Total Marks – 120
Attempt Questions 1-8
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\int x \ln 2x \, dx$ **2**

(b) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^5 x} dx$ **3**

(c) By completing the square, find $\int \frac{dx}{\sqrt{11-10x-x^2}}$ **2**

(d) (i) Find A and B such that $\frac{x^2-3x+14}{(x+3)(x-1)^2} = \frac{A}{x+3} + \frac{B}{(x-1)} + \frac{3}{(x-1)^2}$ **2**

(ii) Hence find $\frac{x^2-3x+14}{(x+3)(x-1)^2}$ **2**

(e) Use the substitution $x = 3\sin\theta$ to evaluate $\int_0^3 \frac{x^3}{\sqrt{9-x^2}} dx$ **4**

Question 2 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Let $z = 2 + 3i$ and $w = 4 - i$.
Find in the form $x + iy$,

(i) zw **1**

(ii) $\overline{\left(\frac{z}{w}\right)}$ **2**

(b) Find the real numbers a and b such that $(a + bi)^2 = 16 + 30i$ **3**

(c) Sketch the locus of z satisfying the following:

(i) $\arg(z - 4) = \frac{3\pi}{4}$ **2**

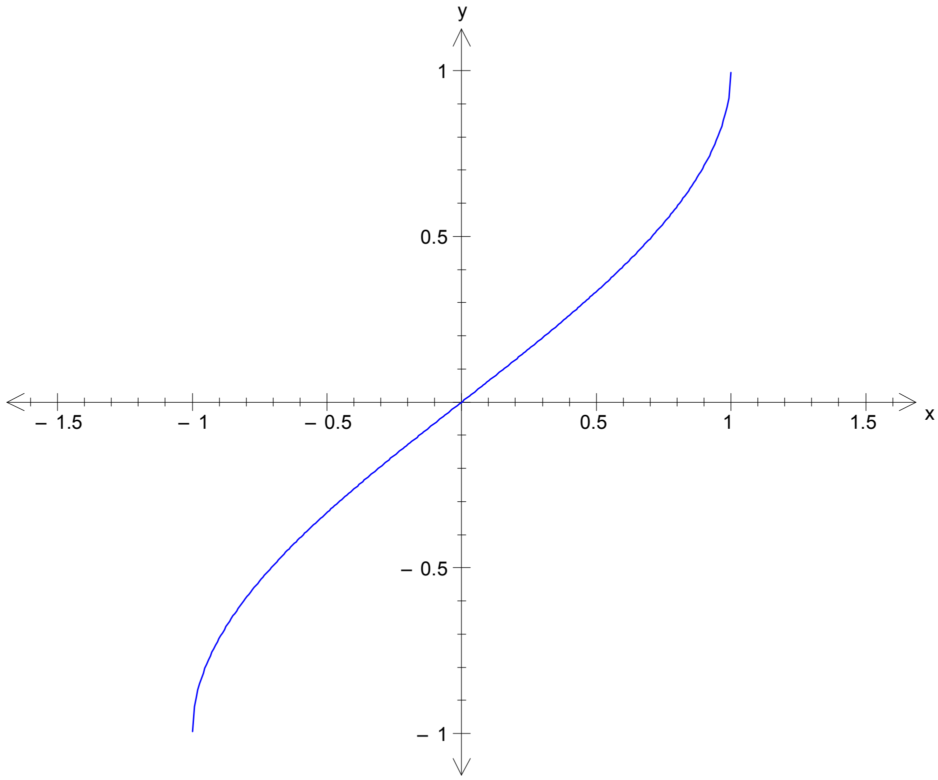
(ii) $\operatorname{Im} z = |z|$ **3**

(d) (i) Express $1 + i$ in modulus-argument form. **2**

(ii) Given that $(1 + i)^n = x + iy$,where x and y are real and n is an integer, show that $x^2 + y^2 = 2^n$ **2**

Question 3 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The diagram shows $y = f(x)$ which is defined in the domain $-1 \le x \le 1$



Draw a neat separate sketch of each of the following graphs.
Use about one third of a page for each graph. Show all significant features.

(i) $y = \left(f(x)\right)^2$ **2**

(ii) $y = \left|f(x)\right|$ **2**

(iii) $y = f(|x|)$ **2**

(iv) Draw $y = f(x)$ and $y = \sqrt{f(x)}$ on the same number plane. **2**

(v) $y = e^{f(x)}$ **2**

(vi) $y = \dfrac{1}{f(x)}$ **2**

(vii) $y = f'(x)$ **3**

Question 4 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) A polynomial is such that $P(x) = x^3 - x^2 + 6x + 4$ has roots α, β and γ.

(i) Find $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$ **2**

(ii) Evaluate $\alpha^2 + \beta^2 + \gamma^2$ **2**

(iii) Use your answer to (ii) to determine the number of real roots of $P(x)$. Justify your answer. **2**

(b) The equation $x^3 - 12x + m = 0$ has a double root. Find the possible values of m. **3**

(c) Let roots α, β and γ be the roots of $x^3 - 2x^2 + 10 = 0$

(i) Find the polynomial equation with integer coefficients whose roots are $\alpha - 2$, $\beta - 2$ and $\gamma - 2$ **2**

(ii) Find the polynomial equation with integer coefficients whose roots are α^2, β^2 and γ^2 **2**

(iii) Evaluate $\alpha^3 + \beta^3 + \gamma^3$ **2**

Question 5 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the equation of the tangent to $x^2 \sin y + 2x = 4$ at the point (2,0) **3**

(b) (i) Show that $\frac{d}{dx}\ln(\sec x) = \tan x$ **1**

(ii) The length of an arc joining two points whose x-coordinates are a and b on the curve $y = f(x)$ is given by **2**

$$\text{arc length} = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx.$$

Consider the curve defined by $y = \ln(\sec x)$.

Find the length of the arc between $x = 0$ and $x = \frac{\pi}{4}$

(c) In a game, two players take turns at drawing, and immediately replacing, a marble from a bag containing two green and three red marbles. The game is won by player A drawing a green marble or player B drawing a green marble. A goes first. Find the probability that:

(i) A wins on her first draw. **1**

(ii) B wins on her first draw. **1**

(iii) A wins in less than four of her turns. **2**

(iv) A wins eventually. **2**

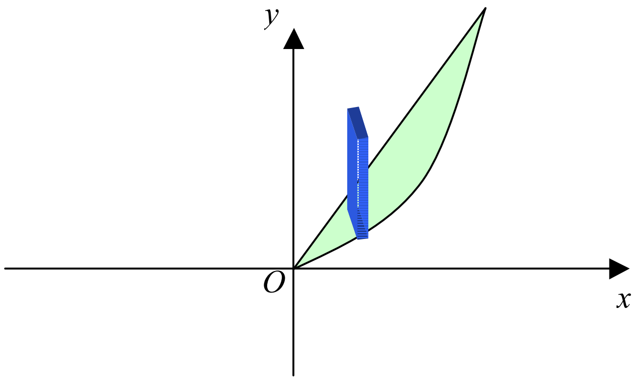
(d) A sequence is defined such that $T_1 = 5, T_2 = 7$ and $T_{n+2} = 3\times T_{n+1} - 2\times T_n$ **3**
Prove by mathematical induction that $T_n = 3 + 2^n$.

Question 6 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Use the identity $2\sin A\cos B = \sin(A+B)+\sin(A-B)$ to show that: **3**

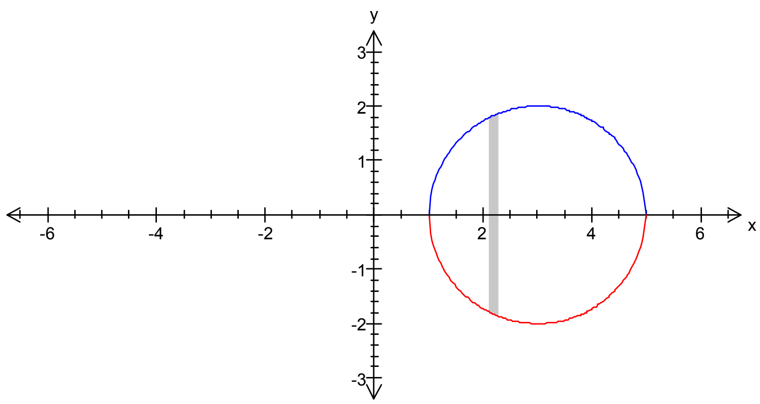
$\int_0^t \sin(\alpha x)\cos(\alpha(t-x))\,dx = \frac{t}{2}\sin(\alpha t)$, where α and t are constants

(b)



The base of a solid is the region contained by $y = x$ and $y = x^2$. Cross-sections, perpendicular to the x–axis are rectangles, with height four times the length of the base. Find the volume of the solid. **4**

(c) The graph below is of the circle $(x-3)^2 + y^2 = 4$.
The circle is to be rotated around the y-axis. Consider a strip is of width δx.



(i) Copy the diagram and draw an appropriate cylindrical shell. **1**

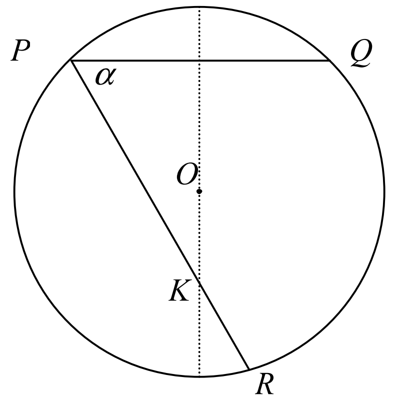
(ii) Use the method of cylindrical shells to show that the volume of the doughnut formed when the region inside the circle is rotated about the y-axis is given by **2**

$$V = 4\pi \int_1^5 x\sqrt{4-(x-3)^2}\,dx$$

(iii) Hence find the volume of the doughnut using the substitution $x-3 = 2\sin\theta$ **5**

Question 7 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the general solution of $\tan 2x = 2\sin x \cos x$ **4**



(b) PQ is a chord of a circle. The diameter of the circle perpendicular to PQ meets another chord PR at K such that $OK = KR$ and $\angle QPR = \alpha$

(i) Prove that $OKRQ$ is a cyclic quadrilateral **3**

(ii) Hence deduce that KQ bisects $\angle OQR$. **3**

(c) (i) Show that $\sin\left(\sin^{-1} x - \cos^{-1} x\right) = 2x^2 - 1$ **2**

(ii) Hence solve $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(1-x)$ **3**

Question 8 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The diagram shows a regular pentagon *ABCDE* with all sides 1 unit in length. The pentagon is inscribed in a rectangle *KLMN*.

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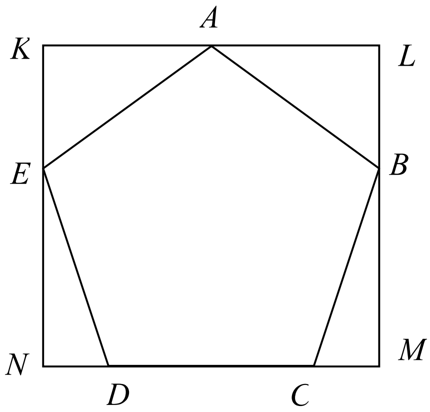

(i) Deduce from the diagram that $\Delta NED \equiv \Delta BMC$ **2**

(ii) Prove that $ND = \cos 72°$ **1**

(iii) Given that opposite sides of a rectangle are equal, show that $2\cos 36° = 1 + 2\cos 72°$ **2**

(iv) Hence show that $\cos 36° = \dfrac{1+\sqrt{5}}{4}$ **3**

(v) Hence calculate the exact value of $\cos 72°$ **2**

(b) (i) Using the binomial theorem write down the expansion of $(1 + i)^{2m}$,where $i = \sqrt{-1}$, and m is a positive integer. **2**

(ii) Hence prove that ${}^{2m}C_0 - {}^{2m}C_2 + {}^{2m}C_4 - {}^{2m}C_6 \ldots (-1)^m\ {}^{2m}C_{2m} = 2^m \cos\dfrac{m\pi}{2}$ **3**