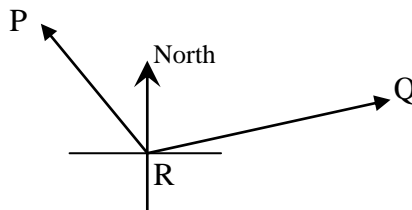
i) Draw a diagram to illustrate this information.

1

ii) Calculate the distance between Ally and Bella.

2

e) Peta and Quentin are pilots of two light planes which leave Resthaven station at the same time. Peta flies on a bearing of 330° at a speed of 180 km/h and Quentin flies on a bearing of 080° at a speed of 240 km/h. Copy the diagram below onto your answer page and mark the information on the diagram.



i) How far apart are Peta and Quentin after 2 hours?

2

ii) What is the bearing of Quentin from Peta after 2 hours.

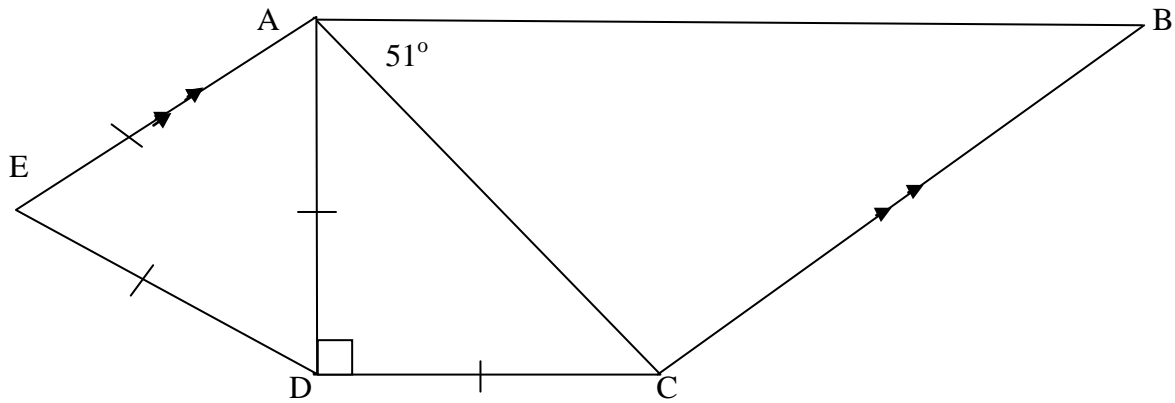
2

End of Question 4

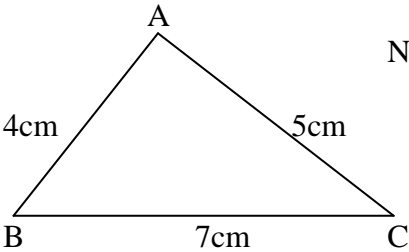
Question 5 (12 marks) Use a SEPARATE writing booklet.

Marks

- a) In the diagram below $AE = ED = AD = DC$, $\angle ADC = 90^\circ$ and $AE \parallel BC$.
 $\angle BAC = 51^\circ$



- i) Find the size of $\angle EAB$. Give reasons for your answer. 3
- ii) Find the size of $\angle ABC$. Give reasons for your answer. 1
- b) Solve, giving your answer(s) in exact form : $2x^2 - 5x - 4 = 0$. 2

- c) 2
- 

NOT TO SCALE

In the diagram above, find the size of the largest angle.
 Give your answer correct to the nearest degree.

- d) Simplify: $\frac{5}{m-2} - \frac{2}{m-3}$ 2
- e) Solve the pair of simultaneous equations 2

$$3x - y = 10$$

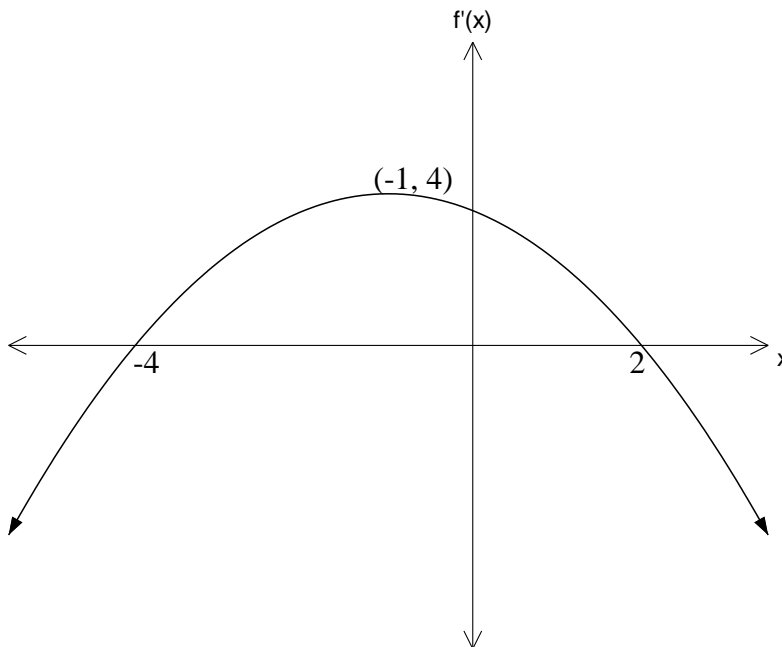
$$x = y + 2$$

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet.

Marks

- a) For the function $y = x^6 - 6x^4$
- i) Find the x coordinates of the points where the curve crosses the axes. **2**
 - ii) Find the coordinates of the stationary points and determine their nature. **4**
 - iii) Find the coordinates of the points of inflexion. **2**
 - iv) Sketch the graph of $y = x^6 - 6x^4$ indicating clearly the intercepts, stationary points and points of inflexion. **2**
- b) For a certain function $y = f(x)$, the sketch of $y = f'(x)$ is shown.



Give the x coordinates of the stationary points on $y = f(x)$ and indicate if they are maxima or minima. **2**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet.

Marks

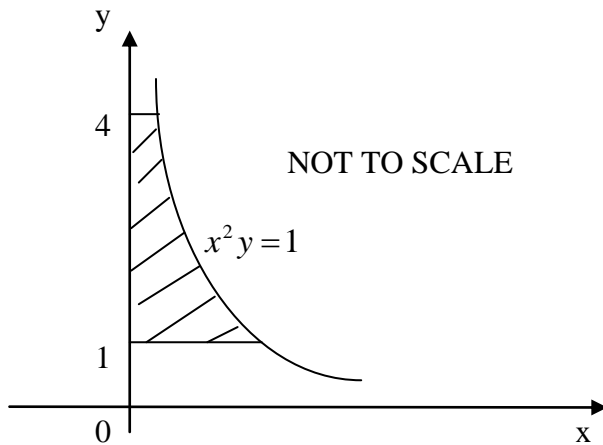
- a) For the parabola with equation $x^2 = -8y$.
- i) Find the coordinates of the focus (S) of the parabola. **1**
 - ii) Find the equation of the directrix of the parabola. **1**
 - iii) Show that the point A(-8, -8) lies on the parabola. **1**
 - iv) Find the equation of the focal chord of the parabola which passes through A. **2**
 - v) Find the equation of the tangent to the parabola at A. **2**
- b)
- i) Show that the curves $y = x^2 - 3x$ and $y = 5x - x^2$ intersect at the points (0, 0) and (4, 4). **2**
 - ii) Find the area enclosed between the two curves. **3**

End of Question 7

Question 8 (12 marks) Use a SEPARATE writing booklet.

Marks

a)

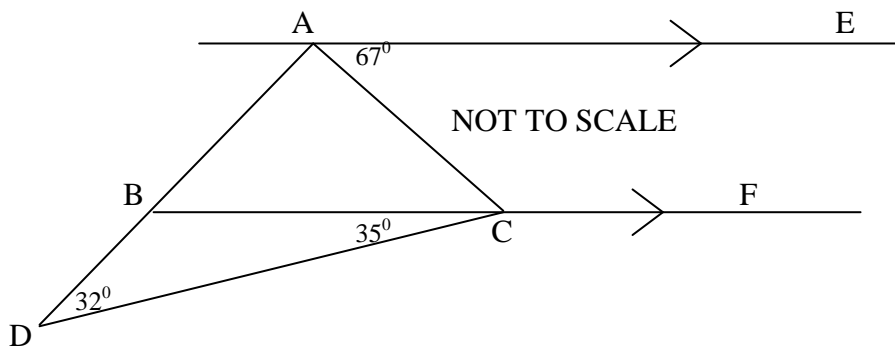


The shaded region above shows the area bounded by the graph $x^2 y = 1$, ($x > 0$), the y-axis and the lines $y = 1$ and $y = 4$.

Find the volume of the solid of revolution formed when the shaded region is rotated about y-axis. Give your answers in exact form. **4**

b) Use the change of base rule to evaluate $\log_8 4$. **1**

c)



In the diagram above AE is parallel to BF.
 $\angle ADC = 32^\circ$, $\angle BCD = 35^\circ$ and $\angle CAE = 67^\circ$

i) Show that $\triangle ABC$ is isosceles. **3**

ii) Find the size of $\angle BAC$. **1**

Question 8 continues on page 11

Question 8 continued**Marks**

- d) Find an approximation for $\int_1^3 g(x)dx$ by using Simpson's Rule with the values in the table below.

2

x	1	1.5	2	2.5	3
$g(x)$	12	8	0	3	5

- e) Evaluate $\sum_{n=2}^5 n^2 - 1$

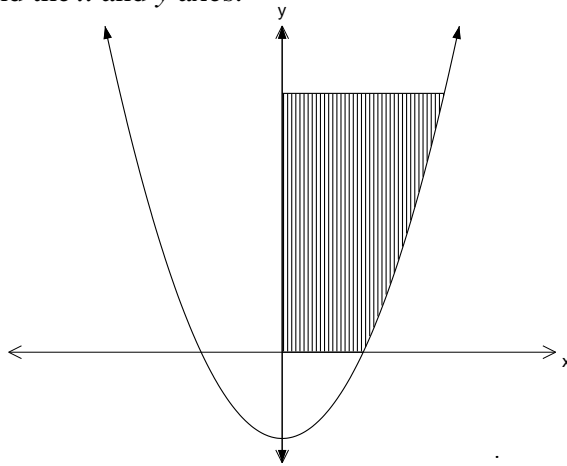
1**End of Question 8**

Question 9 (12 marks) Use a SEPARATE writing booklet.

Marks

- a) The diagram shows the region bounded by the curve $y = 2x^2 - 2$ the line $y = 6$ and the x and y axes.

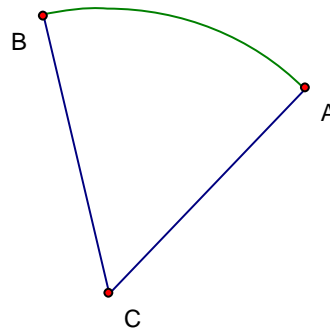
3



Find the volume of the solid of revolution formed when the region is rotated about the y axis.

- b) The sector ABC has $r = 5\text{cm}$ and the arc length $BA = 5\text{cm}$.

Calculate the area of the sector.



1

- c) Find the equation of the tangent to $y = 2e^x$ at the point $(0, 2)$.

2

- d) Bernice contributes to a superannuation fund. She contributes \$250 at the start of every quarter. The investment pays 8%pa interest, compounding quarterly. She continues making contributions for 30 years.

(i) How much does she contribute altogether?

1

(ii) What is the value of her initial \$250 investment at the end of the 30 years?

1

(iii) Find the total value of her superannuation.

3

(iv) How much of her superannuation lump sum is interest?

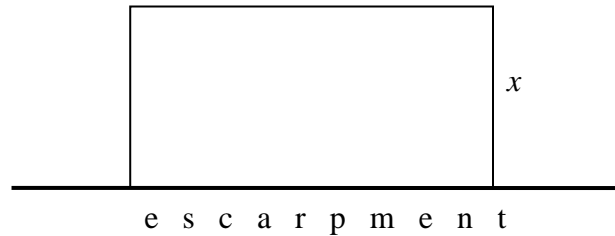
1

End of Question 9

Question 10 (12 marks) Use a SEPARATE writing booklet.

Marks

- a) A farmer wishes to build a rectangular enclosure for his sheep. Fortunately he can use a sandstone escarpment as one side of his rectangle.



He has sufficient fencing material for 200m of fence.

- (i) If we let one side of the rectangle be x , write an expression for the area of the enclosure in terms of x . 2
- (ii) Find the maximum area enclosure the farmer can build. 3
Be sure to justify that this area is a maximum.
- b) Consider the parabola $2y = x^2 - 4x$.
- i) Rewrite it in the form $4a(y - k) = (x - h)^2$ 2
- ii) Give the coordinates of the focus. 1
- iii) Give the equation of the directrix. 1
- c) (i) Find $\int \cos(4x)dx$ 1
- (ii) Evaluate $\int_1^{e^4} \frac{x}{x^2 + 4} dx$ 2

End of Examination

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

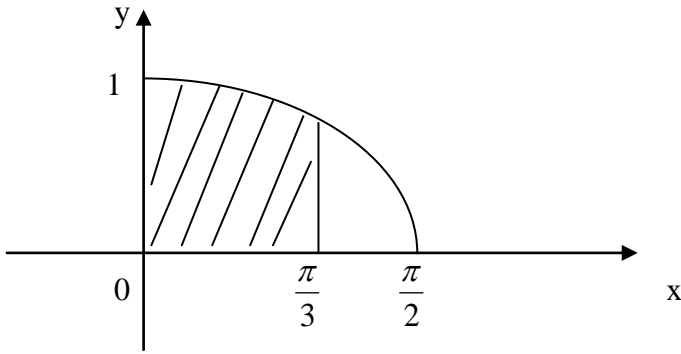
$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

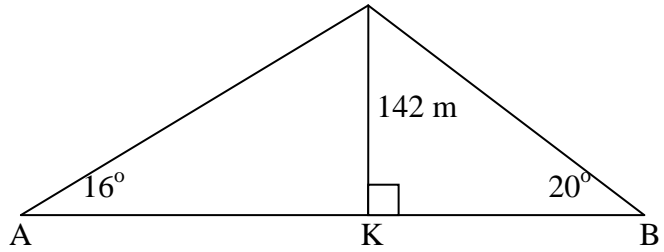
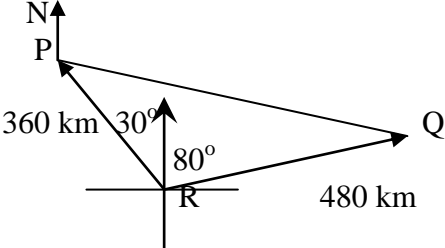
NOTE : $\ln x = \log_e x, \quad x > 0$

**EPPING BOYS' HIGH SCHOOL
YEAR 12 MATHEMATICS (2U)
2008 TRIAL HSC EXAMINATION
SOLUTIONS**

Question 1		Trial HSC Examination- Mathematics
Part	Solution	Marks
(a)	$e^{2.4} - 1 \approx 10.023 \approx 10.0$ (3 sig fig)	2
(b)	$ 2x - 4 \leq 2$ $-2 \leq 2x - 4 \leq 2$ $2 \leq 2x \leq 6$ $1 \leq x \leq 3$	2
(c)	$\frac{4}{2 - \sqrt{3}} = a + b\sqrt{3}$ $\frac{4}{2 - \sqrt{3}} = \frac{4}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$ $= \frac{8 + 4\sqrt{3}}{4 - 3}$ $a + b\sqrt{3} = 8 + 4\sqrt{3}$ $a = 8$ and $b = 4$	2
(d)	$4\frac{1}{2} + 3 + 1\frac{1}{2} + \dots$ Series is arithmetic with $a = 4\frac{1}{2}$ and $d = -1\frac{1}{2}$ $S_n = \frac{n}{2}(2a + (n - 1)d)$ $S_{10} = \frac{10}{2}(9 + (9)(-1\frac{1}{2}))$ $= 5(-4\frac{1}{2})$ $= -22\frac{1}{2}$	2
(e)	$2z^2 + 6zy + xz + 3xy = 2z(z + 3y) + x(z + 3y)$ $= (2z + x)(z + 3y)$	2
(f)	$d = \frac{ 6(1) - 8(3) + 5 }{\sqrt{6^2 + (-8)^2}}$ $= \frac{ -13 }{\sqrt{100}}$ $= 1.3$	2

Question 2		Trial HSC Examination- Mathematics	
Part	Solution		Marks
(a) i)	$\frac{d}{dx}(2x^3 + x^{-3}) = 6x^2 - 3x^{-4}$		2
ii)	$\frac{d}{dx}\left(\frac{1}{e^{2x}} - \sin x\right) = \frac{d}{dx}(e^{-2x} - \sin x)$ $= -2e^{-2x} - \cos x$		2
(b) i)	$\int \sec^2 x - e^{4x} dx = \tan x - \frac{e^{4x}}{4} + c$		2
ii)	$\int_1^e x^2 + \frac{2}{x} dx = \left[\frac{x^3}{3} + 2 \ln x\right]_1^e$ $= \frac{e^3}{3} + 2 \ln e - \frac{1}{3} - 2 \ln 1$ $= \frac{e^3}{3} + 2 - \frac{1}{3} - 0$ $= \frac{e^3 - 1}{3} + 2$		3
(c)	<p>$y = \cos x$</p>  <p>Area = $\int_0^{\pi/3} \cos x dx$</p> <p>$= [\sin x]_0^{\pi/3}$</p> <p>$= \sin \frac{\pi}{3} - \sin 0$</p> <p>$= \frac{\sqrt{3}}{2} \text{ units}^2$</p>		3

Question 3		Trial HSC Examination- Mathematics
Part	Solution	Marks
(a)	Midpoint of (1, 6) and (5, 0).	1
i)	$MP = \left(\frac{1+5}{2}, \frac{6+0}{2} \right) = \left(\frac{6}{2}, \frac{6}{2} \right) = (3, 3)$	
ii)	Show that (3,3) lies on $2x - 3y + 3 = 0$ $LHS = 2(3) - 3(3) + 3$ $= 6 - 9 + 3$ $= 0 = RHS$ So M lies on BD.	1
iii)	Gradient AC = $m_1 = \frac{6-0}{1-5} = \frac{6}{-4} = -\frac{3}{2}$	1
iv)	Find gradient m_2 of BD $2x - 3y + 3 = 0$ $2x - 3y + 3 = 0$ $3y = 2x + 3$ $y = \frac{2}{3}x + 1$ $\therefore m_2 = \frac{2}{3}$ $m_1 \cdot m_2 = -\frac{3}{2} \cdot \frac{2}{3} = -1$ $\therefore \text{BD is perpendicular to AC}$	2
v)	$AC = \sqrt{(5-1)^2 + (0-6)^2} = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$	1
vi)	The lines AC and BD would form the diagonals of the quadrilateral ABCD. BD is the perpendicular bisector of AC from ii and iv above.. The diagonals of a kite meet at right angles and one diagonal bisects the other, so ABCD meets the criteria for a kite.	1
(b)		2
i)	$S_n = \frac{n}{2}(a + l)$ Integers: $1 + 2 + 3 + \dots + 200$ $S_{200} = \frac{200}{2}(1 + 200) = 20\,100$	
ii)	Mult of 2 : $2 + 4 + 6 + \dots + 200$ $S_{100} = \frac{100}{2}(2 + 200) = 10\,100$ Mult of 3: $3 + 6 + 9 + \dots + 198$ $S_{66} = \frac{66}{2}(3 + 198) = 6\,633$ Mult of 6: $6 + 12 + 18 + \dots + 198$ $S_{33} = \frac{33}{2}(6 + 198) = 3\,366$ $\therefore \text{Total Sum} = \text{Integers} - (\text{Mult 2} + \text{Mult 3} - \text{Mult 6})$ $= 20\,100 - 10\,100 - 6\,633 + 3\,366$ $= 6\,733$	3

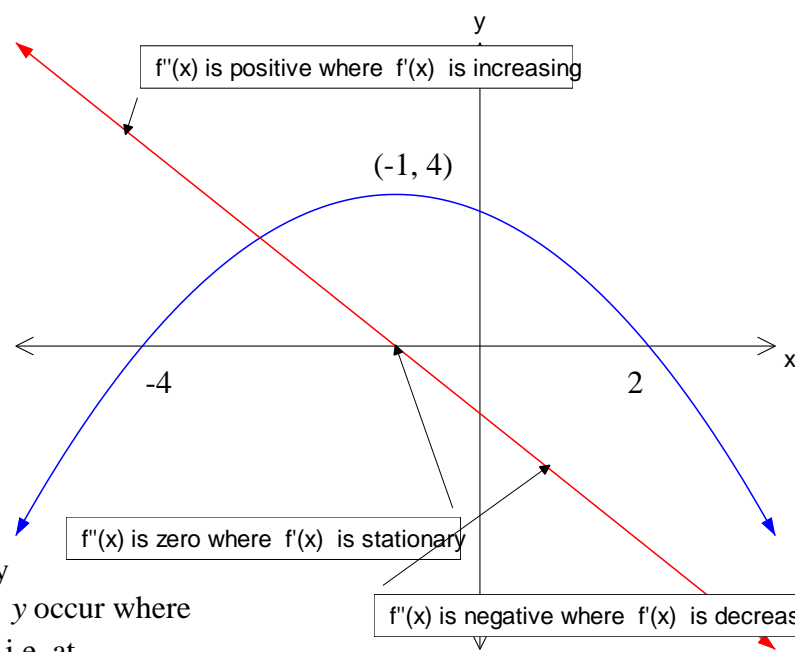
Question 4		Trial HSC Examination- Mathematics
Part	Solution	Marks
(a)	$\sqrt{\frac{\operatorname{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = \sqrt{\frac{1 - \cos^2 x}{\cos^2 x}}$ $= \sqrt{\frac{\sin^2 x}{\cos^2 x}}$ $= \frac{\sin x}{\cos x}$ $= \tan x$	2
(b)	$2.12 \times 180 \div \pi = 121.467\dots = 121^\circ 28'$	1
(c)	Domain $-3 \leq x \leq 3$ Range $0 \leq y \leq 3$	2
(d) i)		1
ii)	$\tan 16^\circ = \frac{142}{AK} \qquad \tan 20^\circ = \frac{142}{KB}$ $AK = \frac{142}{\tan 16^\circ} \qquad KB = \frac{142}{\tan 20^\circ}$ $= 495.2 \qquad = 390.1$ <p>Distance AB = 495.2 + 390.1 = 885 m (nearest m)</p>	2
(e) i)	 <p> $PQ^2 = 360^2 + 480^2 - 2 \times 360 \times 480 \cos 110^\circ$ $PQ^2 = 478202$ $PQ = 692 \text{ km (nearest km)}$ </p>	2

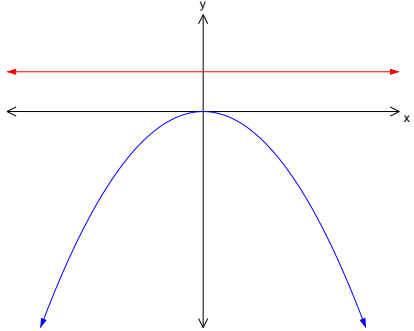
Question 4		Trial HSC Examination- Mathematics
Part	Solution	Marks
(e) ii)	<p>First find $\angle QPR$</p> $\frac{\sin \angle QPR}{480} = \frac{\sin 110^\circ}{692}$ $\sin \angle QPR = \frac{480 \times \sin 110^\circ}{692}$ $\sin \angle QPR = 0.652$ $\angle QPR = 41^\circ$ $\angle NPR = 150^\circ$ $\text{Bearing}(\angle NPQ) = 150^\circ - 41^\circ$ $= 109^\circ$	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Can also be found using cos rule using the 3 sides..</p> </div> <p style="text-align: center; margin-top: 20px;">2</p>

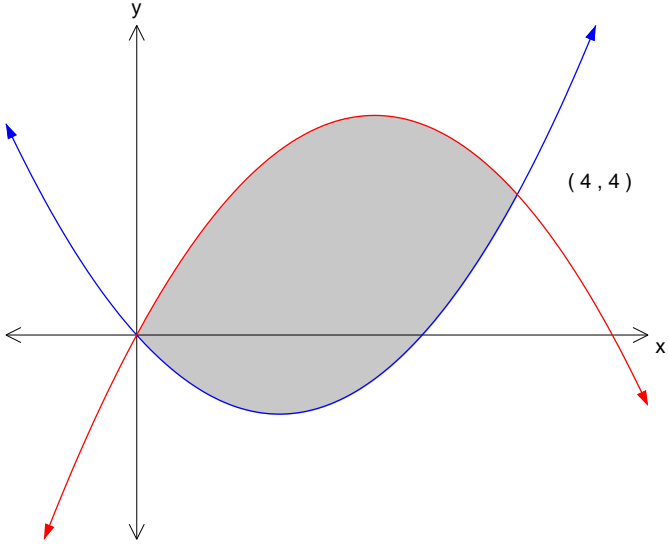
Question 5		Trial HSC Examination- Mathematics
Part	Solution	Marks
(a) i)	$\angle EAD = 60^\circ$ (equilateral Δ) $\angle DAC = 45^\circ$ (isosceles right Δ) $\therefore \angle EAB = \angle EAD + \angle DAC + \angle CAB$ $= 60^\circ + 45^\circ + 51^\circ$ $= 156^\circ$	3
ii)	$\angle ABC = 180^\circ - 156^\circ$ (cointerior \angle on \parallel lines AE and BC) $= 24^\circ$	1
(b)	$x = \frac{+5 \pm \sqrt{5^2 - 4 \times 2 \times (-4)}}{2(2)}$ $= \frac{5 \pm \sqrt{57}}{4}$	2
(c)	$\cos A = \frac{4^2 + 5^2 - 7^2}{2 \times 4 \times 5} = -\frac{1}{5}$ $A = 101.53\dots$ $= 102^\circ$	2
(d)	$\frac{5(m-3) - 2(m-2)}{(m-2)(m-3)}$ $= \frac{5m - 15 - 2m + 4}{(m-2)(m-3)}$ $= \frac{3m - 11}{(m-2)(m-3)}$	2
(c)	$3x - y = 10 \quad (1)$ $x = y + 2 \quad (2)$ $3(y + 2) - y = 10 \quad (3) \text{ sub } (2) \text{ in } (1)$ $3y + 6 - y = 10$ $2y = 4$ $y = 2$ $x = (2) + 2 \quad \text{sub } y \text{ in } (2)$ $x = 4$ solution (4, 2)	2

Question 6		Trial HSC Examination- Mathematics
Part	Solution	Marks
(a) (i)	$y = x^6 - 6x^4$ <p>Crosses axis where</p> $x^6 - 6x^4 = 0$ $x^4(x^2 - 6) = 0$ $x^4(x - \sqrt{6})(x + \sqrt{6}) = 0$ <p>Crosses axis where $x = 0$ and $x = \pm\sqrt{6}$</p>	2
(a) (ii)	$y = x^6 - 6x^4$ $y' = 6x^5 - 24x^3$ $= 6x^3(x^2 - 4)$ $= 6x^3(x - 2)(x + 2)$ $y'' = 30x^4 - 72x^2$ <p>Stationary points where</p> $x = 0, y = 0, y'' = 0$ $x = 2, y = -32, y'' = 192$ $x = -2, y = -32, y'' = 192$ <p>Stationary points $(-2, -32), (0, 0), (2, -32)$</p> $y'' = 30x^4 - 72x^2$ <p>At $x = 0$ $y'' = 0$ so test either side At $x = 1$ $y'' = -42 \therefore$ concave down. At $x = -1$ $y'' = -42 \therefore$ concave down \therefore maximum at $(0, 0)$ At $x = 2$ $y'' = 192 \therefore$ minimum at $(2, -32)$. At $x = -2$ $y'' = 192 \therefore$ minimum at $(-2, -32)$.</p>	4

Question 6		Trial HSC Examination- Mathematics	
Part	Solution		Marks
(a) (iii)	$y'' = 30x^4 - 72x^2$ $= 6x^2(5x^2 - 12)$ $= 6x^2(\sqrt{5}x - 2\sqrt{3})(\sqrt{5}x + 2\sqrt{3})$ $x = 0 \quad y = 0$ $x = \frac{2\sqrt{3}}{\sqrt{5}} = \frac{2\sqrt{15}}{5} \quad y = -20.736$ $x = -\frac{2\sqrt{3}}{\sqrt{5}} = -\frac{2\sqrt{15}}{5} \quad y = -20.736$ <p>Check for changes of concavity From above, no change at (0, 0) but there is a change at $\left(\pm \frac{2\sqrt{15}}{5}, -20.736\right)$ Inflexions at $\left(\pm \frac{2\sqrt{15}}{5}, -20.736\right)$</p>		2
iv)			2

Question 6	Trial HSC Examination- Mathematics	
Part	Solution	Marks
(b)	 <p>Stationary points on y occur where $f'(x) = 0$ i.e. at $x = -4$ and $x = 2$</p> <p>here $f''(x)$ is positive \therefore min turning point at $x = -4$</p> <p>and $x = 2$</p> <p>here $f''(x)$ is negative \therefore max turning point at $x = 2$</p>	2

Question 7		Trial HSC Examination- Mathematics
Part	Solution	Marks
(a) i)	$x^2 = -8y$ $x^2 = -4(2)y$ $a = 2$  <p>Focus is (0, -2)</p>	1
ii)	Directrix is $y = 2$	1
iii)	$x^2 = -8y$ $x^2 = (-8)^2 = 64$ $-8y = -8(-8) = 64$ $\therefore (-8, -8) \text{ lies on the parabola.}$	1
iv)	Chord through (0, -2) and (-8, -8) $m = \frac{-8+2}{-8-0} = \frac{-6}{-8} = \frac{3}{4}$ $y - y_1 = m(x - x_1)$ $y - (-2) = \frac{3}{4}(x - 0)$ $y = \frac{3}{4}x - 2$ $3x - 4y - 8 = 0$	2
v)	$x^2 = -8y$ $y = -\frac{x^2}{8}$ $y' = -\frac{x}{4}$ At A $y' = -\frac{-8}{4} = 2$ $y - (-8) = 2(x - -8)$ $y + 8 = 2x + 16$ $y = 2x + 8$	2

<p>(b) i)</p>	<p>Substitute $y = x^2 - 3x$ into $y = 5x - x^2$</p> $5x - x^2 = x^2 - 3x$ $2x^2 - 8x = 0$ $2x(x - 4) = 0$ $x = 0, \quad y = 0$ $x = 4, \quad y = 4$ <p>Intersect at $(0, 0)$ and $(4, 4)$.</p>	<p>2</p>
<p>ii)</p>	 $\text{Area} = \int_0^4 5x - x^2 dx - \int_0^4 x^2 - 3x dx$ $= \int_0^4 8x - 2x^2 dx$ $= \left[4x^2 - \frac{2x^3}{3} \right]_0^4$ $= \left(64 - \frac{128}{3} \right) - 0$ $= \frac{64}{3} = 21\frac{1}{3}$	<p>3</p>

Question 8		Trial HSC Examination- Mathematics
Part	Solution	Marks
(a)	$x^2 y = 1 \quad \text{ie } x^2 = \frac{1}{y}$ $V = \pi \int_1^4 x^2 dy = \pi \int_1^4 \frac{1}{y} dy$ $= \pi [\ln y]_1^4 = \pi [\ln 4 - \ln 1]$ $= \pi \ln 4 \quad \text{units}^3$	4
(b)	$\log_8 4 = \frac{\log_2 4}{\log_2 8} = \frac{2}{3}$	1
(c)	$\angle EAC = 67^\circ$ (Data) i) $\angle BCA = \angle EAC$ (Alt \angle 's, $AE \parallel BF$) $\therefore \angle BCA = 67^\circ$ $\angle ABC = \angle BDC + \angle BCD$ (Ext $\angle =$ sum of 2 interior opp \angle s) $= 32^\circ + 35^\circ = 67^\circ$ $\therefore \triangle ABC$ is isosceles (Base \angle of an isosceles \triangle)	3
ii)	$\angle BAC + 67^\circ + 67^\circ = 180^\circ$ $\angle BAC = 46^\circ$	1
(d)	$\int_1^3 g(x) dx \approx \frac{1}{6} \{12 + 4(8) + 2(0) + 4(3) + 5\}$ $\approx \frac{61}{6}$ $\approx 10\frac{1}{6}$	2
(e)	$\sum_{n=2}^5 n^2 - 1 = (2^2 - 1) + (3^2 - 1) + (4^2 - 1) + (5^2 - 1)$ $= 3 + 8 + 15 + 24$ $= 50$	1

Question 9		Trial HSC Examination- Mathematics
Part	Solution	Marks
(a)	$y = 2x^2 - 2$ $V = \pi \int_0^6 x^2 dy$ $= \pi \int_0^6 \frac{y+2}{2} dy$ $= \pi \left[\frac{y^2}{4} + y \right]_0^6$ $= \pi \left[\left(\frac{36}{4} + 6 \right) - (0) \right]$ $= 15\pi \text{ u}^3$	3
(b)	$\theta = 1 \text{ and } A = \frac{1}{2} r^2 \theta$ $A = 0.5 \times 5^2 \times 1 = 12.5 \text{ cm}^2$	1
(c)	$f'(x) = 2e^x \quad f'(0) = 2 \text{ slope of tangent}$ $y - 2 = 2x \quad y = 2x + 2$	2
(d)	$4 \times 250 \times 30 = \$30\,000$	1
i)		
ii)	$250 \times 1.02^{120} = \$2\,691.29$	1
iii)	$250 \times (1.02^{120} + 1.02^{119} + 1.02^{118} + \dots + 1.02)$ $250 \times \frac{a(r^n - 1)}{r - 1} = 250 \times \frac{1.02(1.02^{120} - 1)}{1.02 - 1}$ $250 \times 498.02328 = \$124\,505.83$	3
iv)	$\$124\,505.83 - \$30\,000 = \$94\,505.83$	1

Question 10		Trial HSC Examination- Mathematics
Part	Solution	Marks
a) i)	in this case the other side equals $200 - 2x$ Then $A = x(200 - 2x) = 200x - 2x^2$	2
ii)	$\frac{dA}{dx} = 200 - 4x$ which = 0 when $x = 50$ then $A = 100 \times 50 = 5\,000\text{m}^2$ this is a maximum since if we take $w = 49$ then $l = 102$ and $A = 4998$	3
b) i)	$2y = x^2 - 4x$ $2y + 4 = x^2 - 4x + 4$ $4 \times \frac{1}{2}(y + 2) = (x - 2)^2$	2
ii)	Focus is $(2, -1.5)$	1
iii)	Directrix $y = -2.5$	1
c) i)	$I = \frac{1}{4} \sin(4x) + c$	1
ii)	$\int_1^{e^4} \frac{x}{x^2 + 4} \cdot dx = \frac{1}{2} \int_1^{e^4} \frac{2x}{x^2 + 4} \cdot dx$ $\frac{1}{2} [\ln(x^2 + 4)]_1^{e^4} = \frac{1}{2} (\ln(e^8 + 4) - \ln 5)$ $= \ln \sqrt{\frac{e^8 + 4}{5}}$	2

End of Examination