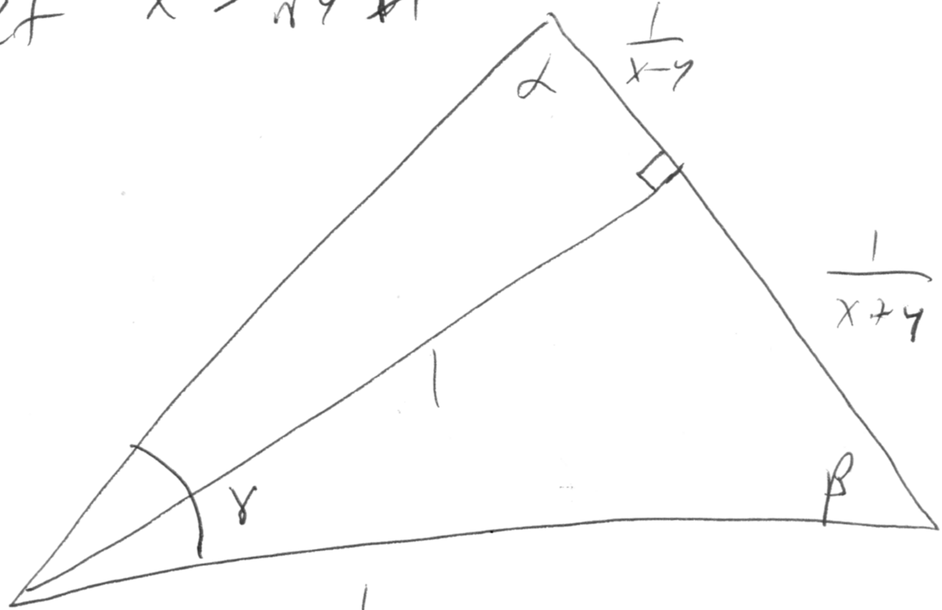
Alternative solution to James Ruse 2011 Q 8 b

Let $x > \sqrt{y^2+1}$



$$\tan\alpha = \frac{1}{1/(x-y)} = x-y$$

$$\tan\beta = \frac{1}{1/(x+y)} = x+y$$

$$\tan\gamma = \tan\left(\tan^{-1}\frac{1/(x-y)}{1} + \tan^{-1}\frac{1/(x+y)}{1}\right)$$

$$= \frac{\frac{1}{x-y} + \frac{1}{x+y}}{1 - \frac{1}{x-y}\cdot\frac{1}{x+y}}$$

$$= \frac{2x}{x^2-y^2-1}$$

$$\therefore \alpha+\beta+\gamma = \tan^{-1}(x-y) + \tan^{-1}(x+y) + \tan^{-1}\frac{2x}{x^2-y^2-1}$$

$$= \pi$$

(angle sum of triangle)

Letting $x=4$, $y=1$

$$\tan^{-1}3 + \tan^{-1}5 + \tan^{-1}\frac{4}{7} = \pi.$$