Macquarie Fields High School

TRIAL

Higher School Certificate

EXAMINATION

2012

Mathematics Faculty

Everyone Counts

MATHEMATICS

EXTENSION 1

Time allowed-Two hours
(Reading time – five minutes)

General Instructions

- Reading Time- 5 minutes.
- Working Time – 2 hours.
- Write using a black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown for every question.
- Begin each question in a new writing booklet.
- **Detach the Multiple choice answer sheet from the back of the question booklet to answer question 1-10**

Total marks – 70

Section I
10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II
60 marks

- Attempt Questions 11 - 14
- Allow about 1 hour 45 minutes for this section.

Examiner: Mrs Hegde

NAME ______________________ TEACHER ______________

Section 1 (10 questions, One mark each):

1. The point P divides the interval $A(-4,2)$ to $B(3,-2)$ externally in the ratio 3:2.

Which one of the following is the coordinates of point P?

(A) $(-17,10)$

(B) $(17,-10)$

(C) $(-16,10)$

(D) $(18,-10)$

2. Given that $f(x) = \cos^{-1}\frac{x}{2} + 2\tan^{-1}x$; which of the following represents $f\left(-\sqrt{3}\right)$?

(A) $\frac{-5\pi}{6}$

(B) 0

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

3. $\lim_{x \to 0} \frac{\sin x \cos x}{2x}$

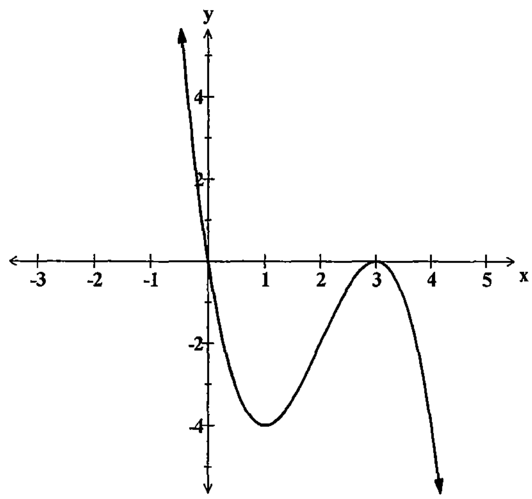
(A) 2

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

4. The graph of $y = P(x)$ is shown below.



Which of the following could be the polynomial $y = P(x)$?

(A) $y = -x(x-3)^2$

(B) $y = x(3-x)^2$

(C) $y = -x(x-3)^3$

(D) $y = -x^2(x+3)^2$

5. The displacement, x metres of a particle moving in simple harmonic motion along the *x-axis* is given by $x = 2 + 4\cos(2t + \pi)$.

Which of the following is the maximum displacement of the particle?

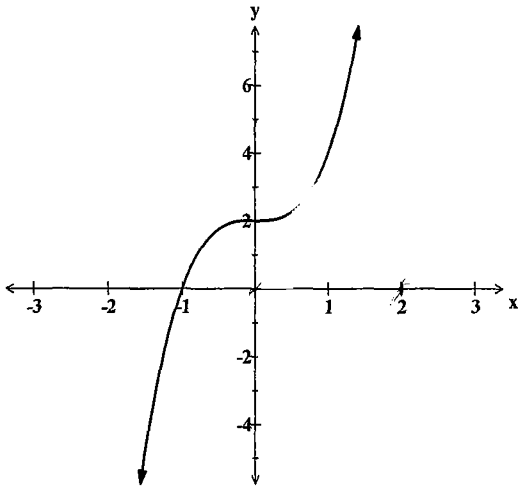
(A) 4 metres

(B) $\frac{\pi}{6}$ metres

(C) $\frac{\pi}{4}$ metres

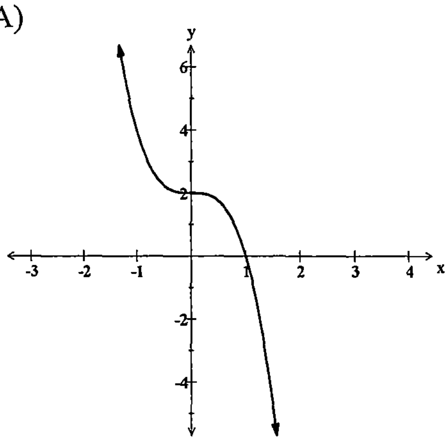
(D) 6 metres

6. The graph of $y = f(x)$ is shown below.

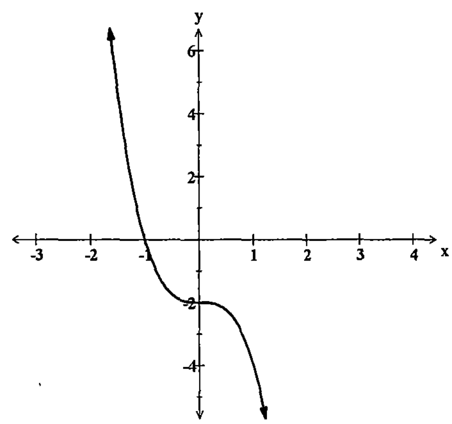


Which of the following is the graph of the inverse function of $y = f(x)$

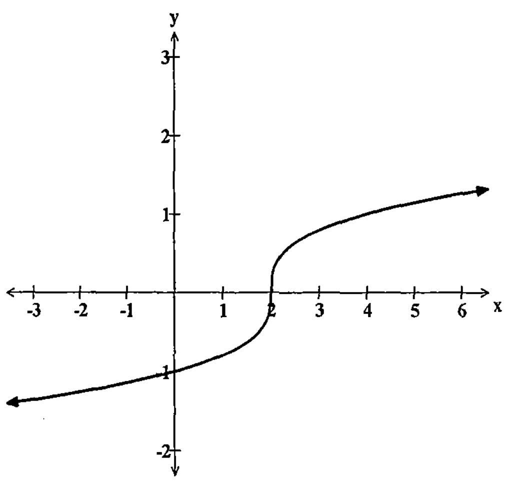
(A)



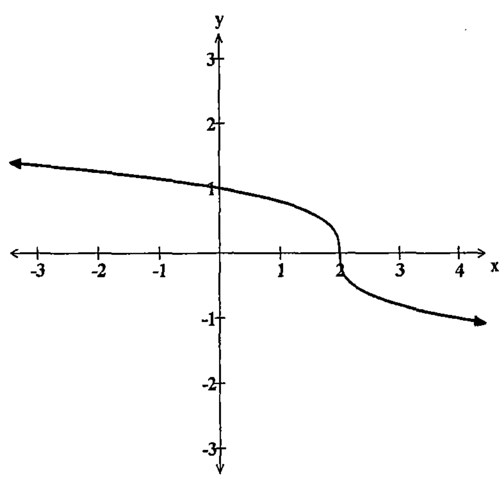
(B)



(C)



(D)



7. The volume of a sphere is increasing at a rate of 240π $\text{cm}^3/\text{minute}$. What is the rate of increase of the radius of the sphere, in cm/minute, when the radius is 2cm?

(A) 1.25

(B) 15

(C) 24

(D) $38\pi^2$

8. $\int\left(\frac{1}{(1-x)^2}-\frac{2}{1-x}\right)dx$

(A) $2\ln(1-x)+\frac{1}{x-1}+c$

(B) $-2\ln(1-x)+\frac{1}{x-1}+c$

(C) $2\ln(1-x)-\frac{1}{x-1}+c$

(D) $-2\ln(1-x)-\frac{1}{x-1}+c$

9. What is the value of $\int\cos^2 3x\,dx$?

(A) $x+\frac{1}{6}\sin 3x+c$

(B) $x+\frac{1}{6}\sin 6x+c$

(C) $\frac{x}{2}+\frac{1}{12}\sin 6x+c$

(D) $\frac{x}{2}+\frac{1}{12}\sin 3x+c$

10. The value of $\binom{50}{1}+2\binom{50}{2}+3\binom{50}{3}+\ldots+50\binom{50}{4}=$

(A) 2^{49}

(B) 2^{50}

(C) $2^{50}-1$

(D) $2^{50}\times 5^{2}$

Section II continued on the next page

Section II: (4 questions, 15 marks each)

Question 11 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) State the Domain of $y=\cos^{-1}(2-x)$. **1**

(b) Solve $2\ln(x+2)=\ln(x+8)$. **2**

(c) The graph of $y=x^2$ and $y=(x-2)^2$ intersect at $x=1$. Find the size of the acute angle (*to the nearest minute*) between these curves at $x=1$. **3**

(d) Solve the following inequality for x: **3**

$$\frac{5x}{x-3}\geq x+4$$

(e) Using the substitution $u=\cos x$, or otherwise, evaluate **3**

$$\int_{\frac{\pi}{2}}^{0}\left(\frac{\sin x}{\sqrt{\sin^2 x+3}}\right)dx$$, leaving your answer in simplified exact form.

(f) $P(2at,at^2)$ is a variable point on the parabola $x^2=4ay$, where the focus is S. Q divides the interval from P to S in the ratio $t^2:1$, where $x=\dfrac{2at}{t^2+1}$ and $y=\dfrac{2at^2}{t^2+1}$

(i) Show that $\dfrac{y}{x}=t$ **1**

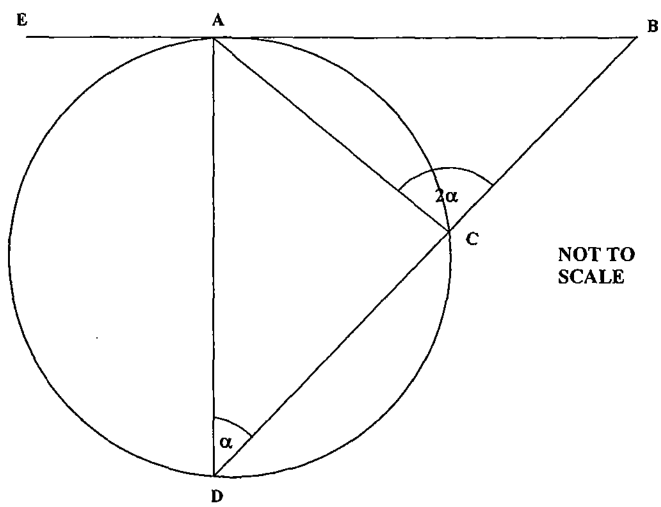
(ii) Prove that as P moves, Q moves in a circle. **2**

Question 12 continued on the next page

Question 12 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the value of the term independent of x in the expansion of $\left(\frac{4}{x^2}-\frac{x}{2}\right)^9$. **3**

(b) In the diagram below, AD lies within the circle touching at both ends. The line EB is a tangent to the circle, touching the circle at A and the line BD cuts the circle at C. $\angle ADC = \alpha$ and $\angle ACB = 2\alpha$.



(i) Prove that AC bisects $\angle BAD$. **2**

(ii) If $\alpha = 45^o$, show that AD is a diameter. **1**

(c) (i) Find the equation of the tangent to the curve $y = x^3 - 4x^2 + 2x + 3$ at $P(1, 2)$. **2**

(ii) The tangent at P meets the curve $y = x^3 - 4x^2 + 2x + 3$ again at Q. Find the coordinates of Q. **2**

(d) $e^{-2x} = 2x - 2$ has a root between $x = 1$ and $x = 2$. Taking $x_1 = 1.5$ as a first approximation to this root, use Newton's method once to find a better approximation correct to 2 decimal places. **2**

(e) Given that $2\cos^2 2x = \sin 4x + 1$.

(i) Show that $\tan 4x = 1$ **2**

(ii) Hence or otherwise solve $2\cos^2 2x = \sin 4x + 1$, for $0 \le x \le \frac{\pi}{2}$ **1**

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) In a herd of 300 horses, the number N infected with a disease at time t **years** is given by $N = \dfrac{300}{P + 299e^{-300t}}$, where P is a constant.

(i) Initially there was one horse infected with the disease. Find P and show that eventually, all the horses will be infected. **2**

(ii) Find the time in **days** when half the herd will be infected. **2**

(iii) Show that $\dfrac{dN}{dt} = N(300 - N)$. **2**

(b) Use mathematical induction to prove that $(n+3)! > 4^{n+1}$ for all integers $n \geq 1$ **3**

(c) Consider the function $f(x) = (e^{2x} - 1)(e^x - 1)$.

(i) Find the coordinate of the stationary point and determine its nature. **3**

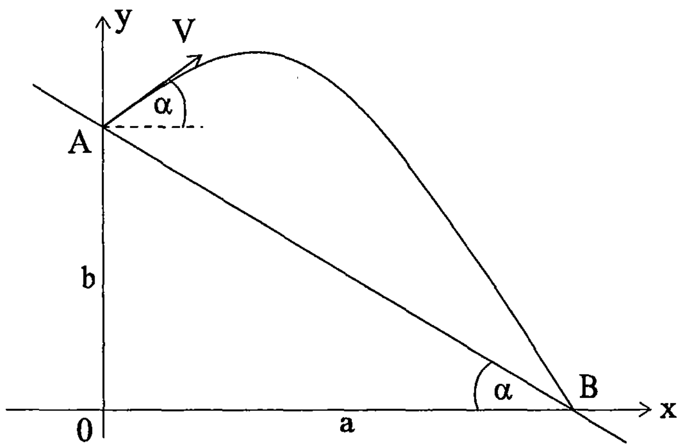
(ii) Describe the behaviour of $f(x)$ as $x \to \infty$ **1**

(iii) Sketch the curve $y = f(x)$ showing all the above information. You are not required to find any point of inflexion. **2**

Question 14 continued on the next page

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

a)



From a point A on a road a golf ball is projected with velocity V at an angle α to the horizontal.
It lands on the road at the point B.

Given that the road AB is inclined at an angle α to the horizontal and using the axes as shown, such that OA = b and OB = a:

i) Show that the cartesian equation of the path in terms of α is **2**

$$y = x\tan\alpha - \frac{gx^2}{2V^2}(1+\tan^2\alpha)+b$$

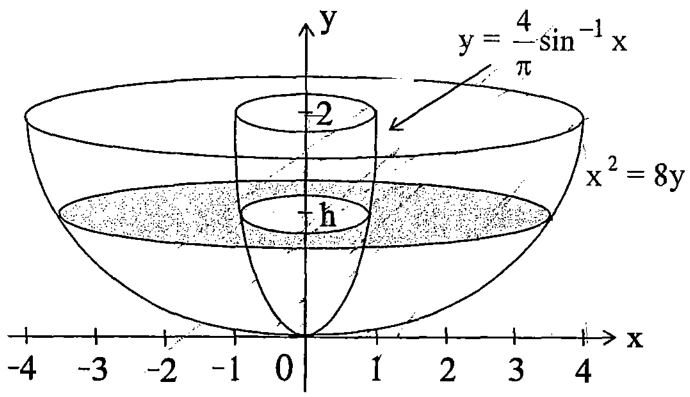
ii) Show that the distance AB travelled by the ball along the road is **3**

$$AB = 2V\sqrt{\frac{b}{g}}$$

Question 14 continued on the next page

b) The area bounded by the curve $y = \frac{4}{\pi}\sin^{-1} x$, the curve $x^2 = 8y$ and the line $y = 2$ is rotated about the y axis to make a tank.

The tank is being filled with water at constant rate of $\frac{9\pi}{2} m^3 s^{-1}$.



i) Show that the volume of the water in cubic metres when the depth is h m can be expressed by **3**

$$V = 4\pi h^2 - \frac{\pi}{2}h + \sin\frac{\pi h}{2}$$

ii) Calculate the rate at which the water is rising when $h = 1$m. **2**

iii) Calculate the rate at which the area of the surface of the water is increasing when $h = 1$m **2**

c) By considering the expansion of $x^2(1+x)^n$, show that

$$\frac{1}{3}\,{}^nC_0 - \frac{1}{4}\,{}^nC_1 + \frac{1}{5}\,{}^nC_2 + \ldots - \frac{(-1)^{n+3}}{n+3}\,{}^nC_n = \frac{2(n!)}{(n+3)!}$$ **3**

END OF EXAMINATION