Name:	
Class:	

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2014

MATHEMATICS EXTENSION 1

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 2 hours.
- Write in black or blue pen.
- Board approved calculators & templates may be used
- A Standard Integral Sheet is provided.
- In Question 11 - 16, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks 70

Section I: **10 marks**

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II: **60 Marks**

- Attempt Question 11 - 14
- Answer on blank paper unless otherwise instructed. Start a new page for each new question.
- Allow about 1 hours & 45 minutes for this section.

The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 11, Question 12, etc. Each question must show your Candidate Number.

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice sheet for Questions 1-10

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1. The focus of the parabola $(x-3)^2 = -8y$ is

a) $(3\,,-2)$ b) $(3\,,2)$ c) $(0\,,-2)$ d) $(-2\,,3)$

2. Find the acute angle between the lines $-x+2y+4=0$ and $3x+y+1=0$. Give your answer to the nearest minute.

a) 81°52’ b) 78°41’ c) 54°28’ d) 45°

3. Find the coordinates of the point that divides the interval A (-2, 7) B (12, 0) externally in the ratio 4:3.

a) (-44 , -28) b) (44, -28) c) (54, -21) d) (54, 21)

4. $\sin 2x$ equals to

a) $\frac{1-tan^2x}{1+tan^2x}$ b) $\frac{2\tan x}{1+tan^2x}$ c) $\frac{2\tan x}{1-tan^2x}$ d) $\frac{1+tan^2x}{1-tan^2x}$

5. 8 people are to be seated around a circular table. If 2 people wish to sit next to each other, how many different ways can this be done?

a) 720 b) 10080 c) 1440 d) 5040

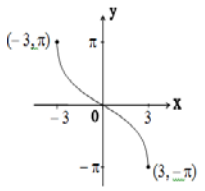
6. Use the trapezoidal rule to find an approximation for $\int_1^3 \log_e x \, dx$ using 2 subintervals.

a) 1.09 b) 1.10 c) 1.24 d) 1.25

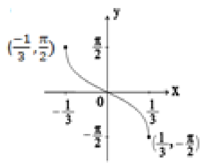
7. Find the $\lim_{x \to 0} \frac{\sin x \cos x}{2x}$

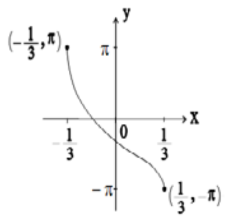
a) 2 b) 1 c) $\frac{1}{2}$ d) $\frac{1}{4}$

8. Which of the following is the graph of $y = -2sin^{-1}3x$?

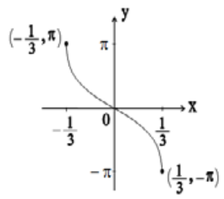


a)


b)



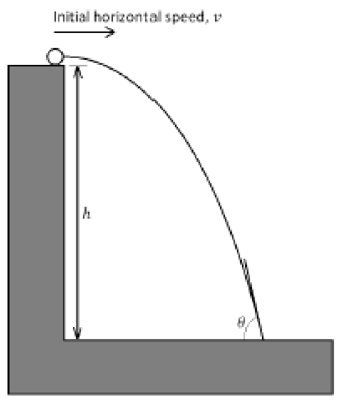
c)



d)

9. The diagram below shows the path of a projectile fired with a horizontal velocity v from a cliff of height h.

Which pair of the following values of v and h will give the greatest value of angle θ ?



a) $v = 10ms^{-1}$ $h = 30m$ b) $v = 30ms^{-1}$ $h = 50m$ c) $v = 50ms^{-1}$ $h = 10m$ d) $v = 10ms^{-1}$ $h = 50m$

10. The solution to

$$\frac{2t+1}{t-2} > 1$$

is

a) $t > -3$ b) $t > 2 \; or \; t < -3$ c) $t > -1$ d) $t > 3 \; or \; t < -2$

Section II

60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE page. Extra writing papers are available.

In questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new page.

a)

Find **3**

$$\int x\sqrt{(x^2+1)^3}\ \ dx$$

using the substitution $u = x^2 + 1$

b)

The area bounded by the curve $y = cos2x$, the x-axis and lines $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$ is rotated about the x-axis. Find the volume of the solid generated in exact form. **3**

c)

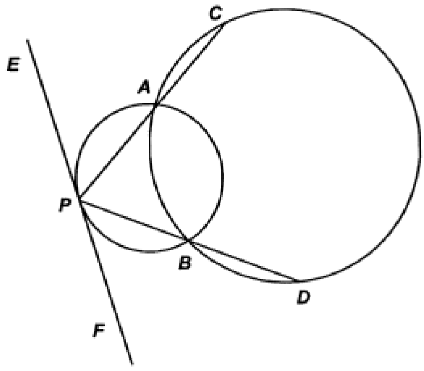
There are 4 families, and each family has exactly 4 children. Assume that the probability of giving birth to a male and giving birth to a female are even. **3**

Determine the probability that exactly 2 of the families will have exactly 2 males and 2 females as children.

Question 11 continues on page 5

d)

Copy the diagram into your answer booklet. *PAC, PBD* are straight lines. *EF* is the tangent at *P*. **3**



Prove CD || EF

e)

Prove by mathematical induction that for any positive integer $n \geq 1$. **3**

$$\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+\ldots\ldots\ldots+\frac{1}{(4n-3)(4n+1)}=\frac{n}{4n+1}$$

End of Question 11

Question 12 (15 marks) Start a new page.

a)

A body is in Simple Harmonic Motion and its position at a time *t* is given by the equation

$$x = R\cos(nt + \alpha) + 1$$

The period of motion is π seconds and $0 \leq \alpha \leq \frac{\pi}{2}$. Initially the body is at rest 3 units to the left of the origin.

i. Find the values of *R, n* and *α.* **3**

ii. Find the velocity of the body when $t = \frac{\pi}{6}$ **1**

b)

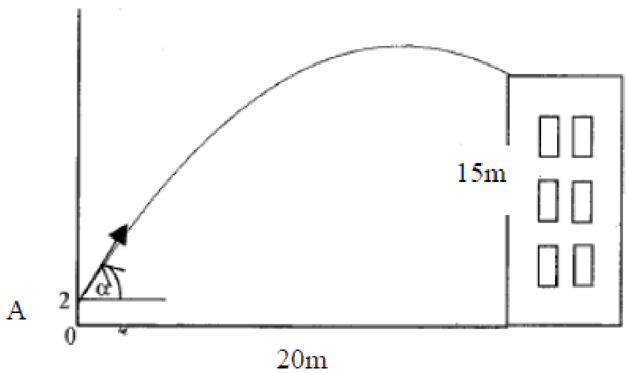
Show that the sum of the Arithmetic sequence **2**

$$\log_{10}(x-2)\,,\ \log_{10}(x-2)^2,\ \log_{10}(x-2)^3, \ldots \log_{10}(x-2)^n$$

is $\frac{n}{2}\log_{10}(x-2)^{n+1}$

c)

Andrew, whose height is 2 metres, throws a ball from area A in the direction of the Cohen building which is 15 metres high. He throws the ball with an initial velocity u at angle α, and he is 20 metres from the base of the building. (Assume $\ddot{x} = 0$ and $\ddot{y} = -10m/s^2$)



i. Show that $y = x\tan\alpha - \frac{5x^2}{u^2}(1 + tan^2\alpha) + 2$, at any time *t.* **2**

ii. Hence, find between which two angles of projection must he throw the ball to ensure that it lands on the roof of the building, or over, given that $u = 25m/s$. (Answer to the nearest degrees). **3**

d)

i. Differentiate $x\tan^{-1}x$ **2**

ii. Hence, or otherwise, find $\int \tan^{-1}x\,dx$ **2**

End of Question 12

Question 13 (15 marks) Start a new page.

a)

At any time *t* minutes, the rate of cooling of a body with temperature *T*, when the surrounding temperature is *S*, is given by the differential equation

$$\frac{dT}{dt} = -k(T - S)$$

for some constant k.

i. Show that $T = S + Ae^{-kt}$, for some constant *A*, satisfies this differential equation. **1**

ii. A metal rod has an initial temperature of 1390°C and cools to 1060°C in 10 minutes when the surrounding temperature is 30°C. **4**

Find how much longer it will take the rod to cool to 110°C, giving your answer to the nearest minute.

b)

Find the monic cubic equation whose roots are the squares of the roots of **3**

$$x^3 + 2x + 1 = 0$$

c)

The acceleration of a particle *P* is given by the equation

$$\ddot{x} = 18x^3 + 27x^2 + 9x$$

Initially $x = -2$ and the velocity, $v = -6$.

i. Show that $v^2 = 9x^2(1 + x)^2$ **2**

ii. Hence, or otherwise, show that **2**

$$\int \frac{1}{x(1+x)} dx = -3t + C\text{, for some constant C}$$

iii. Find the derivative of $\log_e(1 + \frac{1}{x})$ **1**

iv. Using your result in part iii and the initial conditions, find x as a function of t. **2**

End of Question 13

Question 14 (15 marks) Start a new page.

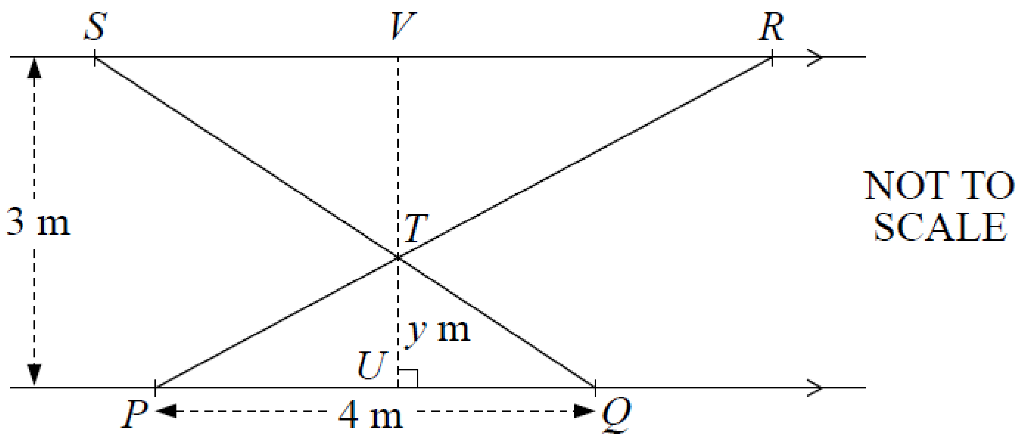
a)

Show that $x = 5sin\theta$ and $y = 5cos\theta + 1$ satisfies the equation **2**

$$y^2 + x^2 - 2y - 24 = 0$$

b)

In the diagram, PQ and SR are parallel railings which are 3 metres apart. The points P and Q are fixed 4 metres apart on the lower railing. Two crossbars PR and QS intersect at T as shown in the diagram. The line through T Perpendicular to PQ intersects PQ at U and SR at V. The length of UT is y metres.



i. Deduce that $SR = \frac{12}{y} - 4$ **3**

ii. Hence show that the total area A of ΔPTQ and ΔRTS is **1**

$$A = 4y - 12 + \frac{18}{y}$$

iii. Find the value of y that minimises A. Justify your answer. **3**

c)

The coefficient of x^k in $(1 + x)^n$, where n is a positive integer, is denoted by c_k (so $c_k = {}^nC_k$)

i. Show that $c_0 + 2c_1 + 3c_2 + \cdots + (n+1)c_n = (n+2)2^{n-1}$ **3**

ii. Find the sum, **3**

$$\frac{c_0}{1 \times 2} - \frac{c_1}{2 \times 3} + \frac{c_2}{3 \times 4} - \cdots + (-1)^n \frac{c_n}{(n+1)(n+2)}$$

Showing all necessary working.

End of paper