Hunters Hill High School

Extension 2, Mathematics

Trial Examination, 2015

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- The marks for each question are shown on the paper
- Show all necessary working in questions 11-16

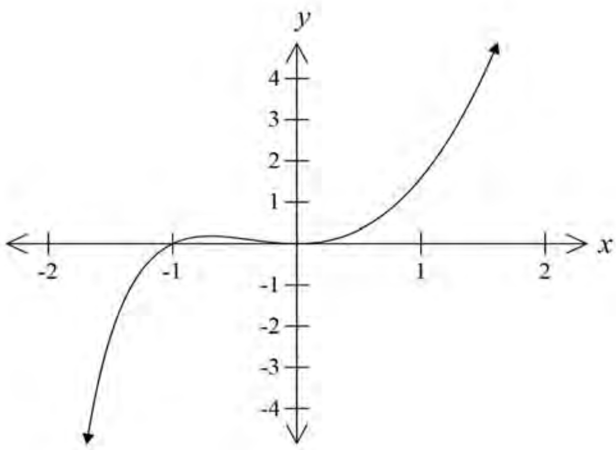
Total Marks: 100

Section I Page 3 – 7
10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II Pages 8 – 14
90 marks
- Attempt Questions 11-16
- Allow about 2 hour 45 minutes for this section

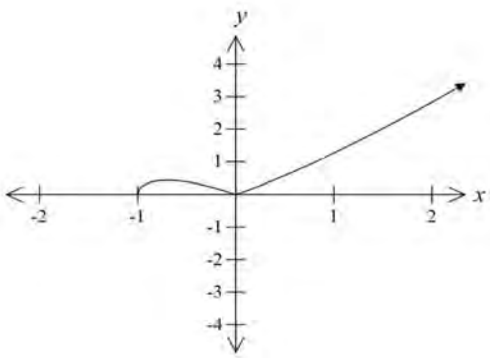
Section I MULTIPLE CHOICE: Write the correct alternative on your writing paper.

1 The diagram shows the graph of the function $y = f(x)$.

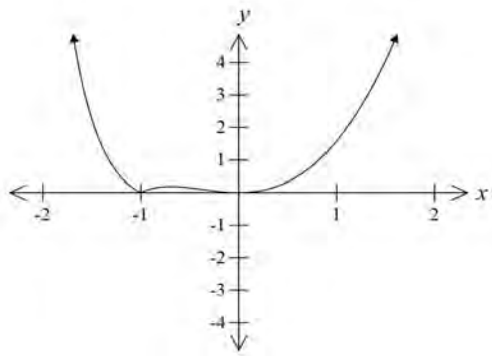


Which of the following is the graph of $y = f(x)^2$?

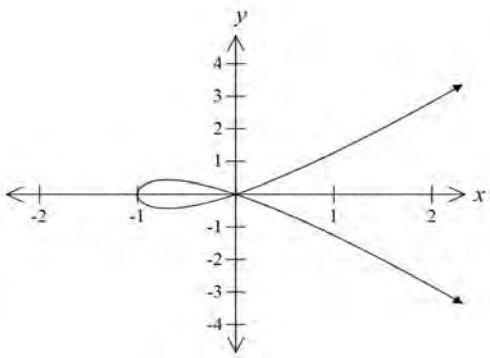
(A)



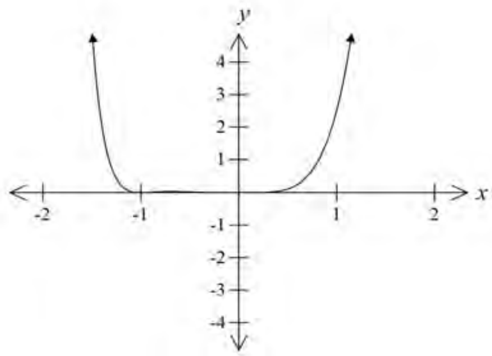
(B)



(C)



(D)



2 Consider the hyperbola with the equation $\frac{x^2}{144} - \frac{y^2}{25} = 1$.

What are the equations of the directrices?

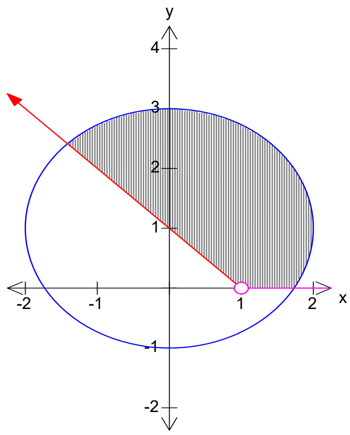
(A) $x = \pm\frac{13}{144}$

(B) $x = \pm\frac{13}{25}$

(C) $x = \pm\frac{25}{13}$

(D) $x = \pm\frac{144}{13}$

3 Consider the Argand diagram below.



Which inequality could define the shaded area?

(A) $|z-i| \le 2$ and $0 \le \arg(z-1) \le \frac{3\pi}{4}$

(B) $|z+i| \le 2$ and $0 \le \arg(z-1) \le \frac{3\pi}{4}$

(C) $|z-i| \le 2$ and $0 \le \arg(z-1) \le \frac{\pi}{4}$

(D) $|z+i| \le 2$ and $0 \le \arg(z-1) \le \frac{\pi}{4}$

4 Which of the following is an expression for $\int \frac{2}{x^2+4x+13}dx$?

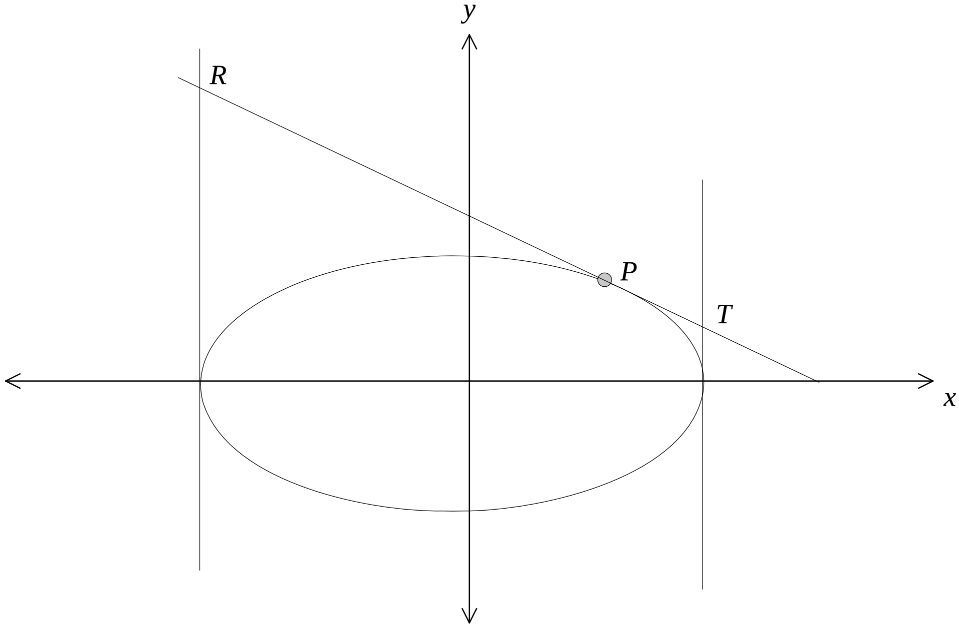
(A) $\frac{1}{3}\tan^{-1}\frac{(x+2)}{3}+c$

(B) $\frac{2}{3}\tan^{-1}\frac{(x+2)}{3}+c$

(C) $\frac{1}{9}\tan^{-1}\frac{(x+2)}{9}+c$

(D) $\frac{2}{9}\tan^{-1}\frac{(x+2)}{9}+c$

5 The point P lies on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $a>b>0$. The tangent at P meets the tangents at the ends of the major axis at R and T.



What is the equation of the tangent at P?

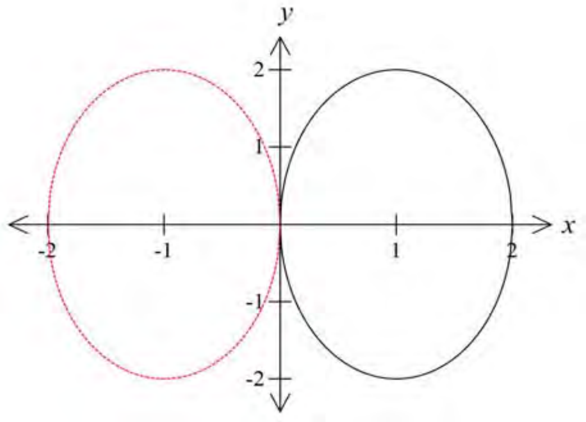
(A) $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

(B) $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$

(C) $\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$

(D) $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

6 The region enclosed by the ellipse $(x-1)^2+\frac{y^2}{4}=1$ is rotated about the y axis to form a solid.



What is the correct expression for volume of this solid using the method of slicing?

(A) $V=\int_{-2}^{2}\pi\sqrt{1-y^2}\,dy$

(B) $V=\int_{-2}^{2}2\pi\sqrt{1-y^2}\,dy$

(C) $V=\int_{-2}^{2}\pi\sqrt{4-y^2}\,dy$

(D) $V=\int_{-2}^{2}2\pi\sqrt{4-y^2}\,dy$

7 The polynomial equation $x^3-3x^2-x+2=0$ has roots α, β and γ. Which of the following polynomial equations have roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$?

(A) $x^3-x^2-3x+1=0$

(B) $x^3-2x^2-3x+1=0$

(C) $2x^3-x^2-3x+1=0$

(D) $2x^3-2x^2-3x+1=0$

8 What is the derivative of $\sin^{-1} x - \sqrt{1-x^2}$?

(A) $\dfrac{\sqrt{1+x}}{\sqrt{1-x}}$

(B) $\dfrac{\sqrt{1+x}}{1-x}$

(C) $\dfrac{1+x}{\sqrt{1-x}}$

(D) $\dfrac{1+x}{1-x}$

9 What are the values of real numbers p and q such that $1-i$ is a root of the equation $z^3 + pz + q = 0$?

(A) $p = -2$ and $q = -4$

(B) $p = -2$ and $q = 4$

(C) $p = 2$ and $q = 4$

(D) $p = 2$ and $q = 4$

10 A particle of mass m is moving in a straight line under the action of a force.

$$F = \frac{m}{x^3}(6-10x)$$

What of the following is an expression for its velocity in any position, if the particle starts from rest at $x = 1$?

(A) $v = \pm\dfrac{1}{x}\sqrt{(-3+10x-7x^2)}$

(B) $v = \pm x\sqrt{(-3+10x-7x^2)}$

(C) $v = \pm\dfrac{1}{x}\sqrt{2(-3+10x-7x^2)}$

(D) $v = \pm x\sqrt{2(-3+10x-7x^2)}$

Total Marks – 90
Section II
Attempt Questions 11-16
All Questions are of equal value

Marks

Question 11 (15 marks) Begin a NEW sheet of paper.

(a) By using the method of partial fractions, show that **4**

$$\int \frac{dx}{x^2-1} = \ln\sqrt{\frac{x-1}{x+1}} + c$$

(b) Use the substitution $u = \cos x$ to evaluate **4**

$$\int_0^1 \sqrt{1-x^2}\ dx$$

(c) If $I = \int e^x \sin x\ dx$

Find *I* using the method of integration by parts. **4**

(d) Evaluate

$$\int_0^{\frac{\pi}{2}} \cos x \sin^3 x\ dx$$

3

End of Question 11

Question 12 (15 marks) Begin a NEW sheet of paper. **Marks**

(a) Given $A = 3 - 4i$ and $B = \sqrt{3} + i$.

(i) Find AB in $x + iy$ form **1**

(ii) Find $\dfrac{A}{B}$ in $x + iy$ form **1**

(iii) Find $\sqrt{A}$ in $x + iy$ form **3**

(iv) Find B in modulus- argument form **2**

(v) Hence find B^4 in $x + iy$ form **1**

(b) On separate Argand diagrams sketch the following loc

(i) $2 \geq |z| \geq 1$ **1**

(ii) $\dfrac{3\pi}{4} > \arg z > \dfrac{\pi}{4}$ **1**

(iii) $3 \geq \operatorname{Re} Z \geq 0$ and $3 \geq \operatorname{Im} Z \geq 1$ **2**

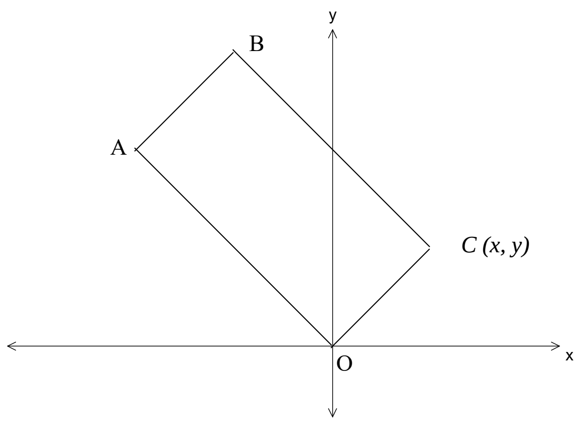
(c) On the Argand diagram shown OABC is a rectangle with the length OA being twice OC. OC represents the complex number $x + iy$.

Find the complex number represented by

(i) OA **1**

(ii) OB **1**

(iii) BC **1**



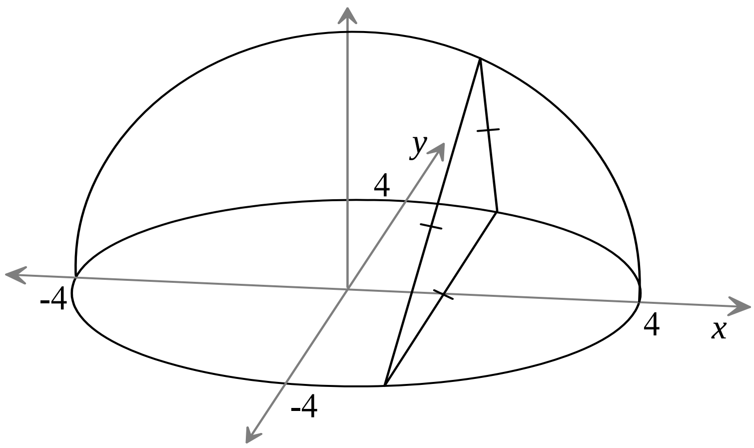
End of Question 12

Question 13 (15 marks) Begin a NEW sheet of paper.

Marks

(a) For the curve with equation $x^2 + 3xy - y^2 = 13$, determine the gradient of the tangent at the point (2, 3) on the curve. **4**

(b)



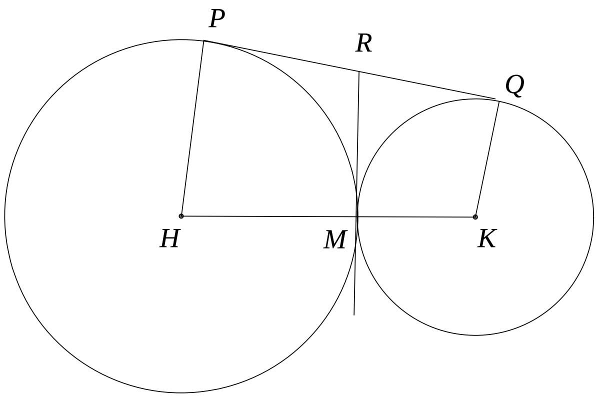
The diagram above shows a solid which has the circle $x^2 + y^2 = 16$ as its base. The cross-section perpendicular to the x axis is an equilateral triangle.

(i) Show that the cross-sectional area of the equilateral triangle is given by $A(x) = \sqrt{3}\left(16 - x^2\right)$ **3**

(ii) Calculate the volume of the solid. **3**

(c) Shown are two circles centres H and K which touch at M. PQ and RM are common tangents.

COPY DIAGRAM



(i) Show that quadrilaterals $HPRM$ and $MRQK$ are cyclic. **2**

(ii) Prove that triangles PRM and MKQ are similar. **3**

End of Question 13

Question 14 (15 marks) Begin a NEW sheet of paper. **Marks**

(a) The equation $P(x) = x^3 + 3x^2 - 24x + k = 0$ has a double root. Find the possible values of k. **2**

(b) The roots of $x^3 + 3px + q = 0$ are α, β and γ, (none of which are equal to 0).

(i) Find the monic equation with roots $\frac{\beta\gamma}{\alpha}, \frac{\alpha\gamma}{\beta}$ and $\frac{\alpha\beta}{\gamma}$, giving the coefficients in terms of p and q. **4**

(ii) Deduce that if $\gamma = \alpha\beta$ then $(3p - q)^2 + q = 0$ **2**

(c) The polynomial $P(x)$ leaves a remainder of 9 when divided by $(x-2)$ and a remainder of 4 when divided by $(x-3)$. Find the remainder when $P(x)$ is divided by $(x-2)(x-3)$. **3**

(d) (i) Solve $Z^5 = 1$ over the complex field giving your answers in modulus-argument form. **2**

(ii) Hence write $Z^5 - 1$ as the product of linear and quadratic factors. **2**

End of Question 14

Question 15 (15 marks) Begin a NEW sheet of paper. **Marks**

(a) A mass of 1kg is falling under gravity (g) through a medium in which the resistance to the motion is proportional to the square of the velocity. (k is the constant of proportionality)

(i) Draw a sketch showing all forces acting. **1**

(ii) Write an equation for the acceleration of this mass. **1**

(iii) Show that the mass reaches a terminal velocity given by $v = \sqrt{\frac{g}{k}}$. **1**

(iv) Show that the distance it has fallen when it reaches a velocity v m/s is given by $x = \frac{1}{2k}\ln\left(\frac{g}{g-kv^2}\right)$ **4**

(b) (i) Show that the recurrence (reduction)formula for

$$I_n = \int \sec^n x dx$$ **4**

is $I_n = \frac{1}{n-1}\tan x \sec^{n-2} x + \frac{n-2}{n-1} I_{n-2}$

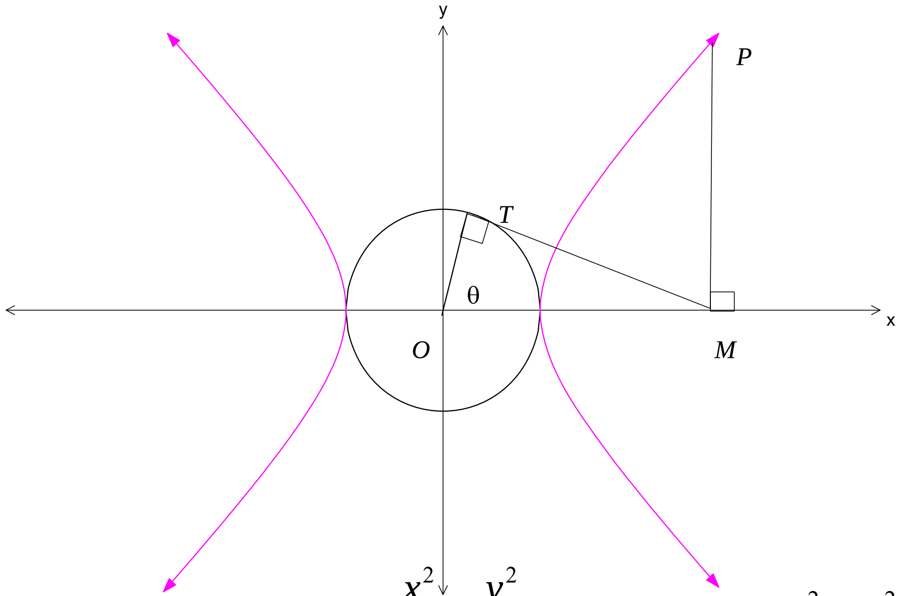
(ii) Hence evaluate $\int_0^{\frac{\pi}{4}} \sec^4 x\, dx$ **2**

(c) A cubic equation in z has all real coefficients. If two of the roots are 3 and $2+i$ determine the equation. **2**

End of Question 15

Question 16 (15 marks) Begin a NEW sheet of paper. **Marks**

(a)



The sketch shows the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and the circle $x^2+y^2=a^2$ with $a,b\geq 0$. T lies on the circle where $\angle TOX=\theta$ and $0\leq\theta\leq\frac{\pi}{2}$. The tangent at T meets OX at M and MP is perpendicular to OX with P on the hyperbola. Coordinates of T are $(a\cos\theta, a\sin\theta)$

(i) Find the equation of the tangent TM and hence the coordinates of M. **3**

(ii) Hence show that the coordinates of P are $(a\sec\theta, b\tan\theta)$ **1**

(iii) If $Q(a\sec\beta, b\tan\beta)$ is another point on the hyperbola, where $\theta+\beta=\frac{\pi}{2}$ and $\theta\neq\frac{\pi}{4}$, show that the equation of PQ is $ay=b(\cos\theta+\sin\theta)x-ab$. **3**

(iv) Every such chord PQ passes through a fixed point, find its coordinates. **2**

(v) Show that as θ approaches $\frac{\pi}{2}$, PQ approaches a line parallel to an asymptote. **2**

Question 7 continues on page 9.

Question 16 continued.

Marks

(b) (i) By letting $Z = \cos\theta + i\sin\theta$ show that **1**

$$Z^n + \frac{1}{Z_n} = 2\cos n\theta .$$

(ii) Hence express $\cos^4\theta$ in terms of $\cos n\theta$ **3**

End of Question 16

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

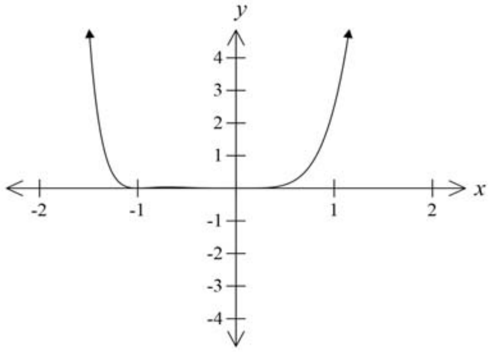
NOTE : $\ln x = \log_e x, \quad x > 0$

HHHS

2015
TRIAL HSC
EXAMINATION

Mathematics
Extension 2

SOLUTIONS

Section I	Trial HSC Examination- Mathematics Extension 2	2015
1		1 Mark: D
2	$b^2 = a^2(e^2 - 1)$ $a^2 = 144$ and $b^2 = 25$. $25 = 144(e^2 - 1)$ $a = 12$ $b = 5$ $(e^2 - 1) = \frac{25}{144}$ or $e^2 = \frac{169}{144}$ or $e = \frac{13}{12}$ Equation of the directrices are $x = \pm\frac{a}{e} = \pm\frac{144}{13}$.	1 Mark: D
3	$\|z - i\| \le 2$ represents a region with a centre is (0, 1) and radius is less than or equal to 2. $0 \le \arg(z-1) \le \frac{3\pi}{4}$ represents a region between angle 0 and $\frac{3\pi}{4}$ whose vertex is (1, 0), not including the vertex $\|z - i\| \le 2$ and $0 \le \arg(z-1) \le \frac{3\pi}{4}$	1 Mark: A
4	$\int \frac{2}{x^2+4x+13}dx = 2\int \frac{dx}{(x+2)^2+3^2}$ $= \frac{2}{3}\tan^{-1}\frac{(x+2)}{3} + c$	1 Mark: B

5	To find the equation of tangent through P $x = a\cos\theta \qquad y = b\sin\theta$ $\dfrac{dx}{d\theta} = -a\sin\theta \qquad \dfrac{dy}{d\theta} = b\cos\theta$ $\dfrac{dy}{dx} = \dfrac{dy}{d\theta}\times\dfrac{d\theta}{dx}$ $= b\cos\theta\times\dfrac{1}{-a\sin\theta} = \dfrac{-b\cos\theta}{a\sin\theta}$ Equation of the tangent $y - y_1 = m(x - x_1)$ $y - b\sin\theta = \dfrac{-b\cos\theta}{a\sin\theta}(x - a\cos\theta)$ $ay\sin\theta - ab\sin^2\theta = -bx\cos\theta + ab\cos^2\theta$ $bx\cos\theta + ay\sin\theta = ab(\sin^2\theta + \cos^2\theta)$ $\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$	1 Mark: D
6	The base is an annulus. $A = \pi(r_2{}^2 - r_1{}^2)$ $= \pi\left(r_2 + r_1\right)\left(r_2 - r_1\right)$ $\left(r-1\right)^2 + \dfrac{y^2}{4} = 1$ $r^2 - 2r + \dfrac{y^2}{4} = 0$ $r = \dfrac{2 \pm \sqrt{(-2)^2 - 4\times1\times\dfrac{y^2}{4}}}{2}$ $r = 1 \pm \sqrt{1 - \dfrac{y^2}{4}}$ Therefore $r_2 + r_1 = 2$ and $r_2 - r_1 = 2\sqrt{1 - \dfrac{y^2}{4}}$ $V = \lim\limits_{\delta y \to 0}\sum\limits_{y=-2}^{2}\pi\times2\times2\sqrt{1 - \dfrac{y^2}{4}}\delta y$ $= \int_{-2}^{2} 2\pi\sqrt{4 - y^2}\,dy$	1 Mark: D

7	$x=\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma}$ $\alpha=\frac{1}{x}$ satisfies $x^3-3x^2-x+2=0$ $\left(\frac{1}{x}\right)^3-3\left(\frac{1}{x}\right)^2-\frac{1}{x}+2=0$ $1-3x-x^2+2x^3=0$ $2x^3-x^2-3x+1=0$	1 Mark: C
8	$y=\sin^{-1}x-\sqrt{1-x^2}$ $\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}-\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\times-2x$ $=\frac{1}{\sqrt{1-x^2}}+\frac{x}{\sqrt{1-x^2}}$ $=\frac{1+x}{\sqrt{1-x^2}}$ $=\frac{1+x}{\sqrt{(1+x)(1-x)}}$ $=\frac{\sqrt{1+x}}{\sqrt{1-x}}$ Result defined for $-1\le x\le 1$	1 Mark: A
9	Using the conjugate root theorem $1+i$ and $1-i$ are both roots of the equation $z^3+pz+q=0$. $(1+i)+(1-i)+\alpha=0$ (sum of the roots) $\alpha=-2$ $(1+i)\times(1-i)\times-2=-q$ (product of the roots) $(1+1)\times-2=-q$ $q=4$ $(1+i)(1-i)+(1-i)-2+(1+i)-2=p$ $p=-2$ Therefore $p=-2$ and $q=4$	1 Mark: B

10	$F=\frac{m}{x^3}(6-10x)$ $ma=\frac{m}{x^3}(6-10x)$ $v\frac{dv}{dx}=\frac{6}{x^3}-\frac{10}{x^2}$ $\int vdv=\int(\frac{6}{x^3}-\frac{10}{x^2})dx$ $\frac{1}{2}v^2=(\frac{6x^{-2}}{-2}-\frac{10x^{-1}}{-1})+c$ $\frac{1}{2}v^2=(\frac{-3}{x^2}+\frac{10}{x})+c$ When $v=0$ and $x=1$ $\frac{1}{2}0^2=(\frac{-3}{1^2}+\frac{10}{1})+c$ $c=-7$ $\frac{1}{2}v^2=(\frac{-3}{x^2}+\frac{10}{x})-7$ $v^2=(\frac{-6}{x^2}+\frac{20}{x})-14$ $=\frac{-6+20x-14x^2}{x^2}$ $v=\pm\frac{1}{x}\sqrt{2(-3+10x-7x^2)}$	1 Mark: C

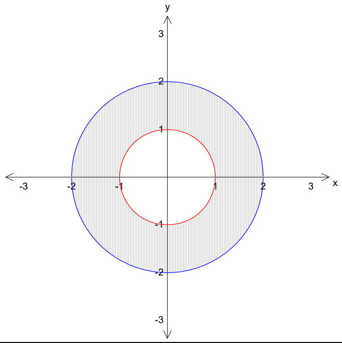
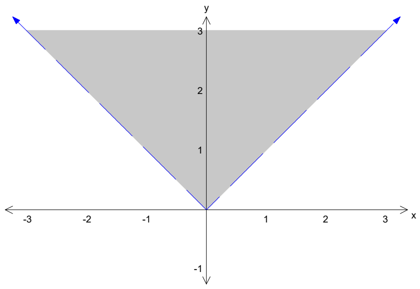
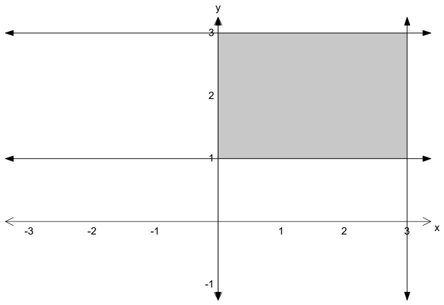
Question 11	Trial HSC Examination- Mathematics Extension 2	2015	
Part	Solution	Marks	Comment
(a)	Let $\dfrac{A}{x+1}+\dfrac{B}{x-1}=\dfrac{1}{x^2-1}$	1	
	$\therefore A(x-1)+B(x+1)=1$		
	If $x=1$ $\quad 2B=1$		
	$B=\dfrac{1}{2}$		Any method to find A and B
	If $x=-1$ $\quad 2A=-1$		
	$\text{A}=-\dfrac{1}{2}$	1	
	$\dfrac{dx}{x^2-1}=\dfrac{1}{2}\int\dfrac{1}{x-1}-\dfrac{1}{x+1}dx$	1	
	$\therefore\int \quad =\dfrac{1}{2}\left[\ln(x-1)-\ln(x+1)\right]+C$		No mark for C
	$=\dfrac{1}{2}\ln\left(\dfrac{x-1}{x+1}\right)+C$	1	
	$=\ln\sqrt{\dfrac{x-1}{x+1}}+c$		Total = 4

Question 11	Trial HSC Examination- Mathematics Extension 2		2015

Part	Solution	Marks	Comment
(b)	Let $x=\cos\theta$ When $x=1$ $\theta=0$ $dx=-\sin\theta d\theta$ $x=0$ $\theta=\frac{\pi}{2}$	1	Can use alternate methods ie $x=\sin\theta$
	$\therefore\int_0^1\sqrt{1-x^2}dx=\int_{\frac{\pi}{2}}^0\sqrt{1-\cos^2\theta}\left(-\sin\theta d\theta\right)$		
	$=\int_{\frac{\pi}{2}}^0-\sin^2\theta d\theta$	1	
	$=\frac{1}{2}\int_{\frac{\pi}{2}}^0(\cos2\theta-1)d\theta$		
	$=\frac{1}{2}\left[\frac{1}{2}\sin2\theta-\theta\right]_{\frac{\pi}{2}}^0$	1	
	$=\frac{1}{2}\left[(0)-\left(0-\frac{\pi}{2}\right)\right]$		
	$=\frac{\pi}{4}$	1	Total = 4
(c)	$I=\int e^x\sin xdx$		
	$=e^x\sin x-\int e^x\cos xdx$	1	
	$=e^x\sin x-(e^x\cos x-\int e^x[-\sin x]dx)$	1	
	$=e^x\sin x-e^x\cos x-I$		
	$2I=e^x(\sin x-\cos x)+C$	1	
	$\therefore I=\frac{e^x}{2}(\sin x-\cos x)+C$	1	Total = 4

Question 11	Trial HSC Examination- Mathematics Extension 2	2015	
Part	**Solution**	**Marks**	**Comment**
(d)	$\int_0^{\frac{\pi}{2}} \cos x \sin^3 x \, dx = \left[\frac{1}{4}\sin^4 x\right]_0^{\frac{\pi}{2}}$	1	May be done by a substitution of $u = \sin x$
	$= \frac{1}{4}\left[\sin^4\left(\frac{\pi}{2}\right) - \sin^4(0)\right]$	1	$du = \cos x \, dx$
	$= \frac{1}{4}[1-0]$		
	$= \frac{1}{4}$	1	Total = 3

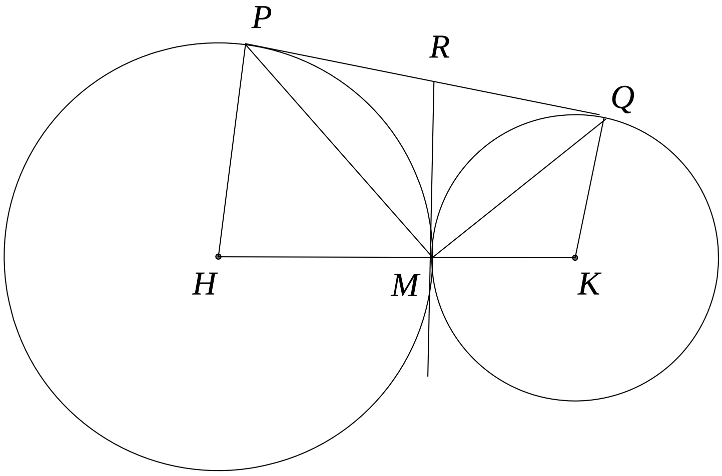
<table>
<tr><td>Question 12</td><td>Trial HSC Examination- Mathematics Extension 2</td><td colspan="2">2015</td></tr>
<tr><td>Part</td><td>Solution</td><td>Marks</td><td>Comment</td></tr>
<tr><td>(a) (i)</td><td>$AB=(3-4i)\left(\sqrt{3}+i\right)$
$=3\sqrt{3}+4+\left(3-4\sqrt{3}\right)i$</td><td>1</td><td></td></tr>
<tr><td>(ii)</td><td>$\frac{A}{B}=\frac{(3-4i)}{\left(\sqrt{3}+i\right)}\times\frac{\left(\sqrt{3}-i\right)}{\left(\sqrt{3}-i\right)}$
$=\frac{3\sqrt{3}-4}{4}+\frac{\left(-4\sqrt{3}-3\right)i}{4}$</td><td>1</td><td></td></tr>
<tr><td>(iii)</td><td>$x+iy=\sqrt{3-4i}\,(x+y\,real)$
$\therefore x^2-y^2+2xyi=3-4i$
$\therefore x^2-y^2=3..........(\alpha)$
$2xy=-4............(\beta)$
Squaring $x^4-2x^2y^2+y^4=9, 4x^2y^2=16$
$x^4+2x^2y^2+y^4=25$
$(x^2+y^2)^2=25$
$x^2+y^2=5...........(\gamma)$
Adding $(\alpha+\gamma)2x^2=8$
$x=\pm2$
$\therefore y=\mp1$
$\therefore\sqrt{A}=\pm(2-i)$</td><td>1
1
1</td><td>Other methods okay
Total = 3</td></tr>
<tr><td>(iv)</td><td>$B=\sqrt{3}+i$
$=2(\frac{\sqrt{3}}{2}+\frac{1}{2}i)$
$|B|=2 \qquad ArgB=\frac{\pi}{6}$
$=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$</td><td>1 for mod
1 for arg</td><td>Total = 2</td></tr>
</table>

Question 12	Trial HSC Examination- Mathematics Extension 2	2015	
Part	Solution	Marks	Comment
(v)	$B^4 = 2^4(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$ $= 16(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$ $= -8 + 8\sqrt{3}i$	1	
(b) (i)		1	
(ii)		1	Dotted lines
(iii)		2	
(c) (i)	$OA = 2(-y + ix)$	1	
(ii)	$OB = OA + AB$ $= -2y + 2ix + x + iy$ $= (x - 2y) + (2x + y)i$	1	
(iii)	$BC = -OA$ $= 2y - 2xi$	1	

Question 13	Trial HSC Examination- Mathematics Extension 2		2015
Part	**Solution**	**Marks**	**Comment**
(a)	$x^2 + 3xy - y^2 = 13$ $2x + 3y + 3x.\frac{dy}{dx} - 2y.\frac{dy}{dx} = 0$ $\frac{dy}{dx}(3x - 2y) = -(2x + 3y)$ $\frac{dy}{dx} = \frac{-(2x+3y)}{(3x-2y)}$ $\therefore$ *at* $(2,3)$ $\quad \frac{dy}{dx} = -\frac{(4+9)}{6-6}$ $\therefore$ Gradient infinite $\therefore$ Tangent is vertical	2 2	Total = 4

(b) (i)	$a - b - 2y \text{ and } \theta - 60^\circ$ $Area = \frac{1}{2} a\, b \sin C$ $Area = \frac{1}{2} \times 2y \times 2y \times \sin 60^\circ$ $Area = \frac{1}{2} \times 4\, y^2 \times \frac{\sqrt{3}}{2}$ $Area = \sqrt{3}\; y^2$ $Area = \sqrt{3}\; (16 - x^2)$	1 1 1	Total = 3

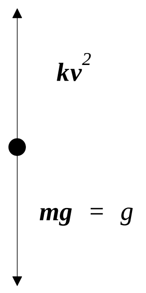
(ii)	$V = \lim_{\delta x \to 0} \sum_{x=-4}^{4} \sqrt{3}(16 - x^2)\delta x$ $V = \int_{-4}^{4} \sqrt{3}(16 - x^2)\,dx$ $V = 2\sqrt{3}\int_{0}^{4} (16 - x^2)\,dx$ $V = 2\sqrt{3}\left[16x - \frac{x^3}{3}\right]_0^4$ $V = \frac{128\sqrt{3}}{3}\ units^3$	1 1 1	Total = 3

(c) (i)	 In $HPRM$ $\angle HPR = \angle RMH = 90^{O}$ (Angle between radius and tangent) $\therefore HPRM$ is a cyclic quadrilateral (Opposite angles supplementary) Similarly for $MRQK$	1 1	Total = 2
(ii)	Join PM and QM. in ΔPRM and MQK $\angle PRM = \angle MKQ$(Exterior angle of cyclic quad $MRQK$) $PR = RM$ (Tangent from external point) $KM = KQ$(radii) $\therefore$ Triangle isoseles. $\therefore \angle RPM = \angle RMP = \angle KMQ$ $\therefore \Delta PRM /// \Delta MQK$(Equiangular)	1 1	Total = 2

Question 14	Trial HSC Examination- Mathematics Extension 2		2015
Part	**Solution**	**Marks**	**Comment**
(a)	$P(x) = x^3 + 3x^2 - 24x + k$ $P'(x) = 3x^2 + 6x - 24$ $P'(x) = 3(x^2 + 2x - 8)$ $P'(x) = 3(x+4)(x-2)$ Therefore $P(-4) = 0 \ and \ P(2) = 0$ So $k = -80, 28$	1 1	Total = 2
(b) (i)	From $x^3 + 3px + q = 0$ $\alpha + \beta + \gamma = 0, \alpha\beta + \alpha + \beta\gamma + = 3p \ \ \alpha\beta\gamma = -q$ $\therefore \frac{\beta\gamma}{\alpha} + \frac{\alpha\gamma}{\beta} + \frac{\alpha\beta}{\gamma} = \frac{(\beta\gamma)^2 + (\alpha\gamma)^2 + (\alpha\beta)^2}{\alpha\beta\gamma}$ $= \frac{(\beta\gamma + \alpha\gamma + \alpha\beta)^2 - 2(\alpha\beta\gamma^2 + \alpha\beta^2\gamma + \alpha^2\beta\gamma)}{\alpha\beta\gamma}$ $= \frac{(\beta\gamma + \alpha\gamma + \alpha\beta)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)}{\alpha\beta\gamma}$ $= \frac{(3p)^2 + 2q(0)}{-q} = -\frac{9p^2}{q}$ $\frac{\beta\gamma}{\alpha}.\frac{\alpha\gamma}{\beta} + \frac{\alpha\gamma}{\beta}.\frac{\alpha\beta}{\gamma} + \frac{\beta\gamma}{\alpha}.\frac{\alpha\beta}{\gamma} = \gamma^2 + \alpha^2 + \beta^2$ $= (\gamma + \alpha + \beta)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$ $= 0 - 2.3p$ $= -6p$ $\frac{\beta\gamma}{\alpha}.\frac{\alpha\gamma}{\beta}.\frac{\alpha\beta}{\gamma} = \alpha\beta\gamma$ $= -q$ $\therefore \text{Required equation is } x^3 + \frac{9p^2}{q}x^2 - 6px + q = 0$	1 1 1 1	Total = 4

Question 14	Trial HSC Examination- Mathematics Extension 2		2015
Part	**Solution**	**Marks**	**Comment**
(b) (ii)	For $\gamma = \alpha\beta$ $\frac{\alpha\beta}{\gamma} = 1$ is a root $\therefore 1 + \frac{9p^2}{q} - 6p + q = 0$ $q + 9p^2 - 6pq + q^2 = 0$ $\therefore (3p-q)^2 + q = 0$	1 1	Total = 2
(c)	$P(x) = (x-2)(x-3)Q(x) + (ax+B)$ $P(2) = 9 \quad P(3) = 4$ Therefore $2a + b = 9$ $3a + b = 4$ $a = -5, b = 19$	1 1 1	Total = 3
(d)	Let $Z = cis\theta$ $Z^5 = cis5\theta$ $\therefore \cos 5\theta + i\sin 5\theta = 0$ $\theta = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$ i.e. $= 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{-4\pi}{5}, \frac{-2\pi}{5}$ $\therefore$ Roots are $Z_1 = 1, \quad Z_2 = cis\frac{2\pi}{5}, \quad Z_3 = cis\frac{4\pi}{5}$ $Z_4 = cis(-\frac{4\pi}{5}) = \bar{Z}_3 \quad Z_5 = cis(-\frac{2\pi}{5}) = \bar{Z}_2$	1 1	Total = 2

Question 14	Trial HSC Examination- Mathematics Extension 2	2015	
Part	Solution	Marks	Comment
(e) ii)	z^5-1		
	$=(z-z_1)(z-z_2)(z-z_5)(z-z_3)(z-z_4)$		
	$=(z-z_1)(z^2-(z_2+z_2z_5)(z^2-(z_3+z_4)z+z_3z_4)$	1	
	$=(z-1)(z^2-2z\cos\frac{2\pi}{5}+1)(z^2-2z\cos\frac{4\pi}{5}+1)$	1	Total = 2

Question 15	Trial HSC Examination- Mathematics Extension 2	2015	
Part	Solution	Marks	Comment
(a) (i)		1	
(a) (ii)	$\ddot{x} = g - kv^2$	1	
(a) (iii)	$When\ \ddot{x} = 0$ $g = kv^2$ $\therefore v = \sqrt{\frac{g}{k}}$	1	
(a) (iv)	$\ddot{x}\frac{d}{dx}(\frac{1}{2}v^2) = g - kv^2$ $= \frac{d}{dv}(\frac{1}{2}v^2).\frac{dv}{dx} = g - kv^2$ $= v.\frac{dv}{dx} = g - kv^2$ $\therefore \frac{dv}{dx} = \frac{g - kv^2}{v}$ $\frac{dx}{dv} = \frac{v}{g - kv^2}$ $\therefore x = -\frac{1}{2k}\ln(g - kv^2) + c$ $When\ x = 0\ v = 0$ $\therefore \quad c = \frac{1}{2k}\ln g$ $\therefore \quad x = -\frac{1}{2k}\ln(g - kv^2) + \frac{1}{2k}\ln g$ $= \frac{1}{2k}\ln(\frac{g}{g - kv^2})$	1 1 1 1	Total = 4

Question 15	Trial HSC Examination- Mathematics Extension 2		2015
Part	**Solution**	**Marks**	**Comment**
(b) (i)	$I_n = \int \sec^n x dx$ $= \int \sec^{n-2} x.\sec^2 x dx$ $= \tan x.\sec^{n-2} x - \int (n-2)\sec^{n-3} x.\tan x.\sec x \tan x dx$ $= \tan x \sec^{n-2} x - (n-2)\int \sec^{n-2} x.\tan^2 x dx$ $= \tan x \sec^{n-2} x - (n-2)\int \sec^{n-2} x.(\sec^2 x - 1)dx$ $= \tan x \sec^{n-2} x - (n-2)\int (\sec^n x - \sec^{n-2} x)dx$ $= \tan x \sec^{n-2} x - (n-2)I_n + (n-2)I_{n-2}$ $I_n + (n-2)I_n = (n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$ $I_n = \frac{1}{n-1}\tan x \sec^{n-2} x + \frac{n-2}{n-1}I_{n-2}$	1 (third line) 1 (fourth line) 1 (seventh line) 1 (final line)	Total = 4
(b) (ii)	$\int_0^{\frac{\pi}{4}} \sec^4 x dx = I_4 = \left[\frac{1}{3}\tan x \sec^2 x + \frac{2}{3}\int \sec^2 x\right]_0^{\frac{\pi}{4}}$ $= \left[\frac{1}{3}\tan x \sec^2 x + \frac{2}{3}\tan x\right]_0^{\frac{\pi}{4}}$ $= \left(\frac{1}{3}\tan\frac{\pi}{4}\sec^2\frac{\pi}{4} + \frac{2}{3}\tan\frac{\pi}{4}\right) - \left(\frac{1}{3}\tan 0 \sec^2 0 + \frac{2}{3}\tan 0\right)$ $= \left(\frac{1}{3}\times 1\times 2 + \frac{2}{3}\times 1\right) - 0$ $= \frac{4}{3}$	1 (third line) 1 (final line)	Total = 2

Question 15	Trial HSC Examination- Mathematics Extension 2		2015
Part	Solution	Marks	Comment
(c)	If one root is $2+i$, another is $2-i$ $\therefore (z-(2+i))(z-(2-i))(z-3)=0$ $(z-2-i)(z-2+i)(z-3)=0$ $(z^2-4z+5)(z-3)=0$ $z^3-7z^2+17z-15=0$	1 1	Total = 2

Question 16	Trial HSC Examination- Mathematics Extension 2		2015
Part	**Solution**	**Marks**	**Comment**
(a) (i)	Coordinates of T are $(a\cos\theta, a\sin\theta)$ $x^2+y^2=a^2$ $\therefore 2x+2y\frac{dy}{dx}=0$ $\therefore \frac{dy}{dx}=-\frac{x}{y}$ at T $\frac{dy}{dx}=-\frac{\cos\theta}{\sin\theta}$ $\therefore$ Equation TM $y-a\sin\theta=-\frac{\cos\theta}{\sin\theta}(x-a\cos\theta)$ $x\cos\theta+y\sin\theta=a$ *When* $y=0$ $x=a\sec\theta$ $\therefore$ Coordinates M are $(a\sec\theta,0)$	1 1 1	Total = 3
(a) (ii)	On $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ when $x=a\sec\theta$ $a^2\frac{\sec^2\theta}{a^2}-\frac{y^2}{b^2}=1$ $\frac{y^2}{b^2}=\sec^2\theta-1$ $\frac{y^2}{b^2}=\tan^2\theta$ $\therefore y=b\tan\theta$ Coordinates of P are $(a\sec\theta, b\tan\theta)$	1	

(a) iii)	$a\sec\beta = a\sec(\frac{\pi}{2}-\theta)$ $= a\cos ec$ $b\tan\beta = b\tan(\frac{\pi}{2}-\theta)$ $= b\cot\theta$ Gradient $PQ = \dfrac{b\cot\theta - b\tan\theta}{a\,\mathrm{cosec}\,\theta - a\sec\theta}$ $= \dfrac{b}{a}\left\{\dfrac{\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{1}{\sin\theta}-\frac{1}{\cos\theta}}\right\}$ $= \dfrac{b}{a}\left\{\dfrac{\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}}{\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}}\right\}$ $= \dfrac{b}{a}\left(\dfrac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}\right)$ $= \dfrac{b}{a}(\cos\theta+\sin\theta)$ $\therefore$ Equation PQ is $y - b\tan\theta = \dfrac{b}{a}(\cos\theta+\sin\theta)(x - a\sec\theta)$ $y - b\tan\theta = \dfrac{b}{a}(\cos\theta+\sin\theta)x - b\cos\theta\sec\theta - b\sin\theta\sec\theta$ $y - b\tan\theta = \dfrac{b}{a}(\cos\theta+\sin\theta)x - b - b\tan\theta$ $y = \dfrac{b}{a}(\cos\theta+\sin\theta)x - b$ $ay = b(\cos\theta+\sin\theta)x - ab$	1 1 1	Total = 3
(a) (iv)	All of the lines have the same y intercept. i.e. $y = -b$ $\therefore$ The fixed point is the intercept $(0,-b)$	1 1	Total = 2

(a) (v)	Equations of Asymptotes are $y=\pm\frac{b}{a}x$ Gradients of asymptotes are $m=\pm\frac{b}{a}$ As $\theta\to\frac{\pi}{2}$ the equation of $PQ\ \to ay=b\left(0+1\right)x-ab$ $\therefore$ Gradient of $PQ\ \to\frac{b}{a}$ $\therefore PQ$ approaches a line which is parallel to an asymptote.	1 1	Total = 2
(b) i)	$z=\cos\theta+i\sin\theta$ $\frac{1}{z}=z^{-1}=\cos\theta-i\sin\theta$ By De Moivres Theorem $z^{n}=\cos n\theta+i\sin n\theta$ $z^{-n}=\cos n\theta-i\sin n\theta$ $z^{n}+z^{-n}=\cos n\theta+i\sin n\theta+\cos n\theta-i\sin n\theta$ $z^{n}+\frac{1}{z^{n}}=2\cos n\theta$	1	
ii)	$\left(z+\frac{1}{z}\right)^{4}=z^{4}+4z^{3}\frac{1}{z}+6z^{2}\frac{1}{z^{2}}+4z\frac{1}{z^{3}}+\frac{1}{z^{4}}$ $=\left(z^{4}+\frac{1}{z^{4}}\right)+4\left(z^{2}+\frac{1}{z^{2}}\right)+6$ $\left(2\cos\theta\right)^{4}=2\cos4\theta+4(2\cos2\theta)+6$ $2^{4}\cos^{4}\theta=2\cos4\theta+8\cos2\theta+6$ $\cos^{4}\theta=\frac{1}{8}\cos4\theta+\frac{1}{2}\cos2\theta+\frac{3}{8}$	1 1 1	Total = 3