GOSFORD HS

Mathematics Trial HSC 2004

Time Allowed - 3 hours plus 5 minutes reading time

Question 1

Marks

(a) Factorise $3x^2 - 5x - 2$ 1

(b) Solve $-3x < 10$ 1

(c) Differentiate $e^{\pi+1}$ 1

(d) Express $\frac{7\pi}{12}$ radians in degrees 1

(e) Solve $|2x - 3| > 5$ 2

(f) Find the primitive of $x^3 - \frac{3}{x}$ 2

(g) Find the values of a and b given that $(ax-3)^2 + b \equiv 4x^2 - 12x + 15$ 2

(h) A weight oscillates up and down at the end of an elastic string. Its depth below the point of suspension, t seconds after it starts, is $(12 + 3\cos 2\pi t)$ cm. At what time will its depth be 9 cm? 2

Question 2

Marks

(a) Find the equation of the normal to the curve $y = x^2 - 6x + 7$ at the point $(2,-1)$. 3

(b) $A(2,2), B(-2,-1), C(2,-1)$ are the vertices of a triangle and E is the point $(1,0)$

(i) Show that the equation of AB is $3x - 4y + 2 = 0$. 2

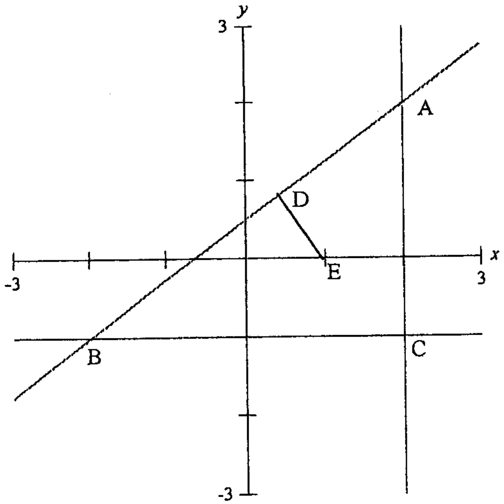
(ii) D is the foot of the perpendicular from $E(1,0)$ to AB. Find the equation of ED. 2

(iii) Prove that the co-ordinates of D are $\left(\frac{2}{5}, \frac{4}{5}\right)$. 2

(iv) Calculate the length of ED 1

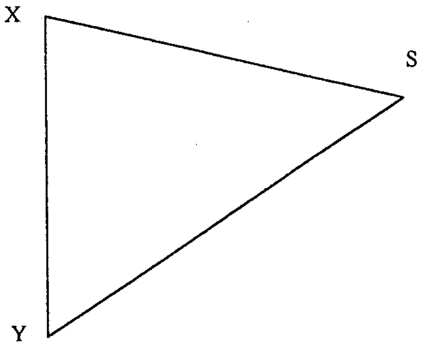
(v) What is the equation of the circle centre E and radius 1 unit? 1

(vi) State briefly why the lines AC, BC and AB are tangents to this circle. 1



Question 3 **Marks**

(a) Two lighthouses, X and Y, 12 kilometres apart observe a ship S, at the same instant. X and Y are on a north-south line. From X, the ship is on a bearing of $105°$ and from Y the bearing of the ship is $058°$.



(i) Copy the diagram onto your own paper

(ii) Show that $\angle XSY = 47°$ 1

(iii) Find, correct to one decimal place the distance of the ship from the nearer lighthouse. 2

(b) Solve $\cos 2\alpha = \frac{1}{2}$, where $0 \le \alpha \le 2\pi$ 3

(c) Differentiate with respect to x

(i) $\sqrt{1-x^2}$ 2

(ii) $x^2 \ln x$ 2

(iii) $\frac{x}{\sin x}$ 2

Question 4 **Marks**

(a) Find the value of $\log_9 243$ 1

(b) Determine the domain of $y = \sqrt{x-5}$ and draw a neat sketch of the curve. 2

(c) Evaluate $\int_0^{\frac{\pi}{2}} \sin 2x \, dx$ 3

(d) The population (P) of the Central Coast is growing over time (t) at a decreasing rate. 2

Explain what this means in terms of $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$

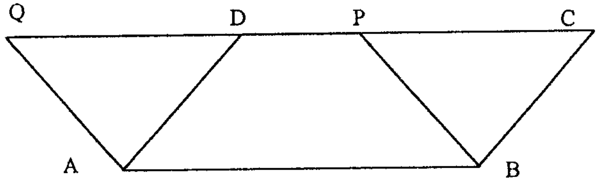
(e) Find the value of k for which the quadratic equation $4x^2 + (1+k)x + 1 = 0$ has real roots. 2

(f) At what points on the curve $y = 3x + 2\ln x$ is the tangent parallel to the line $y - 4x = 3$ 2

Question 5 **Marks**

(a) Find $\int e^{-2x}dx$ 1

(b) ABCD and ABPQ are parallelograms: QDPC is a straight line.



(i) If $\angle QPB = \alpha$ and $\angle ADQ = \beta$ find, with reasons $\angle CPB$, $\angle DQA$ and $\angle BCP$. 2

(ii) Explain why DQ=PC 1

(iii) Prove that $\Delta ADQ \equiv \Delta BCP$ 2

(c) (i) What is the period of $y = 2 + \sin 3x$ 1

(ii) What is the amplitude of $y = 2 + \sin 3x$ 1

(iii) Draw a neat sketch of $y = 2 + \sin 3x$ where $0 \le x \le 2\pi$ 1

(d) Evaluate $\int_0^5 \frac{x}{5+3x^2}dx$ 3

Question 6 **Marks**

(a) A piecemeal function is given by $f(x) = \begin{cases} -x & \text{for } x < 0 \\ x^2 & \text{for } 0 \le x < 2 \\ 4-x & \text{for } x \ge 2 \end{cases}$

(i) Find the value of $f(-4) + f(1) + f(9)$ 2

(ii) Find an expression for $f(a^2)$, where $a \le -\sqrt{2}$ 1

(b) (i) For the curve $y = xe^{-x}$ show that

$$\frac{dy}{dx} = -e^{-x}(x-1)$$ 1

(ii) Find the stationary point(s) and determine the nature of the stationary point(s). 2

(iii) For what values of x is the curve decreasing? 1

(iv) Show that $\frac{d^2y}{dx^2} = e^{-x}(x-2)$ 1

(v) Show that there is a point of inflexion and find the coordinates of this point. 2

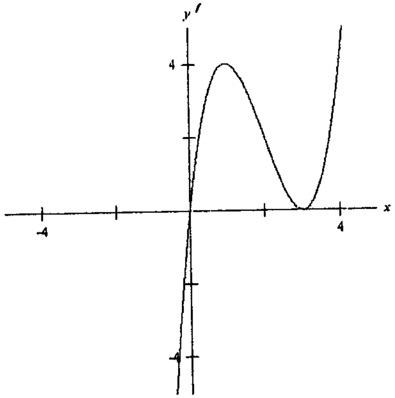
(vi) Sketch the curve, showing the coordinates of the point of inflection and the stationary point(s). 2

Question 7

Marks

(a) The area bounded by the curve $y=\sqrt{x}$, the y-axis and the lines $y=2$ and $y=3$ is rotated about the y-axis. Find the volume of the solid so formed. **3**

(b) The graph below represents the gradient function of the curve $y=f(x)$. Use this graph to draw a neat sketch of the original function. **2**



(c) Amos borrowed $9 000, to buy a second hand car. He has to pay off the loan over 4 years in equal monthly instalments, M, allowing for an interest rate of 18% p.a. which is compounded monthly.

(i) Show that an expression for the amount due after n repayments is given by **3**

$$A_n = 9000(1.015)^n - \frac{200}{3}M\left(1.015^n - 1\right)$$

(ii) Find the amount of each instalment. **1**

(iii) What is the actual interest charged? **1**

(d) Solve $\log_3 x + \log_3(x+8) = 2$ **2**

Question 8

Marks

(a) In a geometric progression the ratio of the sum of the first three terms to the sum to infinity is 7:8. Find the common ratio. **3**

(b) Depths of a 15 metre cross-section of the Namoi river at Boggabri were taken and the results recorded. **3**

Distance	0	3	6	9	19	15
Depth	0	2.3	4.7	5.3	2.7	0

Use the trapezoidal rule to find an approximation to the area of this cross section of the river.

(c) A bushwalker is at a point A, from which branch out two paths. If he takes the path on the **left**, and journeys some distance, the path leads to a point B, from which branches off three paths, one of which leads to safety at C. **2**

However if he takes the path on the **right**, and journeys along a certain distance, this path leads to a point D, from which branch off four paths, one of which leads to C.

Assuming that he has no prior knowledge of any of these facts, except that he wishes to reach safety at C, find the probability that he **does not** reach C on his first try.

(d) (i) Show that the curve $y=2-x^2$ and the straight line $y=-x$ intersect at the points $(-1,1)$ and $(2,-2)$. **1**

(ii) Show that the area enclosed between the curve $y=2-x^2$ and the straight line $y=-x$ is $4\frac{1}{2}$ square units. **3**

Question 9 **Marks**

(a) Simplify $\sqrt{\sin^2 x - \sin^2 x \cos^2 x}$ 1

(b) The point $P(x, y)$ moves so that $PA^2 - PB^2 = 5$, where A and B are the points $(-2,3)$ and $(3,4)$; show that the locus of P has equation $10x + 2y = 17$. 3

(c) Given that $x^2 - 6x - 7 = 8y$ find, by completing the square or otherwise:

- (i) the coordinates of the vertex 1
- (ii) the coordinates of the focus 1
- (iii) the equation of the directrix. 1

(d) An excavator removes V m^3 of soil in t minutes, where 2

$$V = 25t - \frac{t^2}{50}.$$

Find the rate at which the soil is being removed after 5 minutes.

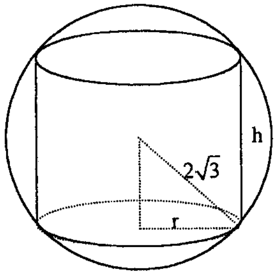
(e) The production rate build up of a colour television is modelled by $\frac{dP}{dt} = 4\,000(t+5)(t-1)$ where t is measured in months. 3

Calculate the total number of televisions produced in the first three months.

Question 10 **Marks**

(a) Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x\,dx + \int_0^{\frac{\pi}{2}} \cos^2 x\,dx$ 2

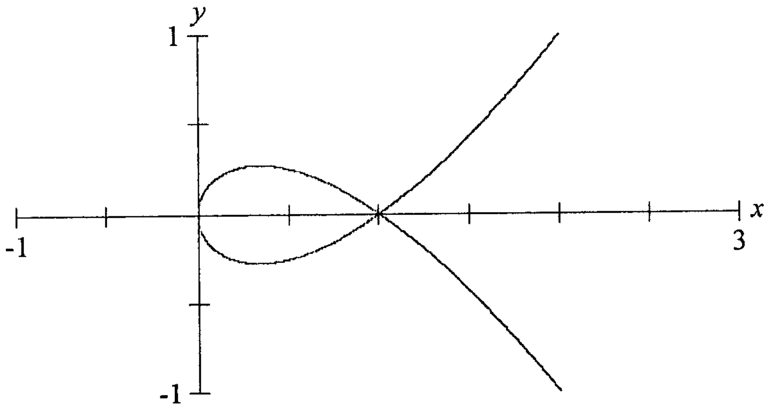
(b) A right circular cylinder of radius r cm and height h cm has to be designed to fit inside a sphere of $2\sqrt{3}$ cm radius so that so that both the bottom and the top touch the sphere completely on the circular rim.



- (i) Show that $r^2 = 12 - \frac{h^2}{4}$ 1
- (ii) If the volume of the cylinder is V show that $V = 12\pi h - \frac{1}{4}\pi h^3$ 1
- (iii) Find the dimensions of the cylinder of maximum volume, and hence determine this volume. 4

Question 10 (c) is on the next page

(c) The graph below represents the curve $2y^2 = x(1-x)^2$. Find the volume of the solid formed by rotating the loop about the x-axis. **4**



End of Examination

GOSFORD HIGH MATHEMATICS TRIAL HSC 2004

Question 1

a) $\underline{(3x+1)(x-2)}$

b) $-3x < 10$

$x > -\frac{10}{3}$

$\underline{x > -3\frac{1}{3}}$

c) $\frac{d}{dx}(\text{constant}) = 0$

$\therefore \frac{d}{dx}(e^{\pi+1}) = 0$

d) $\frac{7\pi}{12} = \frac{7 \times 180^0}{12}$

$\underline{= 105^0}$

e) $|2x-3| > 5$

$2x-3 < -5$ OR $2x-3 > 5$

$2x < -2$ OR $x > 8$

$\underline{x < -1 \text{ OR } x > 4}$

f) $\int (x^3 - \frac{3}{x})\,dx$

$= \int (x^3 - 3 \cdot \frac{1}{x})\,dx$

$\underline{= \frac{x^4}{4} - 3\ln x + C}$

g) let $x = 0$

$\therefore (0-3)^2 + b = 15$

$9 + b = 15$

$b = 6$

Also if

$(ax-3)^2 + b \equiv 4x^2 - 12x + 15$

$a^2x^2 - 6ax + 9 + b \equiv 4x^2 - 12x + 15$

Equatin co-efficients of x

$-6a = -12$

$a = 2$

$\therefore \underline{a = 2, b = 6}$

h) $12 + 3\cos 2\pi t = 9$

$3\cos 2\pi t = -3$

$\cos 2\pi t = -1$

$\therefore 2\pi t = \pi$

$\underline{t = \frac{1}{2}.}$

Question 2

a) $y = x^2 - 6x + 7$

$\frac{dy}{dx} = 2x - 6$

at $x = 2$, $\frac{dy}{dx} = 2 \times 2 - 6$

$= -2$

Slope of the Tangent is -2

$\therefore$ Slope of normal is $\frac{1}{2}$

Equation of the normal at $(2,-1)$

is $y - y_1 = m(x - x_1)$

$y - -1 = \frac{1}{2}(x-2)$

$2y + 2 = x - 2$

$\underline{x - 2y - 4 = 0}$

b)(i) $m_{AB} = \frac{2--1}{2--2}$

$= \frac{3}{4}$

$\therefore$ Equation of AB is

$y - 2 = \frac{3}{4}(x-2)$

$4y - 8 = 3x - 6$

i.e. $\underline{3x - 4y + 2 = 0}$

(ii) $ED \perp AB$

$\therefore m_{ED} = -\frac{4}{3}$

Equation of ED is

$y - 0 = -\frac{4}{3}(x-1)$

$3y = -4x + 4$

$\underline{4x + 3y = 4}$

(iii) Solve

$4x + 3y = 4$ and

$3x - 4y = -2$

$16x + 12y = 16$

$9x - 12y = -6$

ADD

$25x = 10$

$x = \frac{10}{25}$

$x = \frac{2}{5}$

If $x = \frac{2}{5}$

$4 \times \frac{2}{5} + 3y = 4$

$\frac{8}{5} + 3y = 4$

$3y = 4 - \frac{8}{5}$

$= \frac{12}{5}$

$\therefore y = \frac{4}{5}$

$\therefore$ E is $\underline{(\frac{2}{5}, \frac{4}{5})}$

(iv) $ED = \left| \frac{3 \times 1 - 4 \times 0 + 2}{\sqrt{3^2 + (-4)^2}} \right|$

$= \left| \frac{5}{5} \right|$

$\underline{= 1.}$

OR

$ED = \sqrt{(1 - \frac{2}{5})^2 + (0 - \frac{4}{5})^2}$

$= \sqrt{(\frac{3}{5})^2 + \frac{16}{25}}$

$= \sqrt{\frac{9}{25} + \frac{16}{25}}$

$= \sqrt{1}$

$\underline{= 1.}$

(v) $(x-1)^2 + (y-0)^2 = 1^2$

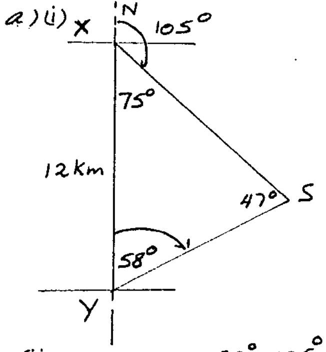
$\underline{(x-1)^2 + y^2 = 1}$

(vi) the perpendicular distance from the centre to each of the lines = radius

$\therefore$ lines are Tangents to the circle

Question 3

a)(i)



(ii) $\angle SXY = 180° - 105°$

$= 75°$

$\angle XSY$

$= 180° - (75° + 58°)$ $\angle$ sum of $\triangle$

$= 47°$

(iii) $\frac{XS}{\sin 58°} = \frac{12}{\sin 47°}$

$XS = \frac{12 \sin 58°}{\sin 47°}$

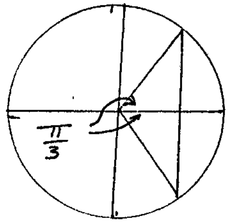
$\underline{= 13 \cdot 9 \text{ km}}$

to the nearest km

b $\cos 2\alpha = \frac{1}{2}$

where $0 \le \alpha \le 2\pi$

$\therefore 0 \le 2\alpha \le 4\pi$



$2\alpha = \pi/3, 5\pi/3, 7\pi/3, 11\pi/3$

$\underline{\alpha = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6}$

c) (i) $\frac{d}{dx}(1-x^2)^{\frac{1}{2}}$

$= \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \times (-2x)$

$\underline{= \frac{-x}{\sqrt{1-x^2}}}$

(ii) $\frac{d}{dx}(x^2 \ln x)$

$= x^2 \cdot \frac{1}{x} + 2x \ln x$

$= x + 2x \ln x$

$= \underline{x(1 + 2\ln x)}$

(iii) $\frac{d}{dx}\left(\frac{x}{\sin x}\right)$

$= \frac{\sin x \cdot 1 - x\cos x}{\sin^2 x}$

$= \underline{\frac{\sin x - x\cos x}{\sin^2 x}}$

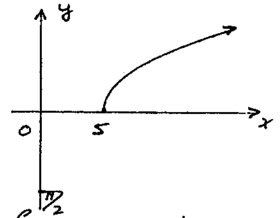
Question 4

a) $\log_9 243 = \frac{\log_{10} 243}{\log_{10} 9}$

$= 2 \cdot 5$

b) Domain $x - 5 \ge 0$

$x \ge 5$



c) $\int_0^{\pi/2} \sin 2x\,dx.$

$= -\frac{1}{2}\left[\cos 2x\right]_0^{\pi/2}$

$= -\frac{1}{2}(\cos\pi - \cos 0)$

$= -\frac{1}{2}(-1-1)$

$= -\frac{1}{2} \times -2$

$\underline{= 1.}$

d) $\underline{\frac{dP}{dt} > 0, \frac{d^2P}{dt^2} < 0}$

e)

Real roots if

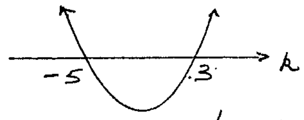
$b^2 - 4ac \ge 0$

i.e. $(1+k)^2 - 4 \times 4 \times 1 \ge 0$

$k^2 + 2k + 1 - 16 \ge 0$

$k^2 + 2k - 15 \ge 0$

$(k+5)(k-3) \ge 0$



$\underline{k \le -5 \text{ OR } k \ge 3}$

f) $y = 3x + 2\ln x$

$\frac{dy}{dx} = 3 + \frac{2}{x}.$

Since the tangent is // to $y = 4x + 3$

$\frac{dy}{dx} = 4$

$3 + \frac{2}{x} = 4$

$\frac{2}{x} = 1$

$x = 2$

at $x = 2$

$y = 6 + 2\ln 2$

$\therefore$ Point is

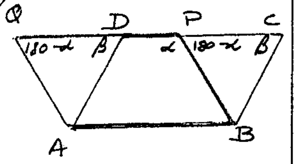
$\underline{(2, 6 + 2\ln 2)}$

Question 5

a) $\int e^{-2x}\,dx = \frac{1}{-2}e^{-2x}+c$

$= -\frac{1}{2}e^{-2x}+c.$

b) (i)



$\angle CPB = 180^\circ - \angle QPB$

since DPC is a st. line

$\angle Q = \angle CPB$ (corr. ∠'s, PB // QA)

$= 180^\circ - \alpha$

$\angle AQD = \angle C$ (corr. ∠'s, AD // BC)

$= \beta$

(ii) $QP = AB = DC$

(opp. sides of //ogram)

$QP = DC$

$QP - DP = DC - DP$

$QD = PC$

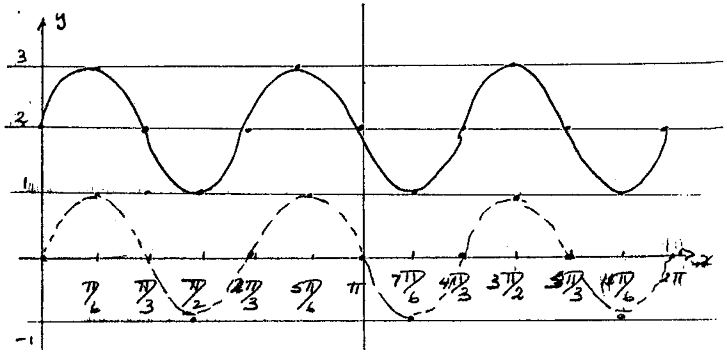
(iii) In $\triangle ADQ$ & $\triangle BCP$

1. $\angle Q = \angle BPC$ (proven above)
2. $\angle QDA = \angle C$ (" ")
3. $QD = PC$ (" ")

$\therefore \triangle ADQ \equiv \triangle BCP$ (AAS)

c) $\overset{*}{(i)}$ ~~[illegible]~~ Period $\frac{2\pi}{3}$

(ii) Amplitude = 1.



d) $\int_0^5 \frac{x}{5+3x^2}\,dx$

$= \frac{1}{6}\int_0^5 \frac{6x}{5+3x^2}\,dx$

$= \frac{1}{6}\left[\ln(5+3x^2)\right]_0^5$

$= \frac{1}{6}\left\{\ln 80 - \ln 5\right\}$

$= \frac{1}{6}\ln\frac{80}{5}$

$= \frac{1}{6}\ln 16$

$= \frac{1}{6}\ln 2^4$

$= \frac{4}{6}\ln 2$

$= \frac{2}{3}\ln 2$

Question 6

(i) $f(-4) = --4$

$= 4$

$f(1) = 1^2$

$= 1$

$f(9) = 4-9$

$= -5.$

$\therefore f(-4)+f(1)+f(9)$

$= 4+1-5$

$= 0$

(ii) $f(a^2)$

where $a \le -\sqrt{2}$

$= 4-a^2$

b) (i)

$y = xe^{-x}$

$\frac{dy}{dx} = x(-e^{-x})+e^{-x}.1$

$= -e^{-x}(x-1)$

(ii) Stationary points occur when $\frac{dy}{dx}=0$

i.e. $-e^{-x}(x-1)=0$

$x-1=0$

$x=1$

∴ Stat. pt at

$(1, 1e^{-1})$

i.e. $(1, \frac{1}{e})$

Note $e^{-x}>0$ for all x

* ~~[illegible]~~ $\frac{d2y}{dx}$

$= -e^{-x}.1+(x-1)(--e^{-x})$

$= -e^{-x}+e^{x}(x-1)$

$= e^{-x}(-1+x-1)$

$= e^{-x}(x-2)$

at $x=1$

$\frac{d2y}{dx^2} = e^{-1}(1-2)$

<0

∴ Maximum turning point at $(1, \frac{1}{e})$

(iii) Curve is decreasing if

$\frac{dy}{dx}<0$

i.e. $-e^{-x}(x-1)<0$

Now $e^{-x}>0$ for all x

$\therefore -(x-1)<0$

$x-1>0$

$x>1.$

Curve is decreasing if $x>1$

(iv) Shown in (ii)

(v) Possible inflexion when $\frac{d2y}{dx^2}=0$

i.e. $e^{-x}(x-2)=0$

$x-2=0$

$x=2$

i.e. at $(2, 2e^{-2})$

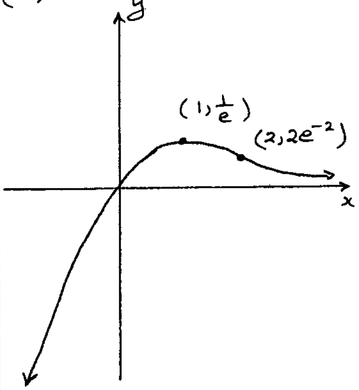
at $x=2^-$, $\frac{d^2y}{dx^2}=(+)(-)$ <0

$x=2^+$, $\frac{d^2y}{dx^2}=(+)(+)$ >0

∴ Change in concavity either side of $x=2$

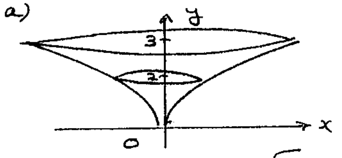
∴ Inflexion at $(2, 2e^{-2})$

(vi)



Question 7.

a)



$y=\sqrt{x}$

$\therefore x=y^2$

$x^2=y^4$

$V = \pi\int_2^3 x^2\,dy$

$= \pi\int_2^3 y^4\,dy$

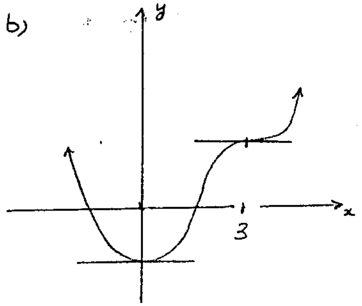
$= \pi\left[\frac{y^5}{5}\right]_2^3$

$= \pi\left(\frac{3^5}{5}-\frac{2^5}{5}\right)$

$= \pi\left(\frac{243-32}{5}\right)$

$= \frac{211\pi}{5}$ cubic units

b)



c) (i)

$A_1 = 9000 + 1{\cdot}5\%$ of $9000 - M$

$= 9000(1{\cdot}015) - M$

$A_2 = A_1 \times 1{\cdot}015 - M$

$= (9000\times 1{\cdot}015 - M)\times 1{\cdot}015 - M$

$= 9000\times 1{\cdot}015^2 - 1{\cdot}015M - M$

$A_3 = A_2\times 1{\cdot}015 - M$

$= 9000\times 1{\cdot}015^3 - 1{\cdot}015^2M$
$\quad - 1{\cdot}015M - M$

$A_n = 9000\times 1{\cdot}015^n$
$\quad - 1{\cdot}015^{n-1}M$
$\quad - 1{\cdot}015^{n-2}M$
$\quad - \dots$
$\quad - 1{\cdot}015^2M$
$\quad - 1{\cdot}015M$
$\quad - M$

$= 9000\times 1{\cdot}015^n - M(1+1{\cdot}015+\dots+1{\cdot}015^{n-1})$

$= 9000\times 1{\cdot}015^n - M\times 1\frac{(1{\cdot}015^n-1)}{(1{\cdot}015-1)}$

$= 9000\times 1{\cdot}015^n - M\frac{(1{\cdot}015^n-1)}{\frac{15}{1000}}$

$= 9000\times 1{\cdot}015^n - \frac{200}{3}(1{\cdot}015^n-1)$

(ii) If $A_{48}=0$

$0 = 9000\times 1{\cdot}015^{48} - \frac{200}{3}(1{\cdot}015^{48}-1)$

$\frac{200}{3}M(1{\cdot}015^{48}-1) = 9000\times 1{\cdot}015^{48}$

$M = \frac{3\times 9000}{200}\times\frac{1{\cdot}015^{48}}{(1{\cdot}015^{48}-1)}$

$= \$264{\cdot}37$

Total Repayments

$= 48 \times \$264.37$

$= \$12689.76$

$\therefore$ Interest

$= \$12689.76 - \$9006$

$= \$3689.76$

Interest Charged

$= \underline{\underline{\$3689.76}}$

Interest Rate

$= \left(\frac{\$3689.76}{\$9000} \times \frac{100}{1} \div 4\right)\%$

$= \underline{\underline{10.2\%}}.$

d) $\log_3 x + \log_3(x+8) = 2$

$\log_3(x(x+8)) = 2$

$x(x+8) = 3^2$

$x^2 + 8x - 9 = 0$

$(x+9)(x-1) = 0$

$x + 9 = 0$ OR $x - 1 = 0$

$x = -9$ OR $x = 1$

But $x > 0$

$\therefore \underline{\underline{x = 1}}.$

Question 8

a) $\frac{a+ar+ar^2}{\frac{a}{1-r}} = \frac{7}{8}.$

$\frac{(1-r)}{a} \times a(1+r+r^2) = \frac{7}{8}$

$(1-r)(1+r+r^2) = \frac{7}{8}$

$1 - r^3 = \frac{7}{8}$

$1 - \frac{7}{8} = r^3$

$\frac{1}{8} = r^3$

$\therefore \underline{\underline{r = \frac{1}{2}}}$

b) $A \doteqdot \frac{h}{2}\{y_0 + y_n + 2y_{rest}\}.$

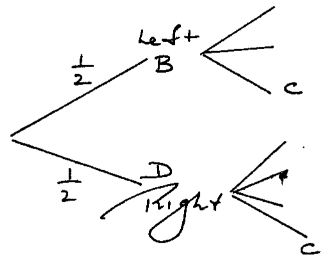
$= \frac{3}{2}\{y_0 + y_5 + 2(y_1 + y_2 + y_3 + y_4)\}$

$= \frac{3}{2}\{0 + 0 + 2(2.3 + 4.7 + 5.3 + 2.7)\}$

$= \frac{3}{2} \times 2(15 \quad)$

$\underline{\underline{= 45 \text{ m}^2}}.$

c)



$BC = \frac{1}{2} \times \frac{1}{3}$

$DC = \frac{1}{2} \times \frac{1}{4}$

$P(\text{reaches } C) = \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{4}$

$= \frac{1}{6} + \frac{1}{8}.$

$= \frac{4+3}{24}$

$= \frac{7}{24}$

$\therefore P(\text{does not reach } C) = 1 - \frac{7}{24}$

$= \underline{\underline{\frac{17}{24}}}$

d) (i) $y = 2 - x^2$ & $y = -x$

$-x = 2 - x^2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ OR $x = -1$

$\therefore$ Curves intersect at $(2, -2)$ and $(-1, 1)$

$A = \left|\int_{-1}^{2} (2 - x^2 - (-x))\,dx\right|$

$= \left|\int_{-1}^{2} (2 - x^2 + x)\,dx\right|$

$= \left|\left[2x - \frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{2}\right|$

$= \left|\left(4 - \frac{8}{3} + 2\right) - \left(-2 - -\frac{1}{3} + \frac{1}{2}\right)\right|$

$= \left|3\frac{1}{3} - (-1\frac{1}{6})\right|$

$= \left|3\frac{1}{3} + 1\frac{1}{6}\right|$

$= \underline{\underline{4\frac{1}{2} \text{ square units}}}.$

Question 9

a) $\sqrt{\sin^2 x - \sin^2 x \cos^2 x}$

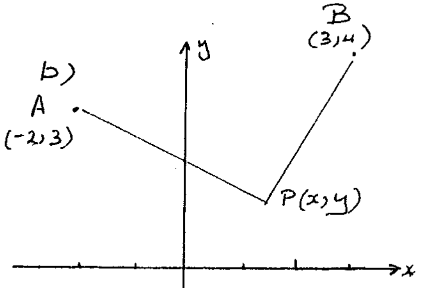
$= \sqrt{\sin^2 x(1 - \cos^2 x)}$

$= \sqrt{\sin^2 x \cdot \sin^2 x}$

$= \sqrt{(\sin^2 x)^2}$

$= \sin^2 x$

b)



$PA^2 - PB^2 = 5$

$(x - -2)^2 + (y-3)^2 - \{(x-3)^2 + (y-4)^2\} = 5$

$x^2 + 4x + 4 + y^2 - 6y + 9 - \{x^2 - 6x + 9 + y^2 - 8y + 16\} = 5$

$\cancel{x^2} + 4x + 4 + \cancel{y^2} - 6y + 9 \cancel{-x^2} + 6x - 9 \cancel{-y^2} + 8y - 16 = 5$

$\underline{\underline{10x + 2y = 17}}$

c) $x^2 - 6x - 7 = 8y$

$x^2 - 6x + 9 = 8y + 7 + 9$

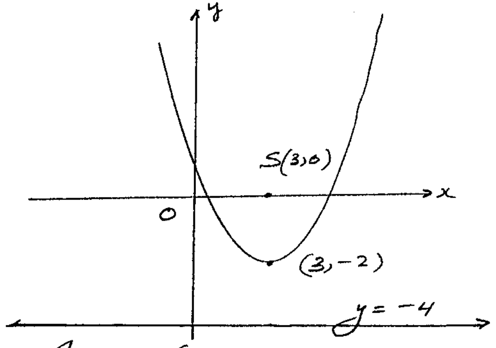
$(x-3)^2 = 8y + 16$

$(x-3)^2 = 8(y+2)$

$(x-h)^2 = 4a(y-k)$ *

Vertex $(3, -2)$

Focal Length $a = 2$.



Focus $(3, 0)$

Directrix $y = -4$

d) $V = 25t - \frac{1}{50}t^2$

$\frac{dV}{dt} = 25 - \frac{1}{50} \times 2t$

$= 25 - \frac{1}{25}t$

at $t = 5$, $\frac{dV}{dt} = 25 - \frac{1}{25} \times 5$

$= 25 - \frac{1}{5}$

$= \underline{\underline{24\frac{4}{5} \text{ m}^3/\text{min}}}.$

c) $\frac{dP}{dt} = 4000\ (t+5)\ t-1)$

$\frac{dP}{dt} = 4000\,(t^2+4t-5)$

$P = \int_0^3 4000(t^2+4t-5)\,dt$

$P = 4000\int_0^3 (t^2+4t-5)\,dt$

$P = 4000\left[\frac{t^3}{3}+\frac{4t^2}{2}-5t\right]_0^3$

$= 4000\left[\frac{t^3}{3}+2t^2-5t\right]_0^3$

$= 4000\left\{\left(\frac{27}{3}+2\times9-15\right)-(0+0-0)\right\}$

$= 4000(9+18-15)$

$= 4000 \times 12$

$= \underline{\underline{48000}}$.

Question 10

a) $\int_0^{\pi/2} \sin^2 x\,dx + \int_0^{\pi/2} \cos^2 x\,dx$

$= \int_0^{\pi/2} (\sin^2 x + \cos^2 x)\,dx$

$= \int_0^{\pi/2} 1\,dx$

$= [x]_0^{\pi/2}$

$= \pi/2 - 0$

$= \underline{\underline{\pi/2}}$.

b) (i) $\left(\frac{h}{2}\right)^2 + r^2 = (2\sqrt{3})^2$

$\frac{h^2}{4} + r^2 = 12$

$r^2 = 12 - \frac{h^2}{4}$

* (ii) $V = \cancel{\tfrac{1}{3}}\pi r^2 h$

$= \cancel{\tfrac{1}{3}}\pi\left(12-\frac{h^2}{4}\right)h$

$V = 12\pi h - \frac{\pi h^3}{4}$

$= 12\pi h - \frac{1}{4}\pi h^3$

$\frac{dV}{dh} = 12\pi - \frac{3\pi}{4}h^2$

Stationary points occur when $\frac{dV}{dh} = 0$

$12\pi - \frac{3\pi}{4}h^2 = 0$

$12\pi = \frac{3\pi h^2}{4}$

$\frac{48\pi}{3\pi} = h^2$

$h^2 = 16$

$h = 4$ Note $h > 0$.

$\frac{d^2V}{dh^2} = -\frac{6\pi}{4}h$

$= -\frac{3\pi}{2}h$

when $h = 4$, $\frac{d^2V}{dh^2} = -\frac{3\pi}{2}\times 4$

< 0

∴ V is a maximum when $h = 4$

If $h = 4$, $r^2 = 12 - \frac{4^2}{4}$

$= 12 - 4$

$= 8$

$r = \sqrt{8}$

∴ If Volume is a maximum height $= 4$ cm & radius $= \sqrt{8}$ cm

∴ $V = \pi r^2 h$

$\cancel{= \pi \times 4^2 \times \sqrt{8}}$ $= \pi(\sqrt{8})^2 \times 4$

$\cancel{= 16\pi \times 2}\sqrt{2}$ $= 32\pi\ \text{cm}^3$.

c) $V = \pi\int_0^1 y^2\,dx$

$= \pi\int_0^1 \frac{1}{2}\,x(1-x)^2\,dx$

$= \frac{\pi}{2}\int_0^1 x(1-2x+x^2)\,dx$

$= \frac{\pi}{2}\int_0^1 (x-2x^2+x^3)\,dx$

$= \frac{\pi}{2}\left[\frac{x^2}{2}-\frac{2x^3}{3}+\frac{x^4}{4}\right]_0^1$

$= \frac{\pi}{2}\left\{\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right)-(0-0+0)\right\}$

$= \frac{\pi}{2}\times\frac{6-8+3}{12}$

$= \frac{\pi}{2}\times\frac{1}{12}$

$= \frac{\pi}{24}$ cubic units.