

Section I**10 marks****Attempt Questions 1-10****Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Questions 1-10.

1. Which of the following is equal to $\frac{x^2 - 36}{x - 6}$?

- (A) $x - 6$
- (B) $x + 6$
- (C) $x - 3$
- (D) $x + 3$.

2. What are the solutions to $3x^2 - 7x - 1 = 0$?

- (A) $x = \frac{-7 \pm \sqrt{37}}{6}$
- (B) $x = \frac{-7 \pm \sqrt{61}}{6}$
- (C) $x = \frac{7 \pm \sqrt{37}}{6}$
- (D) $x = \frac{7 \pm \sqrt{61}}{6}$.

3. What are the exact solutions of $2 \cos x = -\sqrt{3}$ for $0 \leq x \leq 2\pi$?

- (A) $\frac{\pi}{6}$ and $\frac{11\pi}{6}$
- (B) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$
- (C) $\frac{\pi}{3}$ and $\frac{5\pi}{3}$
- (D) $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$.

4. Which of the following define the domain and range of the function $f(x) = \log_e x$?

- (A) Domain: all real x and Range: all real y .
- (B) Domain: $x > 0$ and Range: $y > 0$
- (C) Domain: all real x and Range: $y > 0$
- (D) Domain: $x > 0$ and Range: all real y .

5. What is the derivative of $(e^{3x} + 1)^{-2}$?

- (A) $-2e^{3x}(e^{3x} + 1)^{-3}$
- (B) $-2e^{3x}(e^{3x} + 1)^{-1}$
- (C) $-6e^{3x}(e^{3x} + 1)^{-3}$
- (D) $-6e^{3x}(e^{3x} + 1)^{-1}$.

6. What is the perpendicular distance of the point $(4,5)$ from the line $3x - 2y + 10 = 0$?

- (A) $\frac{12}{\sqrt{13}}$
- (B) $\frac{17}{\sqrt{13}}$
- (C) $\frac{2}{\sqrt{5}}$
- (D) $\frac{12}{\sqrt{41}}$.

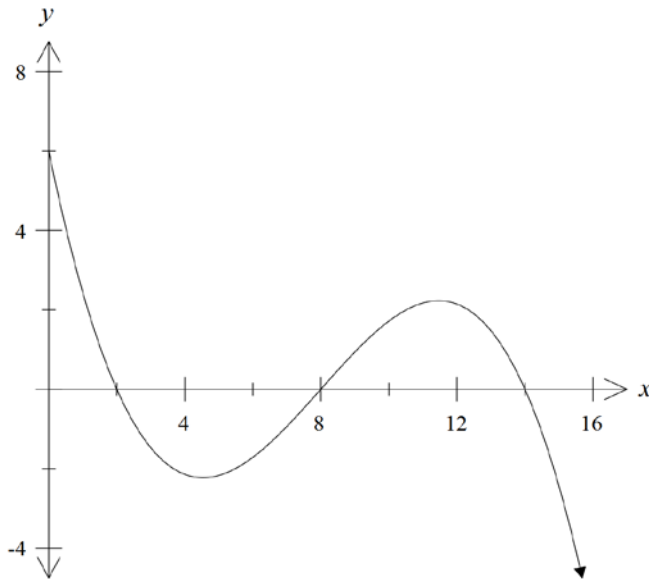
7. What is the solution of $5^x = 20$?

- (A) $\log_4 5$
- (B) $\log_5 4$
- (C) $1 + \log_4 5$
- (D) $1 + \log_5 4$.

8. A parabola has a focus $(3,1)$ and directrix $x = 5$. What is the equation of the parabola?

- (A) $(y-1)^2 = -4(x-4)$
- (B) $(y-1)^2 = 8(x-3)$
- (C) $(x-3)^2 = -8(y-3)$
- (D) $(x-3)^2 = -16(y-1)$.

9. The diagram below shows the graph $y = f(x)$.



Where is the function increasing, at a decreasing rate?

- (A) $(2,0)$
- (B) $(6,-1.8)$
- (C) $(10,1.8)$
- (D) $(14,0)$.

10. What is the value of $\int_3^5 4x^3 dx$

- (A) 192
- (B) 408
- (C) 544
- (D) 706.

Section II**90 marks****Attempt Questions 11-16****Allow about 2 hours and 45 minutes for this section**

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Question 11-16, your responses should include relevant mathematical reasoning and /or calculations.

Question 11 (15 marks)

- a) Evaluate $\frac{\ln 5}{3}$ correct to three significant figures. 1
- b) Evaluate $\lim_{x \rightarrow 3} \frac{x^3 - 3x^2}{x - 3}$ 2
- c) Differentiate $(1 + \tan x)^4$ 2
- d) Differentiate $x \ln x$ 2
- e) Find $\int 4xe^{x^2+1} dx$ 2
- f) Evaluate $\int_0^2 \frac{3x}{x^2+1} dx$ 3
- g) Sketch the region defined by $x^2 + (y-1)^2 \geq 9$ 3

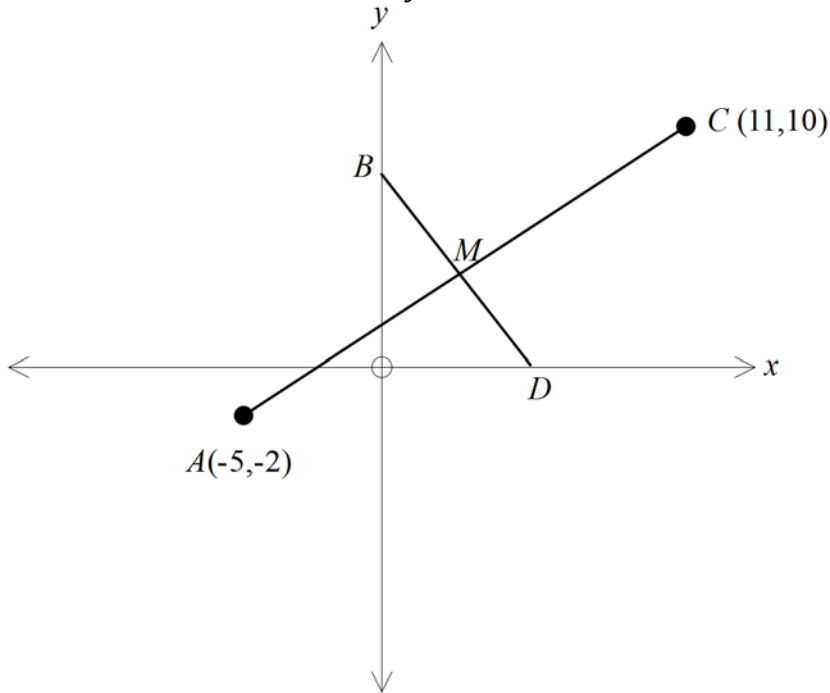
Question 12 (15 marks) Start a new booklet.

a) Given that $\int_0^4 kx + 2 dx = 12$, and k is a constant, find the value of k .

2

b) $A(-5, -2)$ and $C(11, 10)$ are two points on the number plane.

M is the midpoint of AC and the perpendicular bisector of AC meets the x axis at D and the y axis at B .



- i. Find the coordinates of M . 1
- ii. Show that the equation of the perpendicular bisector of AC ,
i.e. line BMD , is $4x + 3y - 24 = 0$ 2
- iii. Hence find the coordinates of the points B and D . 2
- iv. Show that the quadrilateral $ABCD$ is a rhombus. 2

c) Chairs are arranged in rows in front of a stage in a concert hall, so the row closest to the stage is the first row. Each row has two more chairs than the row in front of it. There are forty-two chairs in the tenth row.

- i. How many chairs are in the first row? 2
- ii. The seating arrangement has a total of 680 chairs.
How many rows of chairs are in the concert hall? 3
- iii. How many chairs are in the last row? 1

Question 13 (15 marks) Start a new booklet.

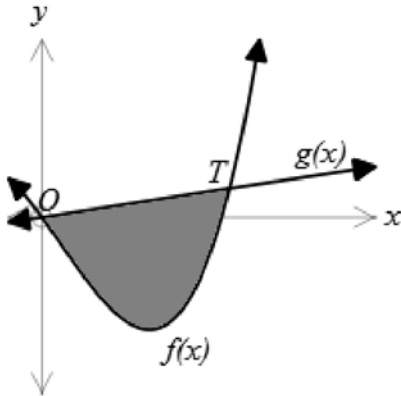
- a) The population $P(t)$ of turtles in a conservation park is given by:

$$P(t) = 200 - 75 \sin\left(\frac{\pi t}{3}\right).$$

where t is time in months.

- i. Find all times during the first 12 months when the population equals 275 turtles. 2
- ii. Sketch the graph of $P(t)$ for $0 \leq t \leq 12$. 2

- b) The diagram shows the graphs of the function $g(x) = 3x$ and $f(x) = 5x^3 - 5x^2 - 27x$. The graphs meet at O and T .



- i. Find the x -coordinate of T . 1
 - ii. Find the area of the shaded region between the graphs of the functions. 3
- c) Tina borrows \$5000 at 1.5% per month reducible interest and pays the loan off in equal monthly instalments. Tina is to repay the loan in 3 years. 3
Calculate the value of each monthly instalment.

- d) The diagram shows two quadrants, centre O .
 $OA = 3$ cm, $OD = 2$ cm, $\angle AOP = \theta$ radians.

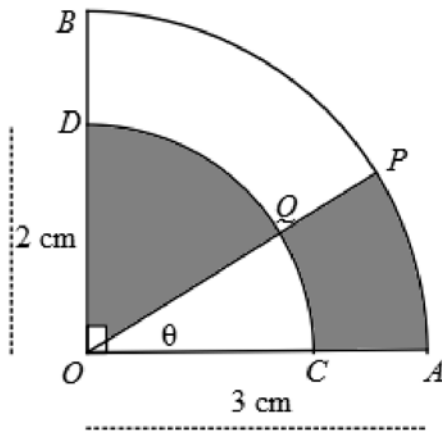


Diagram is NOT drawn to scale.

- i. Show that $\frac{5}{2}\theta$ is an expression for the area of the shaded region $APQC$. 1
- ii. If the area of the shaded region $APQC$ is $\frac{5\pi}{6}$ squared centimetres.
 Find the size of $\angle AOP$. 1
- iii. Hence find the exact area of shaded sector OQD . 2

Question 14 (15 marks) Start a new booklet.

- a) A particle travels so that its displacement (x metres), after t seconds is given by:

$$x = 12t - 3t^2 .$$

- i. Where is the particle 3 seconds after it starts? **1**
- ii. When does the particle turn around? **1**
- iii. How far does the particle travel during the first 5 seconds? **2**
- iv. Find the greatest speed during the first 5 seconds. **1**

- b) A cylinder is to be cut from a solid sphere.

The diagram below shows a cross section of the sphere and cylinder.

The sphere has a diameter of 8 cm. The cylinder has a height of h cm and a radius of r cm.

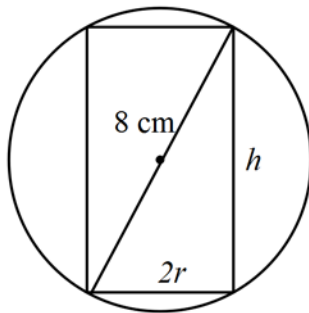


Diagram is NOT drawn to scale.

- i. Show that the volume (V) of the cylinder is given by: **2**
- $$V = \pi \left(\frac{64 - h^2}{4} \right) h$$
- ii. Find the value of h such that the volume of the cylinder is a maximum. **3**

- c) On an island, the population P after t years is given by: $P = P_0 e^{kt}$.

The initial population of the island is halved in 25 years.

- i. Show that $k = \frac{\ln 0.5}{25}$ **1**
- ii. How long will it take for the population to reduce from 5000 people to 2000 people? **2**
- iii. What percentage of the original population will be present after 75 years? **2**

Question 15 (15 marks) Start a new booklet.

- a) i. Copy this table and complete it, leave your answers as fractions. 1

x	1	2	3	4	5
$\frac{2}{x(x+1)}$					

- ii. Use the 5 functional values from part i, and Simpson's rule, to find an approximation to 2

$\int_1^5 \frac{2}{x(x+1)} dx$. Write your approximation using two decimal places.

- iii. Show that $\frac{2}{x} - \frac{2}{x+1} = \frac{2}{x(x+1)}$. 2

- iv. Deduce the value of the integral in part ii, correct to two decimal places. 2

- b) In the diagram, $ABCD$ is a quadrilateral and BD is a diagonal.
 $CB = 8$ cm, $AB = 9$ cm, $AD = 6$ cm and $BD = 12$ cm. $\angle DAB = \angle CBD$.

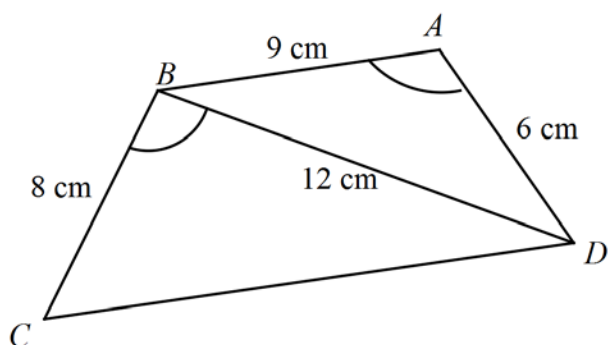
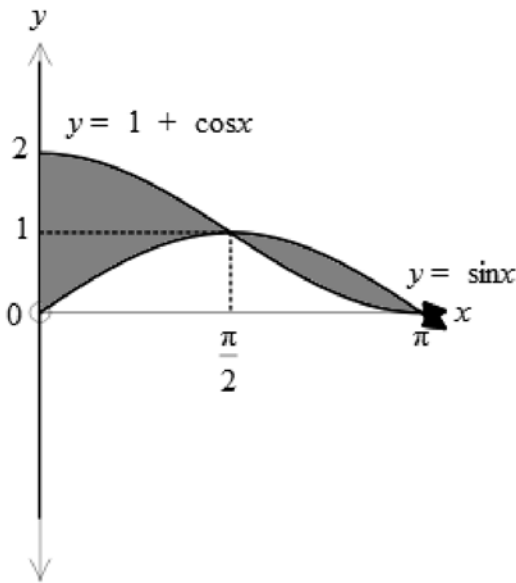


Diagram is NOT drawn to scale.

- i. Prove triangle ABD and BDC are similar. 2
- ii. Find the length of CD . 2
- iii. Prove that AB and CD are parallel 1

- c) The graphs of $y = \sin x$ and $y = 1 + \cos x$ are shown intersecting at $x = \frac{\pi}{2}$ and $x = \pi$ 3

Calculate the total area of the two shaded regions.



Question 16 (15 marks) Start a new booklet.

- a) The region bounded by the curve $y = \sec x$, the lines $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ is rotated through one complete revolution about the x axis. 3

Find the volume of the solid of revolution. Give your answer in exact form.

- b) The acceleration of a particle is given by:

$$\ddot{x} = -12e^{-2t}$$

where x is displacement in metres and t is time in seconds.

Initially its velocity is 7 ms^{-1} and its displacement is 4 m.

- i. Show that the velocity of the particle is given by: $\dot{x} = 6e^{-2t} + 1$ 2
- ii. Graph the velocity with respect to time. 2
- iii. Find the displacement when $t = 3$ seconds. 2

- c) Consider the function $y = 1 + 3x - x^3$, for $-2 \leq x \leq 3$.
- i. Find all stationary points and determine their nature. **3**

 - ii. Find the point of inflexion. **1**

 - iii. Sketch the curve for $-2 \leq x \leq 3$. Do not find the x - intercepts. **1**

 - iv. What is the minimum value for the curve over the stated domain? **1**

END OF THE EXAMINATION