2/3U 1994 TRIAL

Question 1

(a) Find correct to 2 significant figures the value of

$$\sqrt{\frac{7.849x(6.27)^2}{19.863}}$$

(b) Solve $3(x-2)=x+4$

(c) Given $A=2\pi r(r+h)$, find h correct to one deciminal place, given A = 105 , r = 3.2.

(d) Make r the subject of

$$S=\frac{a}{1-r.}$$

(e) Factorize $x^3-8.$

Question 2

(a) Evaluate sin 1.7 correct to 2 decimal places.

(b) Simplify $\frac{1-\cos^2\theta}{\sin\theta\cos\theta}$

(c) Fred receives a pay rise of $4\frac{1}{2}\%$. If his new salary is \$625/week, find his wage before the pay increase.

(d) Find the exact value , showing all working out :

$$27^{-\frac{1}{3}}\times 9^{\frac{3}{2}}$$

(e) Express as a single fraction in simplest form.

$$\frac{2x-3}{2}-\frac{x-2}{5}$$

(2)

Question 3

(a) The points A & B have co-ordinates (2,0) & (0,-2) respectively.

(i) Draw a diagram, clearly marking A & B

(ii) Find the gradient AB

(iii) Show the equation of line ℓ, that passes through A & is perpendicular to AB is given by $x + y = 2$.

(iv) Show that C, the point of intersection of ℓ & the y axis has co-ordinates (0,2).

(v) If D is the point (-2, 0), write down the equation of the circle possing through A, B, C, D.

(vi) Show that the area between the circle ABCD & the quadrilateral ABCD is $4(\pi - 2)$. units^2.

(b) If α & β are the roots of the equation $3x^2 - 15x + 7 = 0$, find

i) $\alpha\beta$ ii) $\frac{1}{\alpha} + \frac{1}{\beta}$.

Question 4.

(a) Differentiate

(i) $x^3 + \frac{1}{x^2}$

(ii) $2xe^{x^2}$

(iii) $\ell n \left(\frac{x}{\cos x} \right)$

(b) Find $\int \sin 2x \; dx$.

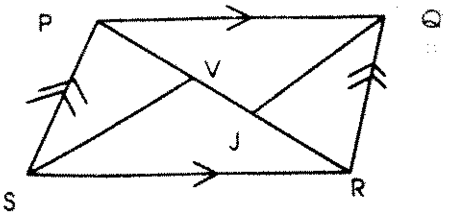
(c) Evaluate the following definite integrals correct to two decimal places.

(i) $\int_1^2 \sqrt{3x - 2} \; dx$

(ii) $\int_1^e \left(\frac{2}{x} + x \right) dx$

Question 5

(a)



PQRS is a parallelogram. TQ bisects angle PQR and VS bisects angle PSR.

i) Copy this diagram onto your answer sheet.

ii) state why $\angle PQR = \angle PSR$

iii) Prove that $\Delta\ PVS$ & $\Delta\ RTQ$ are congruent.

iv) Hence find the length of TV if PR = 20 cm & TR = 8 cm.

(b) The derivative of a function is given by

$$p'(x) = 15(5x-1)^2$$

If $p(0) = 10$, find the equation of $p(x)$.

Question 6

(a) Consider the function $y = \sqrt{9-x^2}$

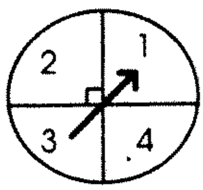
i) State its domain & range

ii) On the number plane shade in the region where the following inequalities hold simultaneously :

$$y < \sqrt{9-x^2} \text{ and } x \geq 0.$$

iii) Does the point (2,-4) lie in this region? Use algebra to justify your answer.

b) The spinner shown below is used in a game.



It is spun twice & the score recorded after each spin. Find the probability that :

i) in each of the two spins the result is 4.

ii) the sum of the two spins is 4.

c) The table below gives the values of f(t) for $0 \leq t \leq 2$.

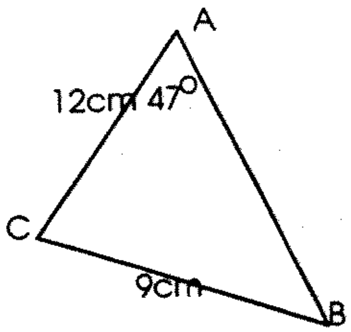
t	0	0.5	1	1.5	2
f(t)	0	0.30	0.37	0.33	0.27

Use the Trapozoidal Rule with 5 function values to evaluate $\int_0^2 f(t)\, dt$ correct to one decimal place.

(4)

Question 7

(a) Triangle ABC is an acute triangle in which AB = 12 cm, BC = 9 cm & angle BAC = 47°.



(i) Find the size of $\angle ACB$, correct to the nearest degree.

(ii) Find the area of ΔABC. Give your answer correct to 2 significant figures.

(b) The first 3 terms of an AP are 50, 43, 36.

(i) Write down the formula for the n^{th} term for this sequence.

(ii) If the last term of the sequence is -27, how many terms are there in this sequence.

iii) Find the sum of the series.

(c) Find the values of k for which the quadratic equation $x^2 - (k+3)x + (k+6) = 0$ has equal and rational roots.

Question 8

(a) For the curve $y = (x-2)^2(x+1)$.

(i) Find the x and y intercepts.

(ii) Find the stationary points, if any and determine their nature.

(iii) Hence, draw a sketch of the curve in the domain $-2 \leq x \leq 3$.

(b) Find the equation of the tangent to the curve $y = x^2 + \frac{2}{x} + 4$ at the point $P(-1,3)$.

(5)

Question 9

(a) For the parabola $x^2 = 8y$ find
(i) the focal length
(ii) the co-ordinates of the focus
(iii) the equation of the directrix.

(b) A stranded motorist on the Nullabor Plains began walking to the nearest watering hole that is located 130 km away. Knowing that she could not cover the entire distance in one day, she planned to walk 75 km the first day & then one third of the previous day's distance each day thereafter. Will she ever reach the watering hole? Justify your answer.

(c) For the function $f(x) = 2\cos x$

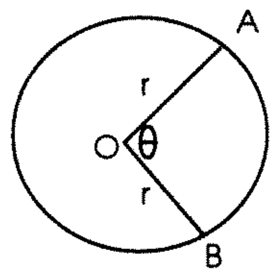
(i) State the range of $f(x)$

(ii) Draw a neat sketch of $f(x) = 2\cos x$ for $0 \le x \le 2\pi$

(iii) Calculate the area enclosed between the curve $y = 2\cos x$, the x axis & the lines $x = \frac{\pi}{2}$ & $x = \frac{3\pi}{2}$

Question 10

(a) In the circle shown arc AB subtends an angle of θ radians at the centre, the radius is r cm., & the perimeter of the sector is 8 cm.



(i) Show, step by step, that $\theta = \frac{8}{r} - 2$ & the area of the sector is given by $A = 4r - r^2$.

(ii) Hence, find the radius of the sector of maximum area when the perimeter of the sector AOB is 8 cm.

(iii) Find the maximum area.

(b) At the beginning of each year for 25 years, Wally invests \$2300 in a superannuation fund, on which he is paid 12% p.a. interest.

Find the amount his investment is worth at the end of the 25th Year.

End of examination

IGHS 2U TRIAL MATHEMATICS 1994

QUESTION 1

(a) $\sqrt{\dfrac{7.849\times(6.27)^2}{19.863}}$

$\doteqdot 3.9414160\$$ ②

$= 3.9$ (to 2 sig fig)

(b) $3(x-2) = x+4$

$3x-6 = x+4$

$2x = 10$

$x = 5$ ②

(c) $A = 2\pi r(r+h)$

$105 = 2\pi\times 3.2(3.2+h)$

$\dfrac{105}{6.4\pi} = 3.2+h$

$h = \dfrac{105}{6.4\pi} - 3.2$

$h \doteqdot 2.02227157$

$h = 2.0$ (1 d.p) ③

(d) $S = \dfrac{a}{1-r}$

$S - Sr = a$

$Sr = S - a$

$r = \dfrac{S-a}{S}$

or

$r = 1 - \dfrac{a}{S}$ ③

(e) $x^3 - 8$

$= x^3 - 2^3$

$= (x-2)(x^2+2x+4)$ ②

⑨

QUESTION 2

(a) $\sin 1.7 \doteqdot 0.99166\$$

$= 0.99$ (2dp) ①

(b) $\dfrac{1-\cos^2\theta}{\sin\theta\cos\theta} = \dfrac{\sin^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$ ③

(c) 104.5% of salary = \$625

1% $= \dfrac{625}{104.5}$

100% $= \dfrac{625}{104.5}\times 100$

$\doteqdot 598.0861$

∴ his wage before the increase was \$598.09 (nearest ¢) ③

(d) $27^{-\frac{1}{3}} \times 9^{\frac{3}{2}}$

$= \dfrac{1}{\sqrt[3]{27}} \times \sqrt{9}^3$

$= \dfrac{1}{3} \times 27$ ②

$= 9$

(e) $\dfrac{2x-3}{2} - \dfrac{x-2}{5}$

$= \dfrac{5(2x-3)-2(x-2)}{10}$

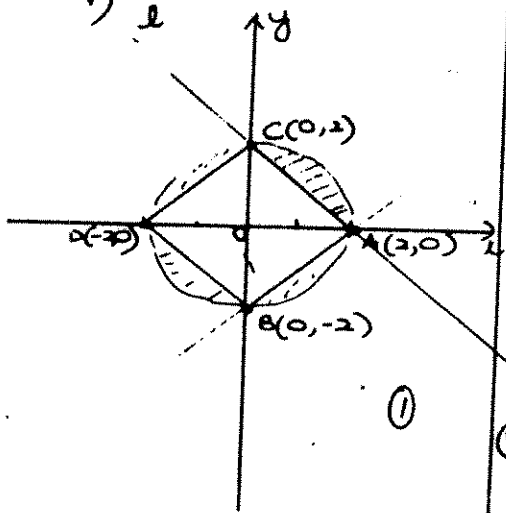
$= \dfrac{10x-15-2x+4}{10}$ ③

$= \dfrac{8x-11}{10}$

QUESTION

(a) A(2,0) B(0,-2)

i)



①

ii) $m_{AB} = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{-2-0}{0-2}$

$m_{AB} = 1.$ ①

iii) ⊥ $m = -1$ at A(2,0)

$m = \dfrac{y-y_1}{x-x_1}$

$-1 = \dfrac{y-0}{x-2}$

$y = -x+2$ ②

l: $x+y=2$ as req'd.

iv) cuts y axis ⇒ x=0

∴ $0+y=2$

$y=2$ ①

⇒ C(0,2) as req'd

v) circle ABCD,

centre (0,0) r=2

∴ $x^2+y^2=4$ ①

vi) Req'd area

= A of circle − A of quad (Rhombus)

$= \pi r^2 - \frac{1}{2}xy$

$= \pi\times 4 - \frac{1}{2}\times 4\times 4$

$= 4\pi - 8$

$A = 4(\pi-2)u^2$ as req'd ②

(b) $3x^2-15x+7=0$

a=3 b=−15 c=7

i) $\alpha\beta = \dfrac{c}{a}$

$= \dfrac{7}{3}$

$\alpha\beta = 2\frac{1}{3}$ ②

ii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta}$

$= \dfrac{-\frac{b}{a}}{\frac{c}{a}}$

$= \dfrac{15}{3}\times\dfrac{3}{7}$

$= \dfrac{15}{7}$ ②

∴ $\dfrac{1}{\alpha}+\dfrac{1}{\beta} = 2\frac{1}{7}$

⑬

QUESTION 4

(a) i) let $y = x^3 + \frac{1}{x^2}$

$= x^3 + x^{-2}$

$\frac{dy}{dx} = 3x^2 - 2x^{-3}$

$= 3x^2 - \frac{2}{x^3}$ (2)

ii) let $y = 2xe^{x^2}$

$\frac{dy}{dx} = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$

$= e^{x^2} \cdot 2 + 2x \cdot e^{x^2} \cdot 2x$

$= 2e^{x^2} + 4x^2e^{x^2}$

$\frac{dy}{dx} = 2e^{x^2}(1 + 2x^2)$ (2)

(iii) let $y = \ln\left(\frac{x}{\cos x}\right)$

$= \ln x - \ln \cos x$

$\frac{dy}{dx} = \frac{1}{x} - \frac{-\sin x}{\cos x}$

$\frac{dy}{dx} = \frac{1}{x} + \tan x$ (2)

(b) $\int \sin 2x \, dx$

$= -\frac{1}{2}\cos 2x + c$ (1)

(c) i) $\int_1^2 \sqrt{3x-2}\, dx$

$= \int_1^2 (3x-2)^{\frac{1}{2}} dx$

$= \left[\frac{(3x-2)^{\frac{3}{2}}}{\frac{3}{2} \times 3}\right]_1^2$

$= \frac{2}{9}\left[\sqrt{(3x-2)^3}\right]_1^2$

c i) cont'd

$= \frac{2}{9}[8 - 1]$

$= \frac{14}{9}$

$= 1\frac{5}{9}$ (2)

ii) $\int_1^e \left(\frac{2}{x} + x\right) dx$

$= \left[2\ln x + \frac{x^2}{2}\right]_1^e$

$= \left(2\ln e + \frac{e^2}{2}\right) - \left(2\ln 1 + \frac{1}{2}\right)$

$= 2 + \frac{e^2}{2} - 0 - \frac{1}{2}$

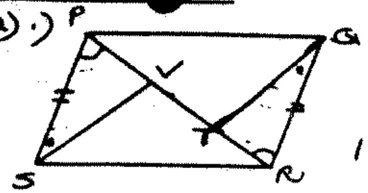
$= \frac{3}{2} + \frac{e^2}{2}$

$= \frac{e^2+3}{2}$ (3)

(7)

QUESTION 5

(a) i)



1

ii) $\angle PQR = \angle PSR$

(opp. $\angle$'s of $\parallel$gram are equal) 1

iii) In Δ's PVS & RTQ

i) PS = QR (opp sides par~grams are =)

ii) $\angle SPR = \angle QRP$

(alt. $\angle$'s, SP $\parallel$ RQ)

iii) $\angle PSV = \angle RQT$ ($\angle PQR = \angle PSR$ from part (ii) & SV, QT bisects these $\angle$'s ~ given)

4

$\therefore \Delta PVS \cong \Delta RTQ$ (AAS)

iv) PR = 20cm & TR = 8cm

Now, TR = PV (corres sides in congruent Δ's)

$\therefore$ PV = 8cm.

$\therefore$ TV = PR − (PV + TR)

= 20 − (8 + 8)

TV = 4 cm 2

(b) $P'(x) = 15(5x-1)^2$

$P(x) = \frac{15(5x-1)^3}{3 \times 5} + c$

$10 = (-1)^3 + c$

$\therefore c = 11$

$\therefore P(x) = (5x-1)^3 + 11$

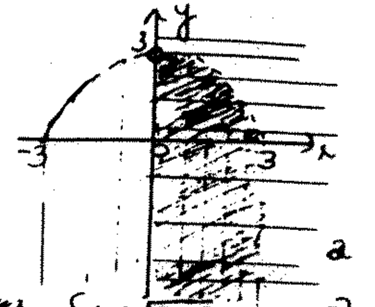
QUESTION 6:

(a) $y = \sqrt{9 - x^2}$

i) D: $-3 \le x \le 3$ 1

R: $0 \le y \le 3$ 1

ii)



2

$= \{y < \sqrt{9-x^2} \cap x \geq 0\}$

iii) (2, −4) lie in region??

sub. in $y < \sqrt{9-x^2}$

$-4 < \sqrt{9-4}$

$-4 < \sqrt{5}$ True.

sub in $x \geq 0$.

$2 \geq 0$ True.

$\therefore$ (2, −4) lies in region 2

$\{y < \sqrt{9-x^2} \cap x \geq 0\}$

(b) i) $P(4,4) = \frac{1}{4} \times \frac{1}{4}$

$P(4+4) = \frac{1}{16}$. 1

ii) P(sum 4) = P(1,3) + P(2,2) + P(3,1)

$= \left(\frac{1}{4} \times \frac{1}{4}\right) \times 3$

P(sum 4) $= \frac{3}{16}$ 2

(N)

…STION 6 (cont'd)

$\int_0^2 f(t)\,dt \doteq \frac{h}{2}\left[f(t_0)+f(t_5) + 2\left[f(t_2)+f(t_3)+f(t_4)\right]\right]$

NB $h=\frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$

$\int^2 f(t)\,dt \doteq \frac{0.5}{2}\left[0+0.27 + 2(0.3+0.37+0.3)\right]$

$\int^2 f(t)\,dt = 0.5675 = 0.6$ (1 dp) (3)

QUESTION 7 :-

i)

ii) $\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin B}{12} = \frac{\sin 47°}{9}$

$\sin B = \frac{12 \sin 47°}{9}$

iii) $\angle B = 77°12'$ (nearest min) $= 77°$ (nearest °)

$\therefore \angle ACB = 180° - (47° + 77°)$

$\angle ACB = 56°$ (nearest °) (2)

iv) $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2}\times 12 \times 9 \times \sin 56$

$\doteq 44.76802892$ (1)

$A = 45\ cm^2$ (2 sig fig)

(b) AP: 50, 43, 36, . . .

$a=50,\ d=-7$

i) $T_n = a+(n-1)d$

$T_n = 50+(n-1)\times -7$

$= 50 - 7n + 7$

$T_n = 57 - 7n$ (2)

ii) $T_n = -27 \quad n=?$

$-27 = 57 - 7n$

$7n = 57+27$

$= 84$

$n = 12$ (2)

∴ 12 terms in sequence

iii) $S_n = \frac{n}{2}(a+l)$

$S_{12} = \frac{12}{2}(50+-27)$

$S_{12} = 138$ (2)

∴ sum of series is 138

(c) $x^2 - (k+3)x + (k+6) = 0$.

$a=1;\ b=-(k+3)\ c=k+6$.

for equal + rational roots, $\Delta = 0$

ie $b^2-4ac = 0$.

$k^2+6k+9-4(k+6)=0$

$k^2+6k+9-4k-24=0$.

$k^2+2k-15=0$.

$(k+5)(k-3)=0$.

$\therefore k=-5$ or $k=3$ (3)

QUESTION 8 -

(a) $y=(x-2)^2(x+1)$

i) cuts y axis ⇒ $x=0$

ie $y=(-2)^2\times 1$.

$y = 4 \Rightarrow (0,4)$ (1)

cuts x axis ⇒ $y=0$

ie $0=(x-2)^2(x+1)$

ie $x=-1$ or $x=2$ (1)

$\Rightarrow (-1,0) + (2,0)$

ii) for staty pts $\frac{dy}{dx} = 0$.

ie $v\frac{du}{dx} + u\frac{dv}{dx} = 0$.

$(x+1)\cdot 2(x-2) + (x-2)^2\cdot 1 = 0$

$(x-2)[2x+2+x-2]=0$

$(x-2)[3x]=0$.

$\therefore x = 2$ or 0 (2)

$\frac{dy}{dx} = 3x^2-6x$

$\frac{d^2y}{dx^2} = 6x-6$.

test $x=2$ $\frac{d^2y}{dx^2} > 0$

∴ min^m T.P at (2,0) (1)

test $x=0$ $\frac{d^2y}{dx^2} < 0$

∴ max^m T.P at (0,4) (1)

iii) when $x=-2$.

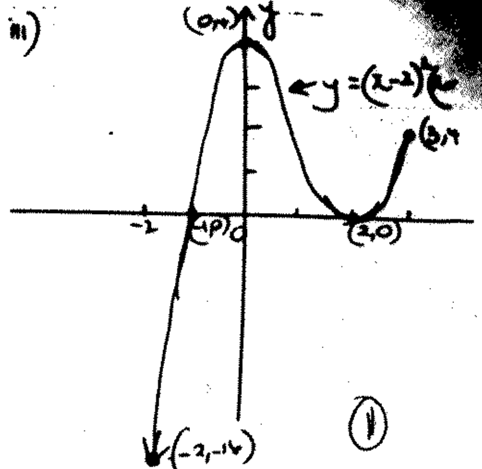
$y=(-4)^2\times -1$

$= -16 \Rightarrow (-2,-16)$ (1)

when $x=3$

$y = 1\times 4$ … (3,4) (1)

iii)



(1)

(b) $y = x^2 + \frac{2}{x} + 4$

$= x^2 + 2x^{-1} + 4$

$\frac{dy}{dx} = 2x - 2x^{-2}$

$= 2x - \frac{2}{x^2}$ (1)

at $x=-1$

$\frac{dy}{dx} = -2 - \frac{2}{1}$

$= -4$ (1)

$\therefore m = -4$ at $P(-1,3)$

$m = \frac{y-y_1}{x-x_1}$

$-4 = \frac{y-3}{x+1}$

$y-3 = -4x-4$ (1)

$4x+y+1=0$ is the eq^n of the tangent

(11)

QUESTION 9 :-

(a) $x^2 = 8y$

cf $x^2 = 4ay$

$\therefore 4a = 8$

$a = 2$

i) focal length $= 2$ u

ii) Focus $(0, 2)$

iii) directrix: $y = -2$. (3)

(b) $S_\infty = ?$ $a = 75$ km $r = \frac{1}{3}$

$S_\infty = \frac{a}{1-r}$

$= \frac{75}{\frac{2}{3}}$

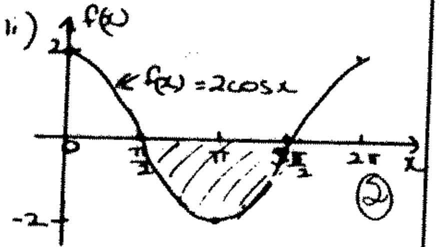
$S_\infty = 112\frac{1}{2}$ km. (2)

No, she will never reach the watering hole as she can only travel $112\frac{1}{2}$ km which is < 130 km. (1)

(c) $f(x) = 2\cos x$.

i) Range: $-2 \le f(x) \le 2$ (1)

ii)



(2)

$A = \left| \int_{\pi/2}^{\frac{3\pi}{2}} 2\cos x \, dx \right|$

$= \left| \left[2\sin x \right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \right|$

$= \left| (2 \times -1) - (2) \right|$

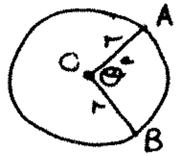
$= |-4|$

$A = 4 \text{ u}^2$ (3)

(9)

QUESTION [illegible] :-

(a)



i) $P_{sector} = 8$ cm

Arc length$_{AB} = r\theta$

$\therefore$ Perimeter $= r\theta +$

$= r(\theta +$

$\therefore 8 = r(\theta +$

$\frac{8}{r} = \theta +$

$\therefore \theta = \frac{8}{r} - 2$ (as req) (2)

Area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} r^2 \left(\frac{8}{r} - 2\right)$

$= \frac{1}{2} \times 8r - r^2$

$A = 4r - r^2$ (as req) (2)

ii) $\frac{dA}{dr} = 4 - 2r$ ($\frac{1}{2}$)

$\frac{d^2A}{dr^2} = -2$. ($\frac{1}{2}$)

For stat. values $\frac{dA}{dr} = 0$

i.e. $4 - 2r = 0$

$2r = 4$

$r = 2$. (1)

test $r = 2$ $\frac{d^2A}{dr^2} = -2 < 0$.

$\therefore$ max value when $r = 2$.

iii) Max A $= 4 \times 2 - 2 \times 2$

$= 8 - 4$

max Area $= 4$ cm^2 (1)

QUESTI[illegible] 10 (cont'd)

(b) $P = \$2300$ $n = 25, 24, 23, \ldots, 1$

$r = .12$

Let A_n be the amnt the investment grows to after n investments

$A_1 = 2300(1.12)^{25}$

$A_2 = 2300(1.12)^{24}$

$A_3 = 2300(1.12)^{23}$

$\vdots$

$A_{24} = 2300(1.12)^2$

$A_{25} = 2300(1.12)^1$

Total Amnt $= A_{25} + A_{24} + \ldots + A_1$

$= 2300(1.12 + 1.12^2 + 1.12^3 + \ldots + 1.12^{25})$

G.P. $a = 1.12$, $n = 25$, $r = 1.1$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{25} = \frac{1.12(1.12^{25} - 1)}{.12}$

Total Amnt $= 2300 \times S_{25}$

$\doteq 343468.0491$

$= \$343\,468.05$ (nearest ¢)

$\therefore$ The investment is worth \$343 468.05 (nearest ¢) after 25 years)

(5)

(4)