Name: ____________________________

Teacher: ___________________________

Class: ___________________________

FORT STREET HIGH SCHOOL

2014

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics Extension 1

Time allowed: 2 hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
	Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
HE2, HE4	Manipulates algebraic expressions to solve problems from topic areas such as inverse functions, trigonometry, polynomials and circle geometry.	11, 12
HE3, HE5 HE6	Uses a variety of methods from calculus to investigate mathematical models of real life situations, such as projectiles, kinematics and growth and decay	13
HE7	Synthesises mathematical solutions to harder problems and communicates them in appropriate form	14

Total Marks 70

Section I 10 marks
Multiple Choice, attempt all questions,
Allow about 15 minutes for this section
Section II 60 Marks
Attempt Questions 11-14,
Allow about 1 hour 45 minutes for this section

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 60	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
	Percent	

General Instructions:

- Questions 11-14 are to be started in a new booklet.
- The marks allocated for each question are indicated.
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used.

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SECTION I (One mark each)
Answer each question by circling the letter for the correct alternative on this sheet.

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1 What is the solution to the inequality $\frac{3}{x-2} \leq 4$?

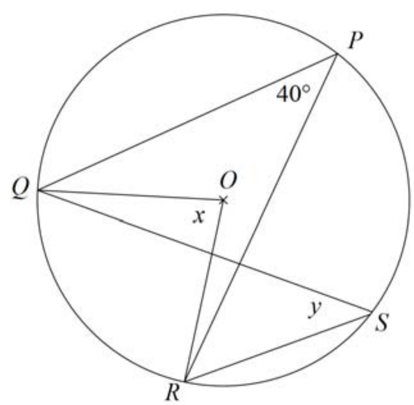
(A) $x < -2$ and $x \geq -\frac{11}{4}$

(B) $x > -2$ and $x \leq -\frac{11}{4}$

(C) $x < 2$ and $x \geq \frac{11}{4}$

(D) $x > 2$ and $x \leq \frac{11}{4}$

2 P, Q, R and S are points on a circle with centre O. $\angle QPR = 40^\circ$.



Why are the values of x and y?

(A) $x = 40^\circ$ and $y = 20^\circ$

(B) $x = 40^\circ$ and $y = 40^\circ$

(C) $x = 80^\circ$ and $y = 20^\circ$

(D) $x = 80^\circ$ and $y = 40^\circ$

3 The point P divides the interval AB joining $A(-4,-3)$ and $B(1,5)$ externally in the ratio 3:2. What are the coordinates of P?

(A) $(-14,-19)$

(B) $(-11,-21)$

(C) $(11,21)$

(D) $(14,19)$

4 How many distinct permutations of the letters of the word 'DIVIDE' are possible in a straight line when the word begins and ends with the letter D?

(A) 12

(B) 180

(C) 360

(D) 720

5 What is the exact value of the definite integral $\int_{\frac{\pi}{2}}^{\pi}\left(\sin^2 x + x\right)dx$?

(A) $\dfrac{3\pi^2+\pi+2}{8}$

(B) $\dfrac{3\pi^2+\pi}{8}$

(C) $\dfrac{3\pi^2+2\pi+2}{8}$

(D) $\dfrac{3\pi^2+2\pi}{8}$

6 What is the value of $f'(x)$ if $f(x)=2x^2\cos^{-1}2x$?

(A) $\dfrac{-8x}{\sqrt{1-2x^2}}$

(B) $\dfrac{-8x}{\sqrt{1-4x^2}}$

(C) $\dfrac{-4x^2}{\sqrt{1-2x^2}}+4x\cos^{-1}2x$

(D) $\dfrac{-4x^2}{\sqrt{1-4x^2}}+4x\cos^{-1}2x$

7 Which of the following is equivalent to the expression $\dfrac{\sin 2\theta+\sin\theta}{\cos 2\theta+\cos\theta+1}$?

(A) $\cot\theta$

(B) $\sec\theta$

(C) $\sin\theta$

(D) $\tan\theta$

8 A point P moves in the xy-plane such that P(tanθ, cotθ) is its parametric presentation with the parameter θ, where θ is any real number. The locus of P then is a

(A) Parabola
(B) Circle
(C) Hyperbola
(D) Straight Line

9 The radius of a balloon is expanding at a constant rate of $1.3 cms^{-1}$. The rate of change of the surface area of the balloon when its radius is $6.3 cm$ is ?

(A) $498.76\ cm^2s^{-1}$
(B) $68.61\ cm^2s^{-1}$
(C) $205.84\ cm^2s^{-1}$
(D) $158.34\ cm^2s^{-1}$

10 Which of the following expressions is correct?

(A) $\tan^{-1} x = \cos^{-1} \dfrac{1}{\sqrt{1-x^2}}$

(B) $\tan^{-1} x = \cos^{-1} \dfrac{1}{\sqrt{1+x^2}}$

(C) $\tan^{-1} x = \cos^{-1} \dfrac{x}{\sqrt{1-x^2}}$

(D) $\tan^{-1} x = \cos^{-1} \dfrac{x}{\sqrt{1+x^2}}$

SECTION II (15 marks each)

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

QUESTION 11: Use a separate writing booklet

(a)

(i) Write down the expansion of $\tan(A+B)$. **[1]**

(ii) Find the value of $\tan\left(\frac{7\pi}{12}\right)$ in simplest surd form. **[2]**

(b) Show that $\lim\limits_{x\to 0}\frac{\sin 4x}{9x}=\frac{4}{9}$. **[1]**

(c) Use Newton's method to find a second approximation to a root of $x-e^{-x}=0$, given that $x=0.5$ is the first approximation. Give the answer correct to three decimal places. **[2]**

(d) The roots α,β and γ of the equation $2x^3+9x^2-27x-54=0$ are in geometric sequence.

(i) Show that $\beta^2=\alpha\gamma$. **[1]**

(ii) Write down the value of $\alpha\beta\gamma$. **[1]**

(iii) Find α,β and γ. **[4]**

(e) By using the substitution $x=\sin\theta$, find $\int_0^{\frac{1}{2}}\left(1-x^2\right)^{\frac{-3}{2}}dx$. **[3]**

QUESTION 12: Use a separate writing booklet

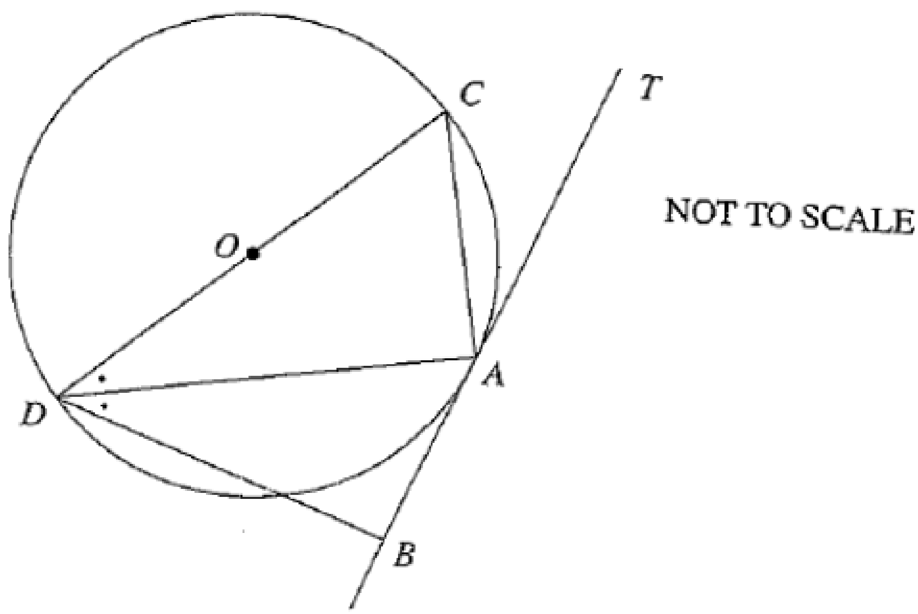
(a) The curves $y = \sin^{-1} x$ and $y = \cos^{-1} x$ intersect at point P. The acute angle between their tangents at that point is θ. Find θ. **[3]**

(b)

(i) Express $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x + \alpha)$ for $R > 0$ and α acute. **[2]**

(ii) Hence, or otherwise, find all solutions to $\cos x - \sqrt{3}\sin x = 2$. **[1]**

(c) **[4]**



O is the centre of a circle. TAB is a tangent to the circle at A.

AD bisects the angle CDB.

Copy or trace the diagram into your Writing Booklet.

Prove that the angle ABD is a right angle.

(d) Ten people arrive to eat at a restaurant. The only seating available for them is at two circular tables, one that seats six persons, and another that seats four.

(i) Using these tables, how many different seating arrangements are there for the ten people? **[2]**

(ii) Assuming that the seating arrangement is random, what is the probability that a particular couple will be seated at the same table? **[3]**

(Question 13 Starts over the page)

QUESTION 13 Use a separate writing booklet

(a) Find $\int \frac{1}{\sqrt{4-9x^2}}\,dx$ **[2]**

(b)

(i) Show that the function $T = R + Ae^{-kt}$ is a solution of the differential equation $\frac{dT}{dt} = -k(T-R)$. **[1]**

(ii) A metal cake tin is removed from an oven at a temperature of $180°C$. If the cake tin takes one minute to cool to $150°C$ and the room temperature is $20°C$, find the time (to the nearest minute) it takes for the cake tin to cool to $80°C$. (Assume that the cake tin cools at a rate proportional to the difference between the temperature of the cake tin and the temperature of the surrounding air.) **[3]**

(c) The acceleration of a particle moving in a straight line is given by $\frac{d^2x}{dt^2} = -\frac{72}{x^2}$, where x metres is the displacement from the origin after t seconds. When $t = 0$, the particle is 9 metres to the right of the origin with a velocity of 4 metres per second.

(i) Show that the velocity, v, of the particle, in terms of x, is $v = \frac{12}{\sqrt{x}}$. **[2]**

(ii) Find an expression for t in terms of x. **[2]**

(iii) How many seconds does it take for the particle to reach a point 35 metres to the right of the origin? **[1]**

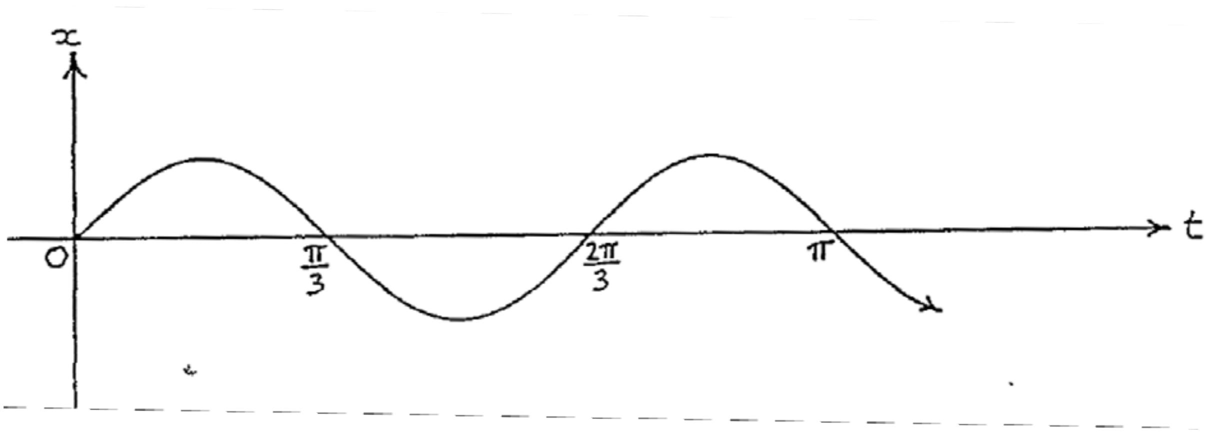
(d)

(i) Show that $\frac{d}{dx}\left(\tan^3 x\right) = 3\sec^4 x - 3\sec^2 x$. **[2]**

(ii) Using (i) or otherwise, evaluate $\int_0^{\frac{\pi}{4}} \sec^4 x\,dx$. **[2]**

QUESTION 14 Use a separate writing booklet

(a)



The diagram shows the displacement $x\,cm$ from the origin at time t seconds of a particle moving in simple harmonic motion.

(i) State the period of the motion. **[1]**

(ii) At what times during the first π seconds is the particle at rest? **[1]**

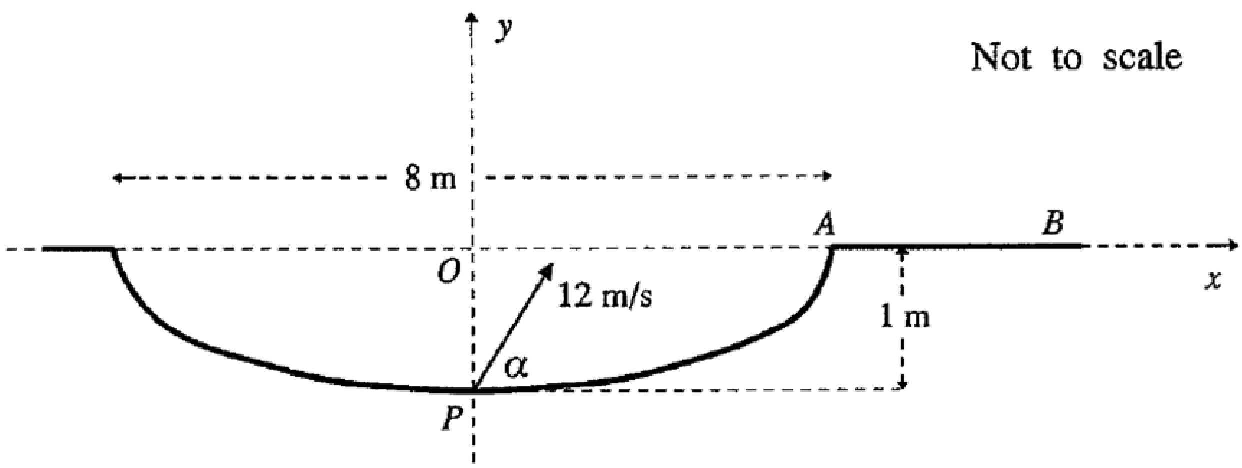
(iii) Show that $\ddot{x} = -9x$. **[1]**

(iv) Given that the particle has initial velocity $4\,cms^{-1}$, find the amplitude of the motion. **[2]**

(v) Write down an equation for x in terms of t. **[1]**

(b) A golf ball is lying at point P, at the middle of the bottom of a sand bunker, which is surrounded by level ground. The point A is at the edge of the bunker, and the line AB lies on the level ground. The bunker is 8 metres wide and 1 metre deep.

The ball is hit towards A with an initial speed of 12 metres per second, and angle of elevation α. You may assume that the acceleration due to gravity is 10 ms^{-2}.



(i) Show that the golf ball's trajectory at time t seconds after being hit may be defined by the equations $x = (12\cos\alpha)t$ and $y = -5t^2 + (12\sin\alpha)t - 1$, where x and y are the horizontal and vertical displacements, in metres, of the ball from the origin O shown in the diagram. **[2]**

(ii) Given $\alpha = 30°$, how far from A will the ball land? **[2]**

(iii) Find the maximum height above the level ground reached by the ball if $\alpha = 30°$. **[2]**

(iv) Find the range of values of α, to the nearest degree, at which the ball must be hit so that it will land to the right of A. **[3]**

END OF EXAMINATION

① C ② D ③ C ④ A ⑤ D ⑥ D ⑦ D ⑧ C ⑨ C ⑩ B

SECTION I MULTIPLE CHOICE ANSWERS (10 Marks)

1) $\frac{3}{x-2} \le 4$

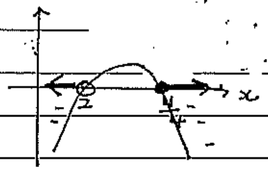
$3(x-2) \le 4(x-2)^2, \; x \ne 2$

$3(x-2) - 4(x-2)^2 \le 0$

$(x-2)(3 - 4x + 8) \le 0$

$(x-2)(11-4x) \le 0$

soln: $x < 2$ & $x \ge \frac{11}{4}$



∴ Ⓒ

2) $x = 80$ (∠ at centre is 2 × ∠ at circumf. on same arc)

$y = 40$ (∠s in same segment of the circle on arc QR)

Ⓓ

3) $k : l = 3 : -2$ $A = (-4, -3)$ $B = (1, 5)$

$x = \frac{kx_2 + lx_1}{k+l}$, $y = \frac{ky_2 + ly_1}{k+l}$

$= \frac{1(3) + (-2)(-4)}{3-2}$, $= \frac{3(5) + (-2)(-3)}{3-2}$

$= 11$, $= 21$

∴ $(11, 21) = P$ Ⓒ

4) D _ _ _ D 2 1's

no. of permutations

$= \frac{4!}{2!}$ Ⓐ

$= 12$

5) $\int_{\frac{\pi}{2}}^{\pi} (\sin^2 x + x)\, dx$

$= \int_{\frac{\pi}{2}}^{\pi} \left[\frac{1}{2}(1 - \cos 2x) + x\right] dx$

$= \left[\frac{1}{2}x - \frac{\sin 2x}{4} + \frac{x^2}{2}\right]_{\frac{\pi}{2}}^{\pi}$

$= \left[\left(\frac{\pi}{2} - 0 + \frac{\pi^2}{2}\right) - \left(\frac{\pi}{4} - 0 + \frac{\pi^2}{8}\right)\right]$

$= \left[\frac{\pi}{2} + \frac{\pi^2}{2} - \frac{\pi}{4} - \frac{\pi^2}{8}\right]$

$= \left[\frac{\pi}{4} + \frac{3\pi^2}{8}\right]$

$= \frac{2\pi + 3\pi^2}{8}$ Ⓓ

6) $f(x) = 2x^2 \cos^{-1} 2x$

$f'(x) = (\cos^{-1} 2x)\, 4x + 2x^2 \left(\frac{-2}{\sqrt{1-4x^2}}\right)$

$= \frac{-4x^2}{\sqrt{1-4x^2}} + 4x \cos^{-1} 2x$

Ⓓ

7) $\frac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1}$

$= \frac{2\sin\theta\cos\theta + \sin\theta}{2\cos^2\theta - 1 + \cos\theta + 1}$

$= \frac{\sin\theta(2\cos\theta + 1)}{\cos\theta(2\cos\theta + 1)}$

$= \tan\theta$ Ⓓ

8. $x = \tan\theta$ ①

$y = \cot\theta = \frac{1}{\tan\theta}$ ②

$xy = 1$ ① × ②

Ⓒ

9. $\frac{dr}{dt} = 1 \cdot 3$ cm/s

$A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

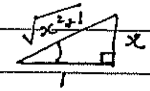
$= 8\pi r \times 1 \cdot 3$

$= 8\pi \times 6 \cdot 3 \times 1 \cdot 3$

$\doteqdot 205 \cdot 84$ cm²/s

Ⓒ

10) let $\alpha = \tan^{-1} x \rightarrow x = \tan\alpha$



∴ $\cos\alpha = \frac{1}{\sqrt{x^2+1}}$

i.e. $\cos^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right) = \alpha$

$= \tan^{-1} x$

Ⓑ

Question 11 (15 marks) — Comments

a) i) $\tan(A+B)$

$= \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ ← ① mark

ii) $\tan\left(\frac{7\pi}{12}\right)$

$= \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \dfrac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}.\tan\frac{\pi}{4}}$ ← ① mark

$= \dfrac{\sqrt{3}+1}{1-\sqrt{3}}$

$= \dfrac{(\sqrt{3}+1)^2}{-2}(1+\sqrt{3})$

Comment: students left the answer as $\dfrac{(\sqrt{3}+1)^2}{-2}$, not simplest form

$= \dfrac{4+2\sqrt{3}}{-2}$

Comment: students forgot the -ve in the denominator.

$= -2-\sqrt{3}$ ← ① mark

b) $\displaystyle\lim_{x\to 0} \frac{\sin 4x}{9x}$

Comment: Well done

$= \displaystyle\lim_{x\to 0} \frac{4}{9}.\frac{\sin 4x}{4x}$ ← ① mark

$= \frac{4}{9} \times 1$

$= \frac{4}{9}$

Q11 ct'd — Comments

c) $x - e^{-x} = 0$

$f(x) = x - e^{-x}$

$f'(x) = 1 + e^{-x}$

Comment: students got formula wrong

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

Comment: calculator error.

$= 0.5 - \left(\dfrac{0.5 - e^{-0.5}}{1 + e^{-0.5}}\right)$ ← ① mark

$\doteqdot 0.566$ ← ① mark

d) $2x^3 + 9x^2 - 27x - 54 = 0$

roots are α, β, γ

i) Since α, β, γ is a G.P:

$\dfrac{\beta}{\alpha} = \dfrac{\gamma}{\beta}$ ← ① mark for $\dfrac{T_2}{T_1} = \dfrac{T_3}{T_2}$

① $\therefore \beta^2 = \alpha\gamma$

② ii) $\alpha\beta\gamma = -\left(\dfrac{-54}{2}\right) = 27$ ← ① mark

iii) sub ① into ②

$\therefore \beta^3 = 27$

$\beta = 3$ ← ① mark

Now $\alpha\beta + \beta\gamma + \alpha\gamma = \dfrac{-27}{2}$

i.e. $3\alpha + 3\gamma + 9 = \dfrac{-27}{2}$

③ i.e. $\alpha + \gamma = \dfrac{-15}{2}$ ← ① mark

Question 11 ct'd. | COMMENTS

Using $\beta = 3$ sub into ①

$\therefore \quad y = \frac{9}{\alpha}$ sub into ③

$\therefore \quad \frac{9}{\alpha} + \alpha = \frac{-15}{2}$

$9 + \alpha^2 = \frac{-15\alpha}{2}$

$18 + 2\alpha^2 = -15\alpha$

$2\alpha^2 + 15\alpha + 18 = 0$

$(2\alpha + 3)(\alpha + 6) = 0$

$\therefore \alpha = \frac{-3}{2}$ or $\alpha = -6$

① mark for obtaining any of these equivalent eqns in terms of α only.

students got lost in the algebra.

$\therefore y = \frac{9}{\frac{-3}{2}} = -6$

or $y = \frac{9}{-6} = \frac{3}{-2}$

① mark for both

Roots are

$\frac{-3}{2}, 3, -6$

e) $\int_0^{1/2} (1 - x^2)^{-3/2} \, dx$ ① mark OR

$x = \sin\theta$

$dx = \cos\theta \, d\theta$

$x = 0, \ \theta = 0$

$x = \frac{1}{2}, \ \theta = \frac{\pi}{6}$

$= \int_0^{\pi/6} (1 - \sin^2\theta)^{-3/2} \cdot \cos\theta \, d\theta$

$= \int_0^{\pi/6} \frac{1}{(\cos^2\theta)^{3/2}} \cdot \cos\theta \, d\theta$

$= \int_0^{\pi/6} \frac{\cos\theta}{\cos^3\theta} \, d\theta$ ① mark

$= \int_0^{\pi/6} \sec^2\theta \, d\theta$

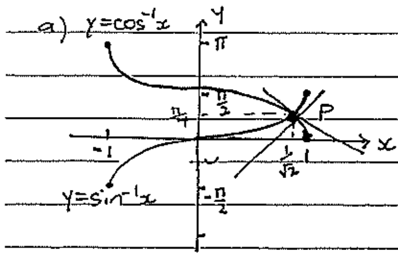
① for answer

$= \left[\tan\theta\right]_0^{\pi/6} = \tan\frac{\pi}{6} - \tan 0 = \frac{1}{\sqrt{3}}$

students forgot that ① $\frac{1}{\cos^2\theta} = \sec^2\theta$ and ② $\int \sec^2\theta \, d\theta = \tan\theta$

QUESTION 12 (15 marks) | COMMENTS

a)



Many students made this question harder by attempting an algebraic solution of simultaneous equations. A graph showing a point of intersection was all that was required.

$y = \sin^{-1}x$ ①

$y = \cos^{-1}x$ ②

$\sin^{-1}x = \cos^{-1}x$

when $x = \frac{1}{\sqrt{2}}$ ← ① mark

$y' = \frac{1}{\sqrt{1-x^2}}$, $\quad y' = \frac{-1}{\sqrt{1-x^2}}$

$= \frac{1}{\sqrt{1-\frac{1}{2}}}$ $\quad = \frac{-1}{\sqrt{1-\frac{1}{2}}}$

$= \sqrt{2}$ $\quad = -\sqrt{2}$ ← ① mark

$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$= \left| \frac{2\sqrt{2}}{1 - 2} \right|$

$= 2\sqrt{2}$

$\therefore$ acute $\angle$: $\theta = 70°32'$ (nearest min) ← ① mark

Question 12 ct'd

	COMMENTS
b) i) $\cos x - \sqrt{3}\sin x = R\cos(x+\alpha)$	Mostly well done

$R = \sqrt{1^2+(\sqrt{3})^2}$

$= 2$

$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \cos x\cos\alpha - \sin x\sin\alpha$

equating coefficients:

$\sin\alpha = \frac{\sqrt{3}}{2}$

$\cos\alpha = \frac{1}{2}$

① mark for R & α

$\therefore \tan\alpha = \sqrt{3}$

$\therefore \alpha = \frac{\pi}{3}$

$\therefore \cos x - \sqrt{3}\sin x = 2\cos(x+\frac{\pi}{3})$ ← ① mark

ii) $\cos x - \sqrt{3}\sin x = 2$

i.e. $2\cos(x+\frac{\pi}{3}) = 2$

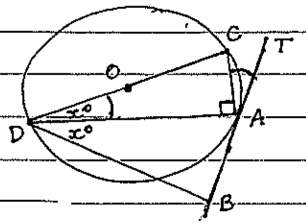
$\cos(x+\frac{\pi}{3}) = 1 = \cos 0$ $\quad \alpha = 0$

when $x+\frac{\pi}{3} = 2n\pi$

$x = 2n\pi - \frac{\pi}{3}$, n = integer ← ① mark

Comments: Many students do not know the general solutions formula. If $\cos x = c$, $x = 2n\pi \pm \cos^{-1}c$

c)



Comments: Mostly well done

Let $\angle CDA = \angle ADB = x°$

$\angle CAD = 90°$ ($\angle$ in a semi-circle) ← ① mark

$\angle CAT = \angle CDA$ ($\angle$ in alternate segment) ← ① mark

$\therefore \angle CAT = x°$

$\therefore \angle ABD = (x°+90°) - x°$ (ext. $\angle$ of $\triangle DBA$) ← ② marks

$= 90°$ as req'd.

QUESTION 12 ct'd.

	COMMENTS
~~10 people in total~~	
d) i) 6 people can be chosen in $^{10}C_6$ ways	Not well done! Most students left out the $^{10}C_6$.

6: 5! 4: 3!

$\therefore$ No. of arrangements

$= {}^{10}C_6 \times 5! \times 3!$

$= 151\,200$

accept either as answer

① mark for $^{10}C_6 \times 5!$

① mark for multiplying by 3!

ii)

If couple are around large table:

no. of ways around BOTH tables

$= ({}^8C_4 \times 5!) \times 3!$ ← ① mark

If couple are around small table:

no. of ways around BOTH tables

$= ({}^8C_2 \times 3!) \times 5!$ ← ① mark

$\therefore$ Prob. req'd

$= \frac{{}^8C_4 \times 5! \times 3! + {}^8C_2 \times 3! \times 5!}{151\,200}$

$= \frac{7}{15}$ ← ① mark.

Comments: Again, many students left out the $^8C_4$.

QUESTION 15.

(a) $\int \frac{1}{\sqrt{4-9x^2}}\,dx$

$= \int \frac{1}{\sqrt{9(\frac{4}{9}-x^2)}}\,dx$

$= \frac{1}{3}\int \frac{1}{\sqrt{(\frac{2}{3})^2-x^2}}\,dx$ ✓

$= \frac{1}{3}\sin^{-1}\frac{x}{\frac{2}{3}} + c$

$= \frac{1}{3}\sin^{-1}\frac{3}{2}x + c$ ✓

Well done

(b) Given $T = R + Ae^{-kt} \Rightarrow Ae^{-kt} = T-R$

now $\frac{dT}{dt} = Ae^{-kt} \times -k$ ✓

$= -kAe^{-kt}$

$= -k(T-R)$ as req'd.

$\therefore T = R + Ae^{-kt}$ is a soln of $\frac{dT}{dt} = -k(T-R)$

Well done

ii) $T = R + Ae^{-kt}$

[$R = 20°C$, $t=0$, $T=180$]

$180 = 20 + A$

$A = 160$

ie $T = 20 + 160e^{-kt}$ ✓

[$t=1$, $T=150$]

$150 = 20 + 160e^{-k}$

$130 = 160e^{-k}$

$e^{-k} = \frac{13}{16}$

$k = -\ln\left(\frac{13}{16}\right)$ or $k = \ln\left(\frac{16}{13}\right)$ ✓

cont'd over

Some students took A to be 180.

(b) cont'd

when $T = 80$

$80 = 20 + 160e^{\ln(\frac{13}{16})t}$

$60 = 160e^{\ln(\frac{13}{16})t}$

$\frac{3}{8} = e^{\ln(\frac{13}{16})t}$

$t = \frac{\ln(\frac{3}{8})}{\ln(\frac{13}{16})}$

$\doteqdot 4.7237$ ✓

$t = 5$ min. (to nearest minute)

$\therefore$ it takes approx. 5 minutes for the cake tin to cool to 80°C.

(c) $\frac{d^2x}{dt^2} = -\frac{72}{x^2}$, $t=0$, $x=9$, $v=4$m/s

ie $a = \frac{-72}{x^2}$

$\therefore \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{-72}{x^2}$

$\frac{1}{2}v^2 = \int -72x^{-2}\,dx$

$= \frac{-72x^{-1}}{-1} + c$

$\frac{1}{2}v^2 = \frac{72}{x} + c$

[when $t=9$, $v=4$]

$8 = \frac{72}{9} + c$

$\therefore c = 0$.

$\therefore \frac{1}{2}v^2 = \frac{72}{x}$ ✓

$v^2 = \frac{144}{x}$

$v = \frac{\pm 12}{\sqrt{x}}$, $x \neq 0$

but since v is initially positive and moving to the right, $x \neq 0$ ✓

then $v = \frac{12}{\sqrt{x}}$ m/s.

Many students did not state why $V = \frac{12}{\sqrt{x}}$ ins of $V = \frac{-12}{\sqrt{x}}$.

13 (c) cont'd

i) $\frac{dx}{dt} = \frac{12}{\sqrt{x}}$

$\frac{dt}{dx} = \frac{\sqrt{x}}{12}$

$t = \frac{1}{12}\int x^{\frac{1}{2}} dx$

$= \frac{1}{12}\times\frac{2}{3}\left[x^{3/2}\right] + k$

$= \frac{1}{18} x^{3/2} + k$

$[t=0, \quad x=9]$

$0 = \frac{1}{18}\times 27 + k$

$= \frac{3}{2} + k$

$k = -\frac{3}{2}$

$\therefore t = \frac{1}{18} x^{3/2} - \frac{3}{2}$

ii) $t = \frac{1}{18} 35^{3/2} - \frac{3}{2}$

$\doteqdot 10.0034 8847$

$= 10$ seconds (nearest sec)

. takes approx 10 seconds to reach 35m to right of origin.

Some students did not make t the subject.

Some students did not calculate the constant.

Some students took x to be 26. This was not the question.

10 (c)

i) $\frac{d}{dx}\tan^3 x = \frac{d}{dx}(\tan x)^3$

$= 3(\tan x)^2 \cdot \sec^2 x$ ✓

$= 3\tan^2 x \sec^2 x$

$= 3(\sec^2 x - 1)\sec^2 x$ ✓

$\therefore \frac{d}{dx}\tan^3 x = 3\sec^4 x - 3\sec^2 x$ (as req'd)

Mostly well done.

ii) $\therefore 3\sec^4 x = \frac{d}{dx}\tan^3 x + 3\sec^2 x$

$\int 3\sec^4 x\, dx = \int_0^{\pi/4} \frac{d}{dx}\tan^3 x\, dx + \int_0^{\pi/4} 3\sec^2 x\, dx$

$\int 3\sec^4 x\, dx = \left[\tan^3 x\right]_0^{\frac{\pi}{4}} + \left[3\tan x\right]_0^{\frac{\pi}{4}}$ ✓

$\int \sec^4 x\, dx = \frac{1}{3}\left\{\left[\tan^3 x\right]_0^{\pi/4} + \left[3\tan x\right]_0^{\pi/4}\right\}$

$= \frac{1}{3}\left[(1-0) + (3-0)\right]$

$= \frac{4}{3}$ ✓

Many students did not multiply by $\frac{1}{3}$.

QUESTION 14

(a) i) $P = \frac{2\pi}{3}$ ✓

(ii) At rest at turning pts.

ie $t = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ seconds ✓ (must have all

iii) Since Motion is given as Simple Harmonic

∴ form is $\ddot{x} = -n^2 x$

$\left[P = \frac{2\pi}{n} = \frac{2\pi}{3} \Rightarrow n = 3 \right]$

$\therefore \ddot{x} = -(3)^2 x$ ✓

$\ddot{x} = -9x$

as req'd.

OR $x = a\sin 3t$

$\dot{x} = 3a\cos 3t$

$\ddot{x} = -9a\sin 3t$

$\ddot{x} = -9x$

as req

(iv) when $t=0$, $x=0$, $v = 4$ cm/s

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -9x.$

$\frac{1}{2}v^2 = -9\int x\,dx$

$= -9\frac{x^2}{2} + c$

$\frac{1}{2}(4)^2 = c$

$\therefore c = 8$

$\therefore \frac{1}{2}v^2 = -\frac{9}{2}x^2 + 8$

$v^2 = -9x^2 + 16.$ ✓

at end pts $v = 0.$

ie $-9x^2 + 16 = 0.$

$x^2 = \frac{16}{9}$

$x = \pm\frac{4}{3}$ ✓

Amp $= 1\frac{1}{3}$ cm (amp is positive)

v) $x = a\sin(nt + \alpha)$

$x = \frac{4}{3}\sin 3t$ $(\alpha = 0)$

OR $x = a\cos(nt + \alpha)$

$x = \frac{4}{3}\cos\left(3t + \frac{\pi}{2}\right)$ $\alpha = \frac{\pi}{2}$ ✓ either.

Comments

* (a) Generally well done.

Note: amplitude is positive. State this.

b)(i)

horizontally

$\ddot{x} = 0$

$\dot{x} = c_1 = v\cos\alpha$

$\dot{x} = v\cos\alpha$

$\dot{x} = 12\cos\alpha$

$x = 12\cos\alpha\, t + c_2$

[when $t=0$, $x=0$]

$c_2 = 0$

$\therefore x = (12\cos\alpha)t$ as req'd

vertically

$\ddot{y} = -10$

$\dot{y} = -10t + c_3$

$\dot{y} = v\sin\alpha$

$= 12\sin\alpha$

$12\sin\alpha = -10t + c_3$

when $t=0$,

$c_3 = 12\sin\alpha$

$\therefore \dot{y} = -10t + 12\sin\alpha$

$y = -\frac{10t^2}{2} + 12\sin\alpha\, t + c_4$

when $t=0$, $y=-1$

$\therefore c_4 = -1$

$\therefore y = -5t^2 + (12\sin\alpha)t - 1$

(ii) $\alpha = 30°$

Hits ground when $y = 0$.

$-5t^2 + 12\sin\alpha t - 1 = 0.$

$5t^2 - 12(\sin 30)t + 1 = 0.$

$5t^2 - 6t + 1 = 0.$

$(5t - 1)(t - 1) = 0$

$t = \frac{1}{5}$, $t = 1$

At $t = \frac{1}{5}$, ball is in line (in the air) with top of bunker ∴ reject $t = 1/5$.

At $t = 1$, ball lands on ground ✓

Distance from A: $x = 12\cos\alpha\, t$

$= 12(\cos 30)1$

$= 6\sqrt{3}$ m

∴ Distance from A $= (6\sqrt{3} - 4)$ m.

$(\doteqdot 6.4$ m$)$

b)

* (i) As this is a show question, you must derive fully the eq showing all integration constants & clearly state their value.

1 mark for $\dot{x}$ /

1 mark for $\ddot{x}$ [

* ii) Many did not know or explain why $t = 1/5$ is rejected (not penalised as long as they only considered t=...)

Many took A to be (8,0) rather than A(4,0).

i) Max height when $\dot{y}=0$

i.e. $-10t + 12\sin 30 = 0$.

$-10t + 6 = 0$.

$t = 0.6$ sec ✓

$y = -5t^2 + 12(\sin 30)t - 1$

$= -5(0.6)^2 + 6(0.6) - 1$

$\therefore h_{max} = 0.8$ m ✓

*(iii) well done.

) land at A $\Rightarrow$ land at $A(4,0)$

when $x=4$: $x = 12(\cos\alpha)t$

$4 = 12(\cos\alpha)t$

$t = \frac{4}{12\cos\alpha}$

$t = \frac{1}{3\cos\alpha}$ ... ① ✓

$y=0$: $0 = -5t^2 + 12(\sin\alpha)t - 1$ ... ②

sub ① into ②

$5\left(\frac{1}{3\cos\alpha}\right)^2 - 12(\sin\alpha)\left(\frac{1}{3\cos\alpha}\right) + 1 = 0$

$5\left(\frac{1}{9\cos^2\alpha}\right) - 4\tan\alpha + 1 = 0$.

[x all terms by 9]

$5\sec^2\alpha - 36\tan\alpha + 9 = 0$.

$5(1+\tan^2\alpha) - 36\tan\alpha + 9 = 0$.

$5 + 5\tan^2\alpha - 36\tan\alpha + 9 = 0$.

$5\tan^2\alpha - 36\tan\alpha + 14 = 0$. ✓

$\tan\alpha = \frac{36 \pm \sqrt{36^2 - 4\times5\times14}}{10}$

$\alpha = 81°37', 22°25'$

To make sure ball lies to the right of the bunker and not fall back in

$23° \leq \alpha \leq 81°$. ✓

*(iv) Many found 't' in terms of x which complicated the working.

- Some managed to give the correct 'α' values to the nearest minute.

BUT all did not realise which way to round so that the ball landed to right of A & not in bunker.

Part (iv) ~ higher order thinking! Asking for answer 'to nearest degree' is what caused the problem.