Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1-10

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1 Which is not the equation of a function?

(A) $y = x^2 + 5$

(B) $y = -\sqrt{25 - x^2}$

(C) $x = y^2 + 3$

(D) $xy = 3$

2 $\frac{1}{\sqrt{3}+2} = a + b\sqrt{3}$, where a and b are integers. Which is correct?

(A) $a = 2$ and $b = 1$

(B) $a = 2$ and $b = -1$

(C) $a = -2$ and $b = -1$

(D) $a = -2$ and $b = 1$

3 $27x^3 - 8y^3 =$

(A) $(3x-2y)(9x^2 + 6xy + 4y^2)$

(B) $(3x-2y)(9x^2 + 12xy + 4y^2)$

(C) $(3x-2y)(9x^2 - 12xy + 4y^2)$

(D) $(3x-2y)(9x^2 - 6xy + 4y^2)$

4 $\log_4 8 - \log_4 2 =$

(A) 4

(B) 2

(C) 1

(D) 0

5 $M\left(-1, \frac{9}{2}\right)$ is the midpoint of $A(2,5)$ and $B(x,y)$. The coordinates of B are:

(A) $(0,4)$

(B) $(-4,-4)$

(C) $(-4,4)$

(D) $(4,-4)$

6 $40°$, expressed in radians in terms of π is:

(A) $\frac{2\pi}{9}$

(B) $\frac{3\pi}{8}$

(C) $\frac{4\pi}{7}$

(D) $\frac{5\pi}{8}$

7 If α and β are roots of $x^2 - 5x - 3 = 0$ then

(A) $\alpha + \beta = 5$ and $\alpha\beta = 3$

(B) $\alpha + \beta = 5$ and $\alpha\beta = -3$

(C) $\alpha + \beta = -5$ and $\alpha\beta = -3$

(D) $\alpha + \beta = -5$ and $\alpha\beta = 3$

8 The period of the graph $y = 2\cos\frac{x}{2}$ is

(A) $\frac{\pi}{2}$

(B) π

(C) 2π

(D) 4π

9 $\dfrac{\sin\theta\cos\theta+\sin^2\theta}{\sin\theta\cos\theta+\cos^2\theta}=$

(A) $\tan\theta$

(B) $\cot\theta$

(C) $\sec^2\theta$

(D) $\text{cosec}^2\theta$

10 What are the coordinates of the focus of the parabola $4y=x^2-8x+4$?

(A) $(4,3)$

(B) $(4,-3)$

(C) $(4,4)$

(D) $(4,-2)$

End of Section I

Section II

90 marks
Attempt Questions 11-16
Allow about 2.5 hours for this section

Begin each question in a new writing booklet, indicating the question number.
Extra writing booklets are available.

Question 11 (15 marks) Start a new writing booklet.

(a) Evaluate $-20(1-20^{20})$, expressing your answer correct to 2 significant figures. **2**

(b) Express $\dfrac{15^m \times 2^{2m}}{5^m}$ in the form a^{bm}, where a and b are integers. **2**

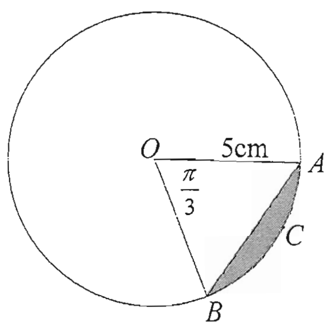
(c) Solve $|x|-2=3x-5$. **3**

(d) Solve $\sec^2\theta=2$ for $0°\leq\theta\leq 360°$. **2**

(e) Evaluate $\sum_{k=8}^{25} 3k-2$. **2**

(f) From the word DIVISION, two letters are to be chosen, without replacement. One letter is chosen and it is an I. If another letter is then chosen, what is the probability of choosing another I? **1**

(g) The diagram below shows a circle of radius 5cm with $\angle AOB=\dfrac{\pi}{3}$.



(i) Find the exact area of the minor sector AOBC. **1**

(ii) Hence, find the exact area of the shaded region. **2**

Question 12 (15 marks) Start a new writing booklet.

(a) For what values of k does the quadratic function $x^2 + kx + (k+3) = 0$ have no real roots? **2**

(b) The limiting sum of a geometric series is 10. Find the common ratio of this series if its first term is 8. **2**

(c) Differentiate the following, with respect to x:

(i) $y = (4 - x^3)^7$. **2**

(ii) $y = \cos x(1 - \sin x)$. **2**

(iii) $y = \frac{\ln x}{x^2}$. **2**

(d) (i) Find $\int 3\cos 5x \, dx$. **1**

(ii) Find $\int \frac{x}{5x^2 - 8} \, dx$. **2**

(e) (i) Differentiate $y = e^{3x^2}$. **1**

(ii) Hence, find $\int x e^{3x^2} \, dx$. **1**

Question 13 (15 marks) Start a new writing booklet.

(a) Evaluate $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \left(\sec^2 x - 1\right) dx$. **3**

(b) (i) Show that the derivative of $f(x) = \dfrac{x+3}{x-4}$ is $f'(x) = \dfrac{-7}{(x-4)^2}$. **2**

(ii) Hence, find the equation of the tangent to the curve $y = f(x)$ at the point $(3, -6)$. **2**

(c) (i) Show that the perpendicular distance from point $(1, -4)$ to the line $4x - 3y + 14 = 0$ is 6 units. **2**

(ii) Find the centre and radius of the circle $x^2 - 2x + y^2 + 8y = 8$. **2**

(iii) Explain why the line $4x - 3y + 14 = 0$ will never intersect the circle $x^2 - 2x + y^2 + 8y = 8$. **1**

(d) The population of a city is increasing at a rate which is proportional to the current population. The population was 1.2 million at the beginning of 1975 and 1.8 million at the beginning of 1995.

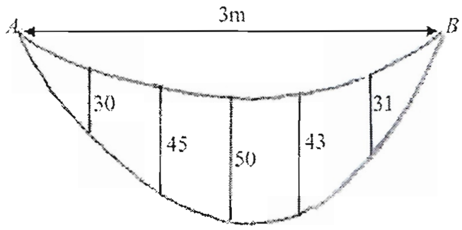
(i) What is the growth rate per year? **2**

(ii) What will be the expected population at the beginning of 2015? **1**

Question 14 (15 marks) Start a new writing booklet.

(a) Cindy needs to estimate the area of the following space for a smile banner.

All vertical measurements are in centimetres and are equally spaced apart.



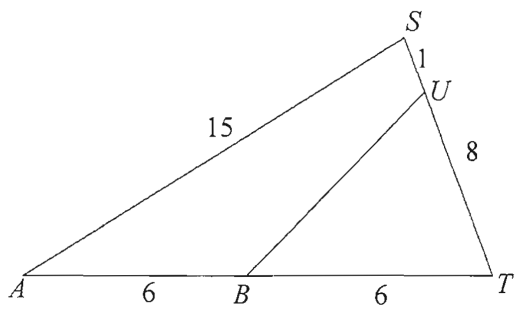
NOT TO SCALE

(i) Copy and complete the table of values in your writing booklet. **1**

Horizontal distance from A (cm)	0						300
Vertical Distance between the curves	0						0

(ii) Use Simpson's Rule and all the values from the table to find an approximation for the area of the smile. **2**

(b)



NOT TO SCALE

(i) Prove ΔBUT is similar to ΔSAT. **3**

(ii) Hence, find the length of BU. **1**

(iii) What type of triangles are ΔBUT and ΔSAT, based on their angle sizes? Justify your answer. **1**

Question 14 continues on page 9

(c) At a Sydney restaurant, the Granny Smith Apple Festival is being honoured by including apple as an ingredient on the menu.

- Course 1: Entrée: 1 of 4 dishes contain apple.
- Course 2: Main: 2 of 5 dishes contain apple.
- Course 3 Dessert: 2 of 4 dishes contain apple.

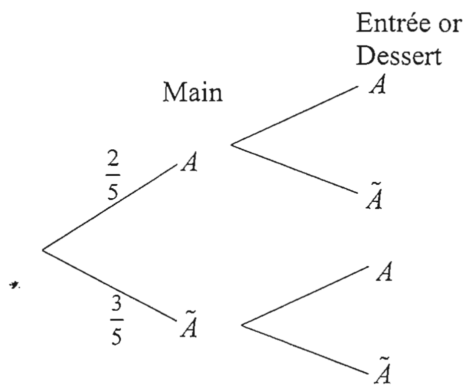
The restaurant only offers a three course meal, or a two course meal comprising of a main and either an entrée or a dessert.

Sarah is having an entrée, main and dessert.

(i) What is the probability that Sarah's 3 course meal has no dishes containing apple? **1**

(ii) What is the probability that Sarah's 3 course meal has at least one dish containing apple? **1**

Lisa only wants a two course meal: She will definitely have a main and will either have an entrée or a dessert.



(iii) Copy and complete the above tree diagram in your writing booklet, and hence find the probability that only one of Lisa's dishes contains apples. **2**

(d) The point $P(x, y)$ moves such that its distance from the point $A(1,0)$ is half its distance from the point $B(4,0)$. Find the equation of the locus of P and hence describe the locus geometrically. **3**

End of Question 14

Question 15 (15 marks) Start a new writing booklet.

(a) A closed cylindrical can, with radius r cm and height h cm is to be made to hold 1000 mL of oil. The manufacturer wants to minimise the surface area of the container to lower manufacturing costs.

(i) Show that $h = \dfrac{1000}{\pi r^2}$. **1**

(ii) Hence, show that the surface area, A, of the can is given by **1**

$$A = 2\pi r^2 + \frac{2000}{r}, \text{ where } r > 0.$$

(iii) Hence, find the exact radius such that the surface area is a minimum. **3**

(b) The acceleration of a particle is given by $\ddot{x} = 6t - 16$, where x is displacement in metres and t is time in seconds. Taking the right as the positive direction, the initial velocity is $5ms^{-1}$ and the initial displacement is 7 metres to the left of the origin.

(i) Show that the displacement of the particle is given by $x = t^3 - 8t^2 + 5t - 7$. **2**

(ii) Find when and where the particle comes to rest. **3**

(iii) Does the particle ever pass through the origin? Justify your answer. **1**

Question 15 continues on page 11

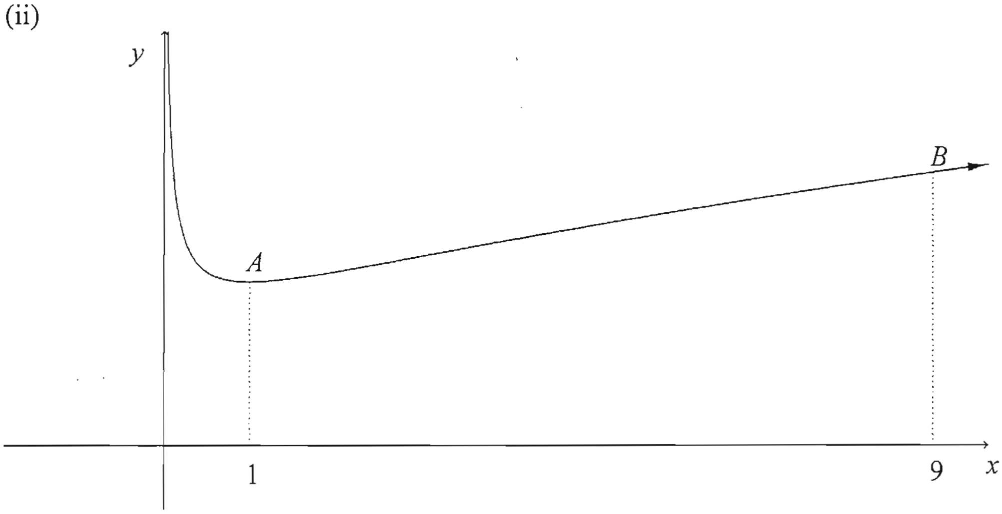
Question 16 (15 marks) Start a new writing booklet.

(a) Expand $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$. **1**

(b) (i) Copy and complete the table of values in your writing booklet. **1**

x	1	4	9
$\sqrt{x}$			
$\sqrt{x}+\frac{1}{\sqrt{x}}$			

(ii)



NOT TO SCALE

The graph of $y=\sqrt{x}+\frac{1}{\sqrt{x}}$ is shown above. Copy or trace the diagram into your writing booklet and write down the coordinates of the points A and B on your graph. **1**

(iii) On the same set of axis, draw $y=\sqrt{x}$ for $1 \le x \le 9$ and shade the region bounded by the curves $y=\sqrt{x}$ and $y=\sqrt{x}+\frac{1}{\sqrt{x}}$, and by the lines $x=1$ and $x=9$. **1**

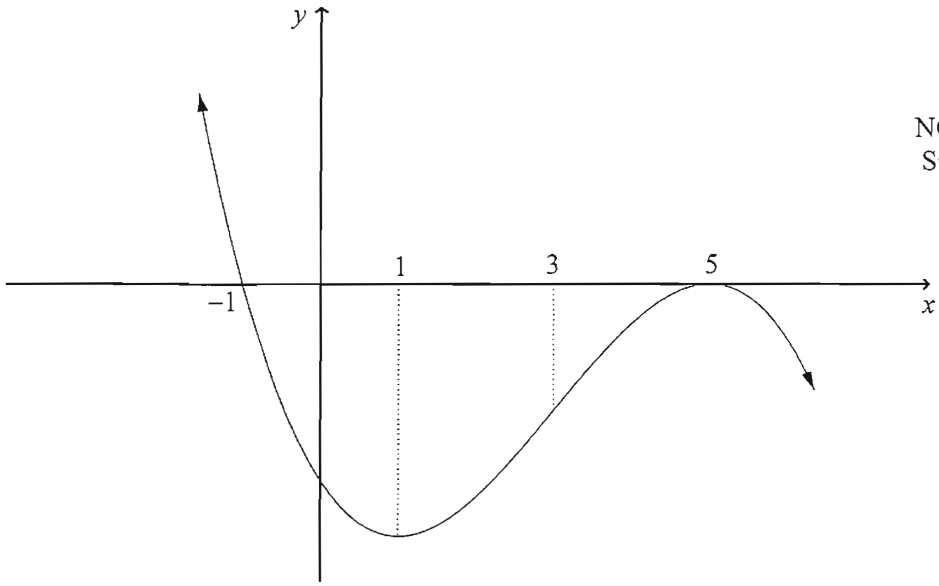
(iv) Find the area of the shaded region, as defined in part (iii). **3**

(v) Find the volume of the solid obtained by rotating the shaded region about the x-axis. **3**

Question 16 continues on page 12

Question 16 (continued)

(c) The graph of $y = g'(x)$ is shown below.



Copy or trace the graph into your writing booklet.

(i) There are two stationary points on $y = g(x)$. **2**

Determine their x-coordinates and their nature.

(ii) On the same diagram, draw a possible sketch of $y = g(x)$, showing all **3**

important features.

End of paper