

Question One

(a) Find:

(i) $\int x\sqrt{x^2-5} dx$ 2

(ii) $\int (1-x^2)^3 dx$ 2

(b) Evaluate:

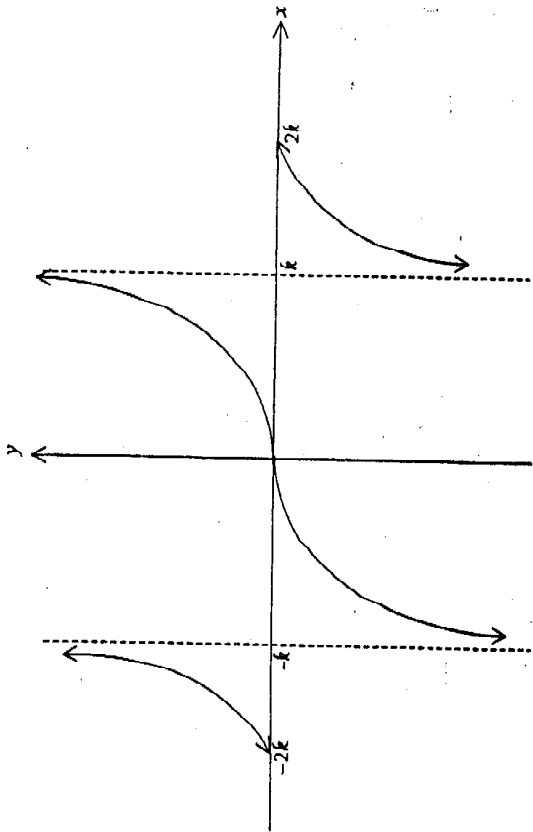
(i) $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin\theta} d\theta$ 3

(ii) $\int_0^1 \frac{x^2-5x-2}{(2-x)(4+x)} dx$ 4

(iii) $\int_1^2 x^2 \cdot e^x dx$ 4

Question Two (Start a new booklet)

(a) The graph of $f(x)$ is shown below:



Draw neat sketches of the following:

(i) $y = f(x-k)$

(ii) $y = [f(x)]^2$

(iii) $y = \frac{1}{f(x)}$

(iv) $y = f'(x)$

(b) Consider the curve $y = 4x^2(2-x^2)$.

(i) Sketch the curve, clearly indicating the important features. 4

(ii) Hence sketch the curve $y^2 = 4x^2(2-x^2)$ 1

(iii) Sketch the curve $y = \log_e 4x^2(2-x^2)$ 2

Question Three (Start a new booklet)

- (a) Express $w = 1 + i$ and $z = \sqrt{3} - i$ in the form $r(\cos\theta + i\sin\theta)$.

Hence find the modulus and argument of

(i) wz 4

(ii) $w^{-1}z$ 4

- (b) An equilateral triangle has its vertices on the circle $|z| = 2$. One vertex is the point representing $\sqrt{3} + i$. Find the other two vertices and make a neat sketch. 4

(c) Solve for z :

$$\frac{z - 2i}{1 + iz} = \frac{4}{3}$$

expressing your answer in modulus-argument form. 4

- (d) Shade the region of the Argand diagram consisting of those points z for which

(i) $R(z) \leq 2$ and $I(z) > -1$ 1

(ii) $|z - 1 - i| \leq 1$, $0 \leq \arg z \leq \frac{\pi}{4}$ 2

Question Four (Start a new booklet)

- (a) Given that $P(x) = (x^4 - 1)(x^2 - 2)$, factorise $P(x)$ completely over:

(i) The real numbers R 3

(ii) The complex numbers C .

- (b) (i) If $x = \alpha$ is a double root of the polynomial equation $Q(x) = 0$, show that $x = \alpha$ is a root of the equation $Q'(x) = 0$. 2

(ii) If the polynomial $P(x) = x^4 + x^2 + 6x + 4$ has a rational zero of multiplicity 2, find all the zeros of $P(x)$ over the complex field. 4

(c) Consider the polynomial $P(x) = x^4 - 4x^3 + 11x^2 - 14x + 10$.

(i) If $P(x)$ has roots $a + bi$ and $a - 2bi$ [where a and b are real], find the values of a and b . 3

(ii) Hence find the zeros of $P(x)$ over the complex field and express $P(x)$ as the product of two quadratic factors. 3

Question Five (Start a new booklet)

- (a) A solid has its base in the shape of an ellipse with major axis 8 units and minor axis 6 units. If every section perpendicular to the major axis is an equilateral triangle, show that the volume of the solid formed is $48\sqrt{3}$ cubic units. 5
- (b) Using the method of cylindrical shells find the volume of the solid of revolution obtained by rotating about the y -axis, the region bounded by the curve $y = \sin x$ and the x -axis from $x = 0$ to $x = \frac{\pi}{2}$. 5
- (c) Find the volume obtained by rotating the area enclosed by the x -axis, the curve $y = \tan^{-1} x$ and the line $x = 1$ about the line $x = 1$. 5

Question Six (Start a new booklet)

- (a) Consider the curves $\frac{x^2}{16} + \frac{y^2}{7} = 1$ and $x^2 - \frac{y^2}{8} = 1$. 4
- (i) Show that both curves have the same foci. 4
- (ii) Find the equation of the circle through the points of intersection of the two curves. 4
- (b) (i) Show that the tangent to the ellipse $\frac{x^2}{12} + \frac{y^2}{4} = 1$ at the point $P(3, 1)$ has equation $x + y = 4$. 3
- (ii) If this tangent cuts the directrix in the fourth quadrant at the point T , and S is the corresponding focus, show that SP and ST are at right angles to each other. 4

Question Seven (Start a new booklet)

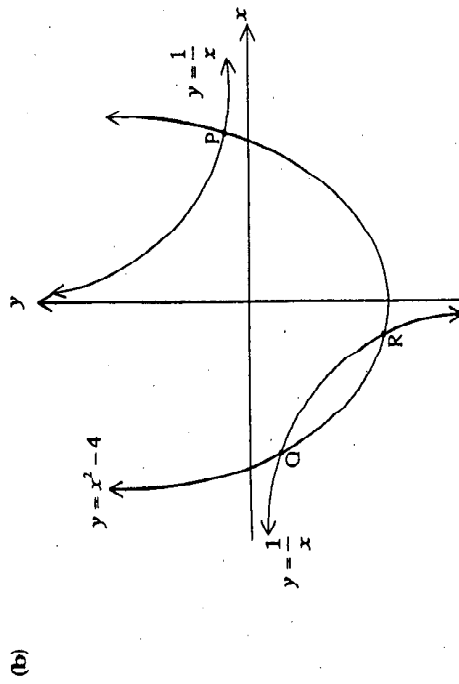
(a) Consider the function $f(x) = \frac{3x}{(x-1)(4-x)}$.

(i) Express $f(x)$ in partial fractions. 2

(ii) Find the co-ordinates and the nature of any turning points of the graph $y = f(x)$. 4

(iii) Sketch the graph of $y = f(x)$ showing clearly the co-ordinates of any turning points and the equation of asymptotes. 3

(iv) Find the area of the region bounded by the curve $y = f(x)$ and the x-axis between the lines $x = 2$ and $x = 3$. 2



The curves $y = x^2 - 4$ and $y = \frac{1}{x}$ intersect at the points P, Q and R whose x-coordinates are α , β and λ respectively. 1

(i) Show that α , β and λ are roots of the equation $x^3 - 4x - 1 = 0$. 1

(ii) Find a polynomial equation which has roots α^2 , β^2 and λ^2 . 3

Question Eight (Start a new booklet)

(a) Use the substitution $x = \frac{2}{3} \sin \theta$ to prove that $\int_0^2 \sqrt{4-9x^2} dx = \frac{\pi}{3}$.

Hence, or otherwise, find the area enclosed by the ellipse $9x^2 + y^2 = 4$. 5

(b) (i) If $z = \cos \theta + i \sin \theta$ use de Moivre's Theorem to show that $z^n + \frac{1}{z^n} = 2 \cos n\theta$. 4

(ii) By expanding $\left(z + \frac{1}{z}\right)^4$ show that $\cos^4 \theta = \frac{1}{8} (\cos 4\theta + 4 \cos 2\theta + 3)$. 5

(c) Show that $\frac{d}{dx} \left[\ln(x + \sqrt{x^2 + 4}) \right] = \frac{1}{\sqrt{x^2 + 4}}$.

Hence or otherwise, prove that $\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{\sec^2 x dx}{\sqrt{\tan^2 x + 4}} = 2 \ln \left(\frac{\sqrt{5} + 1}{2} \right)$. 5

END OF PAPER