Lecture 16
Graphs

(from Patel, 1996: 1.3 Techniques of Graphing.)

1. **Domain:** The set of all values of x that generate real values of y. The graph does not exist in the interval where y is not real.

2. **Range:** The corresponding values of y, once the domain is known.

3. **Intercepts on the axes:** The x-intercepts and the y-intercepts.

4. **Symmetry:** **(a)** The curve $y = f(x)$ is symmetric about the:
   (i) y-axis if $f(x) = f(-x)$ (even)
   (ii) origin if $f(-x) = -f(x)$ (odd) (point symmetry of order 2 about the origin)
   (b) The curve $y^2 = f(x)$ is symetric about the x-axis.

5. **Asymptotes:** Vertical asymptotes of the curve $y = f(x)$ are given by those values of x for which y is not defined. In the case of rational functions such as $f(x) = \frac{x}{x^2-4}$, the vertical asymptotes are easily found by putting the denominator equal to zero, provided the numerator is not equal to zero.
   A function $f(x)$ may be undefined at $x = c$ and yet have no vertical asymptote at $x = c$, e.g., $f(x) = \frac{x^2-4}{x-2}, x \neq 2$.
   If $\lim_{x \to a} f(x) = \pm\infty$, then $x = a$ is the vertical asymptote.
   The graph of $y = f(x)$ has a horizontal asymptote at $y = c$, if $\lim_{x \to \pm\infty} f(x) = c$.
   Oblique asymptote: Occasionally, for a rational function in which the degree of numerator > the degree of the denominator, we obtain an oblique asymptote of the form $y = mx + b$, e.g., $y = \frac{x^2+2x+3}{x+2} = x + \frac{3}{x+2}$.
   As $x \to \pm\infty,\ y \to x$, i.e., $y = x$ is the oblique asymptote.

6. **Stationary Points:** $f'(x) = 0$ gives all the stationary points. If the nature of the stationary point is obvious, there is no need to waste time in ascertaining the signs of $f''(x)$ or the variation of signs of $f'(x)$.

7. **Composition of ordinates:** The sketching of curves can often be simplified by the composition of ordinates. We express an equation for $f(x)$ as the combination of two or more simpler equations and hence sketch the graphs of the component equations; then by adding, subtracting,

multiplying or dividing the ordinates, we can draw the graph of $f(x)$.

8. Given $y = f(x)$, the sketching of $\frac{1}{f(x)}$, $\sqrt{f(x)}$, $[f(x)]^2$ and any other related function can be accomplished by using the properties of these related functions.

9. The following two diagrams, showing the relative positions with respect to the line $y = 1$ of the graphs of $f(x) = x, x^2, \sqrt{x}, \frac{1}{x}$ in particular, and $f(x), [f(x)]^2, \sqrt{f(x)}, \frac{1}{f(x)}$ in general, will be found very useful in sketching functions.