Hunters Hill High School

Extension 1, Mathematics

Trial Examination, 2015

Hunters Hill High School

General Instructions

- Reading Time - 5 minutes.
- Working Time - 2 hours.
- Write using a blue or black pen.
- Board Approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (70)

- **Section I**

  10 Marks

  Attempt Questions 1-10
- **Section II**

  60 Marks

  Attempt Questions 11-14

Section I **(10 Marks)** Circle the correct response on the answer sheet provided

1 Which of the following is the correct expression for $\int \frac{dx}{\sqrt{36-x^2}}$**?**

(A) $\cos^{-1}\frac{x}{6}+c$

(B) $\cos^{-1}6x+c$

(C) $\sin^{-1}\frac{x}{6}+c$

(D) $\sin^{-1}6x+c$

2 What is the domain and range of $y=\cos^{-1}(\frac{3x}{2})$?

(A) Domain: $-\frac{2}{3}\le x\le\frac{2}{3}$. Range: $0\le y\le\pi$

(B) Domain: $-1\le x\le 1$. Range: $0\le y\le\pi$

(C) Domain: $-\frac{2}{3}\le x\le\frac{2}{3}$. Range: $-\pi\le y\le\pi$

(D) Domain: $-1\le x\le 1$. Range: $-\pi\le y\le\pi$

3 What is the solution to the inequality $\frac{3}{x-2}\le 4$?

(A) $x<-2$ and $x\ge-\frac{11}{4}$

(B) $x>-2$ and $x\le-\frac{11}{4}$

(C) $x<2$ and $x\ge\frac{11}{4}$

(D) $x>2$ and $x\le\frac{11}{4}$

4 Which of the following is an expression for $\int\frac{e^x}{1+e^{2x}}dx$?

Use the substitution $u=e^x$.

(A) $e^x\tan^{-1}e^x+c$

(B) $e^x\tan^{-1}e^{2x}+c$

(C) $\tan^{-1}e^x+c$

(D) $\tan^{-1}e^{2x}+c$

5 How many solutions would $\sin 2\theta = \sin\theta$ have in the domain $0 \le \theta \le 360^{\circ}$?

(A) 2

(B) 3

(C) 4

(D) 5

6 When $g(x)$ is divided by $x^2 + x - 6$ the remainder is $7x + 13$.
What is the remainder when $g(x)$ is divided by $x + 3$?

(A) - 8

(B) -5

(C) 34

(D) 55

7 What is the acute angle between the lines $y - \sqrt{3}x - 6 = 0$ and $\sqrt{3}y - x + 2 = 0$?

(A) 30°

(B) 45°

(C) 60°

(D) 90°

8 Let α, β and γ be the roots of $x^3 - 4x + 1 = 0$.
What is the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$?

(A) -4

(B) -1

(C) 1

(D) 4

9 A bowl of soup at temperature Tº, when placed in a cooler environment, loses heat according to the law $\frac{dT}{dt} = k(T - T_0)$ where t is the time elapsed in minutes and T_0 is the temperature of the environment in degrees Celsius. A bowl of soup at 96ºC is left to stand in a room at a temperature of 18ºC. After 3 minutes the soup cools down to 75ºC. What is the value of k correct to 4 decimal places?

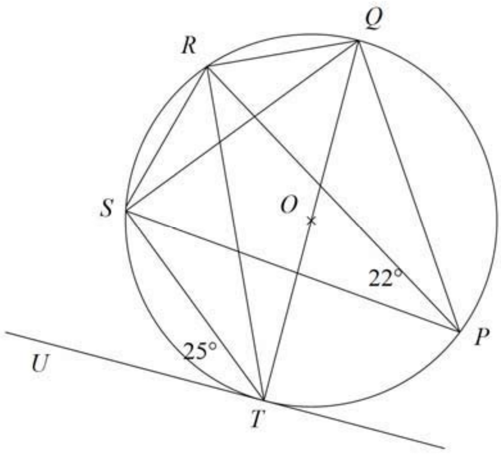
(A) 0.0784

(B) 0.0856

(C) 0.1046

(D) 0.1236

10 A circle with centre O has a tangent TU, diameter QT, $\angle STU = 25^\circ$ and $\angle RPS = 22^\circ$



What is the size of $\angle RTQ$?

(A) 22°

(B) 25°

(C) 43°

(D) 47°

Section II

Question 11 (15 Marks) Use a Separate Sheet of paper **Marks**

(a) Using the substitution $u = x^2 - 2$, or otherwise, find $\int \frac{x}{\sqrt{x^2 - 2}}\,dx$. **3**

(b) Find $\int \sin^2 6x\,dx$. **2**

(c) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.
The equation of the tangents at P and Q respectively are
$y = px - ap^2$ and $y = qx - aq^2$.

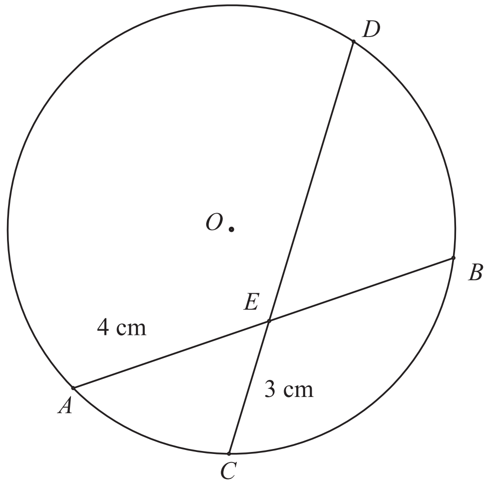
(i) The tangents at P and Q meet at the point R. Show that the coordinates of R are $(a(p+q), apq)$. **2**

(ii) The equation of the chord PQ is $y = \frac{p+q}{2}x - apq$ (Do NOT show this.) **1**
If the chord PQ passes through $(0, a)$, show that $pq = -1$.

(iii) Find the equation of the locus of R if the chord PQ passes through $(0, a)$ **2**

(d) There are eight parking spaces at the front of a motel which are all vacant at 2 pm. Two utilities and six cars arrive in the next hour and park randomly in the eight spaces. What is the probability that the two utilities park side by side? **2**

(e) In the circle centred at O, the chords AB and CD intersect at E. The length of AB is x cm and of CD is y cm. AE = 4 cm and CE = 3 cm. **3**



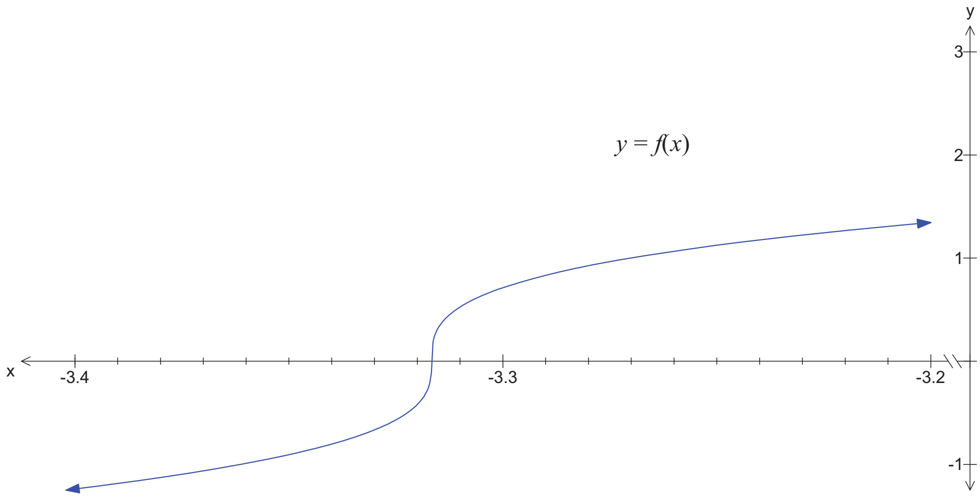
Show that $4x = 3y + 7$

End of Question 11

Question 12 (15 Marks) Use a Separate Sheet of paper **Marks**

(a) The point $P(3, 2)$ divides the interval MN internally in the ratio 3 : 2. If M is the point (6, -1), find the coordinates of N. **3**

(b) The curve $f(x) = (x^3 - 12x)^{\frac{1}{3}}$ is shown below.



(i) Find $f'(x)$. (No need to simplify your answer.) **1**

(ii) Taking an initial estimate of $x_1 = ?.3$, use one application of Newtons' Method to obtain another approximation to the root of $f(x) = 0$. **2**

(iii) Explain why using $x = ?.3$ does not produce a better approximation to the root than the original estimate. **1**

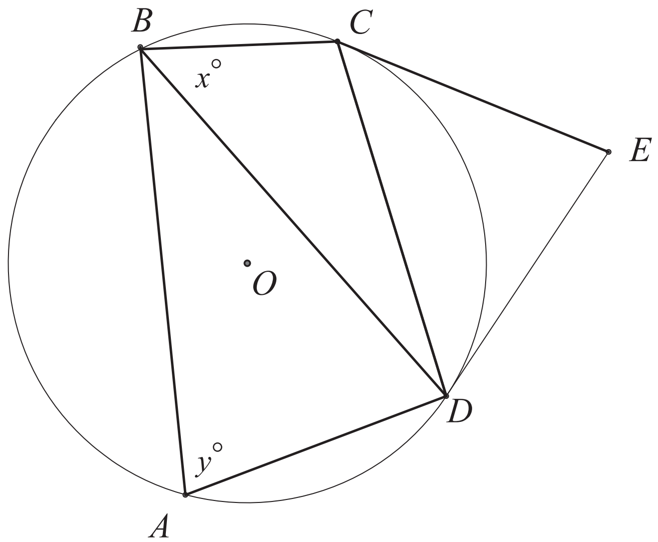
(c) (i) Use the expansion for $\sin(A + B)$ and the exact values for $\cos\frac{\pi}{4}$ and $\sin\frac{\pi}{4}$ to show that $\sin\left(x + \frac{\pi}{4}\right) = \frac{\sin x + \cos x}{\sqrt{2}}$. **2**

(ii) Hence, or otherwise, solve $\frac{\sin x + \cos x}{\sqrt{2}} = \frac{\sqrt{3}}{2}$ for $0 \le x \le 2\pi$. **2**

Question 12 continues on the next page

Question 12 continued **Marks**

(d) The circle *ABCD* has centre *O*. Tangents are drawn from an external point *E* to contact the circle at *C* and *D*. $\angle CBD = x^\circ$ and $\angle BAD = y^\circ$.



(i) Show that $\angle CED = (180 - 2x)^\circ$. **2**

(ii) Show that $\angle BDC = (y - x)^\circ$. **2**

End of Question 12

Question 13 (15 Marks) Use a Separate Sheet of paper

Marks

(a) The velocity of a particle moving along the x axis, in simple harmonic motion is given by:

$$v^2 = 24 + 2x - x^2.$$

(i) What are the endpoints of the motion? **2**

(ii) Write an equation for the acceleration of the particle in terms of x. **2**

(iii) Find the period of the motion. **1**

(b) A particle is moving in a straight line such that its acceleration is given by $\ddot{x} = -x$.

(i) Given that, when $x = 0$, $\dot{x} = 1$, show that $|\dot{x}| = \sqrt{1 - x^2}$. **3**

(ii) Given that, when $x = 0$, $t = 0$, find an expression for x in terms of t. **2**

(c) (i) Using the expansion of $(1 + x)^{n-1}$ show that:

$$\binom{n-1}{1} + \binom{n-1}{2} + \dots \binom{n-1}{n-2} = 2^{n-1} - 2.$$

3

(ii) Find the least positive integer n, such that:

$$\binom{n-1}{1} + \binom{n-1}{2} + \dots \binom{n-1}{n-2} > 1\,000$$ **2**

End of Question 13

Question 14 (15 Marks) Use a Separate Sheet of paper

Marks

(a) Prove, using the method of mathematical induction, that $4^n + 14$ is divisible by 6 for all $n \geq 1$. **3**

(b) A spherical weather balloon is being inflated from empty using a container of helium, so that its' volume is increasing at a constant rate of 0.5 m³/s. (Assume the balloon maintains a spherical shape throughout inflation.)

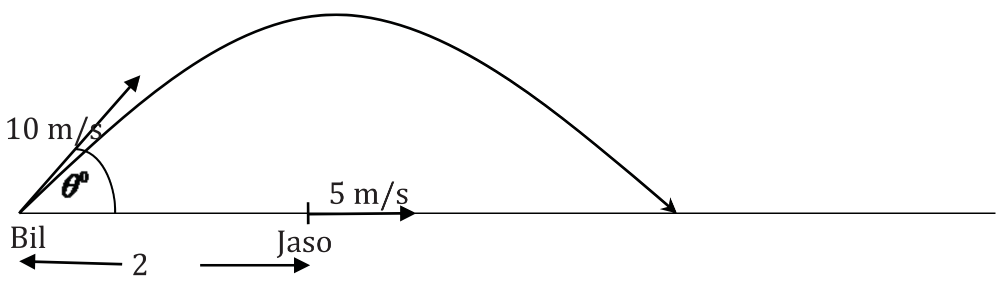
(i) Show that the radius at a time t is given by $r = \sqrt[3]{\dfrac{3t}{8\pi}}$, **2**

(ii) Show that the rate of increase of its surface area after 8 seconds is $\sqrt[3]{\dfrac{\pi}{3}}$ m^2/s. **3**

(iii) If the maximum safe surface area before there is a risk that the balloon will burst is 200 m^2, what is the maximum time that the inflation should be allowed to proceed? **1**

(c) Two players on a basketball court, Bill is standing on the end line with the ball ready to throw to Jason, who is moving directly towards the other end of the court, away from Bill. Jason is 2m away and running at 5m/s when Bill throws the ball in the same direction as Jason is travelling. Bill throws the ball at 10m/s and at an angle of θ^0 to the Horizontal. Assume Jason's velocity is constant, the height at which the ball is thrown and caught is identical and $g = -10m/s^2$. The equation describing the trajectory of the ball are:

$\ddot{x} = 0,\ \dot{x} = 10\cos\theta,\ x = 10t\cos\theta,\ \ddot{y} = -10,\ \dot{y} = -10t + 10\sin\theta,\ y = -5t^2 + 10t\sin\theta.$



(i) Show that $20\sin\theta\cos\theta - 10\sin\theta - 2 = 0$ for Jason to catch the ball. **3**

(ii) Using a suitable method find approximate values for θ. **3**

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Hunters Hill High School

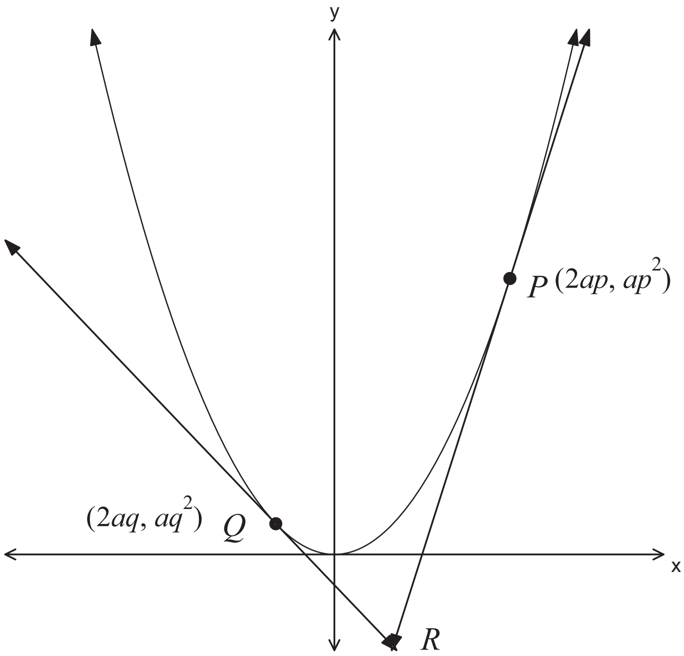
2015
TRIAL HSC
EXAMINATION

Mathematics Extension 1

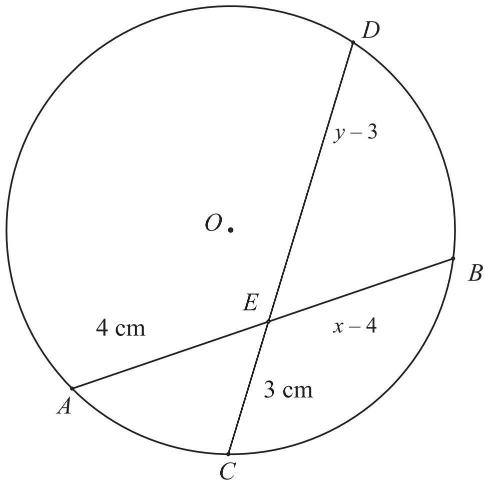
SOLUTIONS

Question 1	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
1		C	
2		A	
3		C	
4		C	
5		D	.
6		A	
7		**A**	
8		**D**	
9		**C**	
10		**C**	

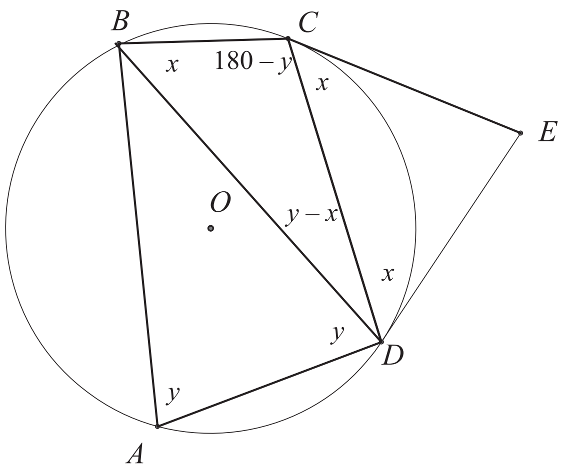
Question 11	Trial HSC Examination - Mathematics Extension 1		2015
Part	**Solution**	**Marks**	**Comment**
a)	$u = x^2 - 2$ $du = 2x\,dx$ $\int \frac{x}{\sqrt{x^2-2}}\,dx = \frac{1}{2}\int \frac{2x}{\sqrt{x^2-2}}\,dx$ $= \frac{1}{2}\int \frac{du}{\sqrt{u}}$ $= \frac{1}{2}\int u^{-\frac{1}{2}}du$ $= u^{\frac{1}{2}} + C$ $= \sqrt{x^2-2} + C$	3	1 For correct method of substitution including du 1 for integral
b)	$\cos 2A = 1 - 2\sin^2 A$ $2\sin^2 A = 1 - \cos 2A$ $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ $\int \sin^2 6x\,dx = \frac{1}{2}\int 1 - \cos 12x\,dx$ $= \frac{1}{2}\left(x - \frac{1}{12}\sin 12x\right) + C$ $= \frac{x}{2} - \frac{\sin 12x}{24} + C$	2	1 for transformation of the integral or recalling a formula 1 for integration

Question 11	Trial HSC Examination - Mathematics Extension 1		2015
Part	**Solution**	**Marks**	**Comment**
c) (i)	 Equations of tangents: $y = px - ap^2$ and $y = qx - aq^2$ To find R, solve simultaneously $px - ap^2 = qx - aq^2$ $px - qx = ap^2 - aq^2$ $x(p - q) = a(p + q)(p - q)$ $x = a(p + q)$ $y = p \times a(p + q) - ap^2$ $= ap^2 + apq - ap^2$ $= apq$ R is the point $(a(p + q), apq)$	2	NB Graph not needed for marks. 1 for reasonable attempt to solve simultaneously 1 for correct result.
c) (ii)	Substitute $(0, a)$ into $y = \frac{p+q}{2}x - apq$ $a = \frac{p+q}{2} \times 0 - apq$ $-apq = a$ $pq = ?$	1 mark	

Question 11	Trial HSC Examination - Mathematics Extension 1		2015
Part	**Solution**	**Marks**	**Comment**
c) (iii)	If chord passes through (0, a) then $pq = ?$ and $p = -\frac{1}{q}$ R is the point $(a(p+q), apq)$. Which becomes $\left(a\left(-\frac{1}{q}+q\right), a \times (?\ )\right)$ $x = a\left(-\frac{1}{q}+q\right)$ and $y = -a$ $x = -y\left(\frac{q^2-1}{q}\right)$ $qx = y(1-q^2)$ $y = \frac{qx}{1-q^2}$ OR SIMILARLY $y = \frac{px}{1-p^2}$	2	1 for introducing pq = -1 to eliminate p or q 1 for relating x and y and obtaining the equation of the locus.
d)	There are $^8\mathbf{P}_8$ (8!) ways the 8 vehicles can park. If the two utes are together, treat them as one, so there are 7 vehicles. These can park in $^7\mathbf{P}_7$ (7!) ways with $^2\mathbf{P}_2$ (2!) ways of arranging the utes among themselves. So the 7 are arranged in $\frac{7!}{2!}$ ways. Probability $= \frac{7!}{2!} \div 8!$ $= \frac{7!}{8!\ 2!}$ $= \frac{1}{8 \times 2}$ $= \frac{1}{16}$	2	1 for arrangement of vehicles with utes together. 1 for probability

Question 11	Trial HSC Examination - Mathematics Extension 1		2015
Part	**Solution**	**Marks**	**Comment**
e)		3	
	Since $AB = x$, $EB = x - 4$ and since $CD = y$, $ED = y - 3$		1
	$AE.EB = CE.ED$ (Ratio of intercepts on chords)		1
	$4(x-4) = 3(y-3)$ $4x - 16 = 3y - 9$ $4x = 3y + 7$		1
			15

Question 12	Trial HSC Examination - Mathematics Extension 1		2015
Part	**Solution**	**Marks**	**Comment**
a)	*let* $N(x, y)$ $\therefore \frac{3x+2(6)}{5} = 3 \quad and \ \frac{3y+2(-1)}{5} = 2$ $\therefore x = 1 \quad y = 4$	3	No need to simplify further
b) (i)	$f(x) = (x^3 - 12x)^{\frac{1}{3}}$ $f(x) = \frac{1}{3}(x^3 - 12x)^{-\frac{2}{3}}.(3x^2 - 12)$	1	No need to simplify further
b) (ii)	$x_1 = ?.3$ $f(x_1) = ((?.3)^3 - 12(?.3))^{\frac{1}{3}}$ ≈ 1.54 $f'(x_1) = \frac{1}{3}((?.3)^3 - 12(?.3))^{-\frac{2}{3}} \times (3(?.3)^2 - 12)$ ≈ 2.90 $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $= ?.3 - \frac{1.54}{2.90}$ ≈ -3.83 (2 dec places)	2	1 for evaluating function and derivative. 1 for substitution into Newtons Method formula.
b) (iii)	As Newtons Method uses the intercept that the tangent makes, from the graph, the tangent at -3.3 is quite flat compared to the sudden drop in the curve to meet the axis. Hence the tangent would meet the axis much further along than the graph, so the second approximation is not as good as the first.	1	Mark for mention of the tangent meeting the axis or similar
c) (i)	$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\sin\left(x + \frac{\pi}{4}\right) = \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$ $= (\sin x) \times \frac{1}{\sqrt{2}} + (\cos x) \times \frac{1}{\sqrt{2}}$ $= \frac{\sin x + \cos x}{\sqrt{2}}$	2	1 correct definition 1 correct evaluation

c) (ii)	$\frac{\sin x + \cos x}{\sqrt{2}} = \frac{\sqrt{3}}{2}$ $\sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$ $x + \frac{\pi}{4} = \frac{\pi}{3}$ or $\frac{2\pi}{3}$ $\left(\frac{\pi}{4} \le x + \frac{\pi}{4} \le \frac{9\pi}{4}\right)$ $x = \frac{\pi}{12}$ or $\frac{5\pi}{12}$ $(0 \le x \le 2\pi)$	2	1 for initial solution of $\frac{\pi}{3}$ and set. 1 for final solution for x
d) (i)	 $\angle EDC = \angle CBD = x$ (Angle between a tangent and a chord is equal to the angle in the alternate segment) Similarly $\angle ECD = \angle CBD = x$ Hence $\angle ECD = \angle EDC = x$ Or $EC = ED$ (Tangents from an external point are equal) Hence $\angle ECD = \angle EDC = x$ $\angle CED = 180 - \angle ECD - \angle EDC$ (Angle sum of triangle) $\angle CED = (180 - 2x)^\circ$	2	1 for partially completed proof with some of the required points or with single error 2 for completely correct proof Or any other valid proof
d) (ii)	$\angle BCD = 180 - y^\circ$ (Opposite angles of cyclic quadrilateral are supplementary) $\angle BDC = 180 - \angle CBD - \angle BCD$ $= 180 - x - (180 - y)$ $= 180 - x - 180 + y$ $= (y - x)^\circ$	2	1 for cyclic quad or similar partial proof. 2 for full proof.
		/15	

Question 13	**Trial HSC Examination - Mathematics Extension 1**		**2015**
Part	**Solution**	**Marks**	**Comment**
a)(i)	Endpoints where $v = 0$ $v^2 = 24 + 2x - x^2 = 0$ $(6 - x)(4 + x) = 0$ $x = 6$ and $x = ?$	2	.
(ii)	$v^2 = 24 + 2x - x^2 = 0$ Acceleration $= \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $= \frac{d}{dx}\left(\frac{24 + 2x - x^2}{2}\right)$ $= \frac{2 - 2x}{2}$ Acceleration $= 1 - x$	2	
(iii)	$a = -n^2 x$ $\ddot{x} = 1 - x$ $\ddot{x} = ?\ ^2(x - 1)$ $n = 1$ Period $T = \frac{2\pi}{n}$ Period $T = 2\pi$ seconds	1	

b) (i)	$\ddot{x} = -x$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -x \quad (v = \dot{x})$ $\frac{1}{2}v^2 = -\frac{x^2}{2} + C$ At $x = 0, \quad \dot{x} = v = 1$ $\frac{1}{2}(1)^2 = 0 + C$ $C = \frac{1}{2}$ $\frac{1}{2}v^2 = -\frac{x^2}{2} + \frac{1}{2}$ $v^2 = 1 - x^2$ $\lvert\dot{x}\rvert = \sqrt{1 - x^2}$	1	
(ii)	Let $x = a\cos(nt + \alpha)$ $a = 1$ and $n = 1$ $x = \cos(t + \alpha)$ $x = 0$, when $t = 0$ $0 = \cos\alpha$ $\alpha = \frac{\pi}{2}$ $x = \cos\left(t + \frac{\pi}{2}\right)$ Let $x = a\sin(nt + \alpha)$ $a = 1$ and $n = 1$ $x = \sin(t + \alpha)$ $x = 0$, when $t = 0$ $0 = \sin\alpha$ $\alpha = 0$ $x = \sin(t)$ OR		

13 (c) (i)	$(1+x)^{n-1} = \binom{n-1}{0} 1^{n-1} + \binom{n-1}{1} 1^{n-2} x + \binom{n-1}{2} 1^{n-3} x^2 + \ldots$ $\ldots + \binom{n-1}{n-2} 1^1 x^{n-2} + \binom{n-1}{n-1} x^{n-1}$ Let $x = 1$ $(1+1)^{n-1} = \binom{n-1}{0} 1^{n-1} + \binom{n-1}{1} 1^{n-2}(1) + \binom{n-1}{2} 1^{n-3}(1)$ $\ldots. + \binom{n-1}{n-2} 1^1 (1)^{n-2} + \binom{n-1}{n-1} (1)^{n-1}$ $\binom{n-1}{1} 1^{n-2}(1) + \binom{n-1}{2} 1^{n-3}(1)^2 + \ldots$ $\ldots. + \binom{n-1}{n-2} 1^1 (1)^{n-2} + 2 = 2^{n-1}$ $\binom{n-1}{1} + \binom{n-1}{2} + \ldots$ $\ldots. + \binom{n-1}{n-2} = 2^{n-1} - 2$	3	
(ii)	$2^{n-1} - 2 > 1000$ $2^{n-1} > 1002$ $(n-1)\ln 2 > \ln 1002$ $n - 1 > \dfrac{\ln 1002}{\ln 2}$ $n - 1 > 9.9$ $n > 10.9$ Least positve integer, $n = 11$	2	

Question 14	Trial HSC Examination - Mathematics Extension 1		2015
Part	**Solution**	**Marks**	**Comment**
a)	Show true for $n=1$ $4^1+14=14+4$ $=18=6\times 3$ $\therefore$ true for $n=1$ Assume true for $n=k$ $4^k+14=6p$ Consider $n=k+1$ $4^{k+1}+14=4\times 4^k+14$ $=4\times 4^k+4\times 14-3\times 14$ $=4(4^k+14)-3\times 14$ $=4\times 6p-6\times 7$ $=6(4p-7)$ $\therefore 4^k+14$ is divisible by 6 Hence if proposition is true for $n=k$ it is true for $n=k+1$ But since true for $n=1$ by induction is true for all $n\geq 1$	3	

Question 14	Trial HSC Examination - Mathematics Extension 1		2015
Part	**Solution**	**Marks**	**Comment**
(b) (i)	$\frac{dV}{dt} = 0.5$ $V = 0.5t + C$ When $t = 0,\ V = 0$ $V = \frac{t}{2}$ $V = \frac{4}{3}\pi r^3$ $\frac{t}{2} = \frac{4}{3}\pi r^3$ $3t = 8\pi r^3$ $r^3 = \frac{3t}{8\pi}$ $r = \sqrt[3]{\frac{3t}{8\pi}}$	2	

Question 14	Trial HSC Examination - Mathematics Extension 1		2015
Part	**Solution**	**Marks**	**Comment**
b) (ii)	$r = \sqrt[3]{\dfrac{3t}{8\pi}}$ $r = \left(\dfrac{3t}{8\pi}\right)^{\frac{1}{3}}$ $r = \left(\dfrac{3}{8\pi}\right)^{\frac{1}{3}} \times t^{\frac{1}{3}}$ $\dfrac{dr}{dt} = \left(\dfrac{3}{8\pi}\right)^{\frac{1}{3}} \times \dfrac{t^{-\frac{2}{3}}}{3}$ $A = 4\pi r^2$ $\dfrac{dA}{dr} = 8\pi r$ $\dfrac{dA}{dt} = \dfrac{dA}{dr} \cdot \dfrac{dr}{dt}$ $= 8\pi\, r \times \left(\dfrac{3}{8\pi}\right)^{\frac{1}{3}} \times \dfrac{t^{-\frac{2}{3}}}{3}$ $= 8\,\pi \left(\left(\dfrac{3t}{8\pi}\right)^{\frac{1}{3}}\right) \times \left(\left(\dfrac{3}{8\pi}\right)^{\frac{1}{3}} \times \dfrac{t^{-\frac{2}{3}}}{3}\right)$ When $t = 8$ $\dfrac{dA}{dt} = 8\,\pi \left(\left(3\dfrac{8}{8\pi}\right)^{\frac{1}{3}}\right) \times \left(\left(\dfrac{3}{8\pi}\right)^{\frac{1}{3}} \times \dfrac{(8)^{-\frac{2}{3}}}{3}\right)$ $= 8\,\pi \times \left(\dfrac{3}{\pi}\right)^{\frac{1}{3}} \times \dfrac{1}{2}\left(\dfrac{3}{\pi}\right)^{\frac{1}{3}} \times \dfrac{1}{12}$ $= \dfrac{\pi}{3}\left(\dfrac{3}{\pi}\right)^{\frac{2}{3}}$ $= \dfrac{\pi}{3}\left(\dfrac{\pi}{3}\right)^{-\frac{2}{3}}$ $= \left(\dfrac{\pi}{3}\right)^{\frac{1}{3}}$ $= \sqrt[3]{\dfrac{\pi}{3}}$	3	

Question 14	Trial HSC Examination - Mathematics Extension 1		2015
Part	**Solution**	**Marks**	**Comment**
b) (iii)	$4\pi r^2 = 200$ $r^2 = \frac{50}{\pi}$ $r = \sqrt{\frac{50}{\pi}}$ $r = \sqrt[3]{\frac{3t}{8\pi}}$ $\sqrt{\frac{50}{\pi}} = \sqrt[3]{\frac{3t}{8\pi}}$ $\left(\sqrt{\frac{50}{\pi}}\right)^6 = \left(\sqrt[3]{\frac{3t}{8\pi}}\right)^6$ $\frac{125000}{\pi^3} = \frac{9t^2}{64\pi^2}$ $t^2 = \frac{125000}{\pi^3} \cdot \frac{64\pi^2}{9}$ $= \frac{8000000}{9\pi}$ $= 282\,942$ $t = 532$ seconds $t = 8$ minutes and 52 seconds	1	1 for using logs or trial and error 1 for answer
c) (i)	ON SEPARATE SHEETS	3	
c) (ii)	ON SEPARATE SHEETS	3	
		/15	