2022	North Sydney Boys High School

HSC Course Assessment Task 3-Trial Examination

MATHEMATICS EXTENSION 1

General instructions

- Reading time - 10 minutes
- Working time - 2 hours.
- A reference sheet is provided
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- For questions in Section II, show relevant mathematical reasoning and/or calculations in every question. Marks may be deducted for illegible or incomplete working.
- Attempt **all** questions.
- At the conclusion of the examination, bundle any additional sheets used in the correct order within this paper and hand to examination supervisors.

Class (please tick)

◯ 12MM4.A2 – Mr Berry

◯ 12MM4.B2 – Ms Lee

◯ 12MM3.B2 – Mr Lin

◯ 12MM3.A2 – Mr Ireland

◯ 12MM3.C2 – Ms Moss

◯ 12MM4.C2 – Mr Umakanthan

Student Number								

Marker's use only.

QUESTION	1-10	11	12	13	14	**Total**
MARKS	$\overline{10}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{70}$

Section I 10 Marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

---

1. A cubic function has only one distinct root. Which of the following could be true about the function.

   I It's local maximum and local minimum values have opposite signs.

   II It's local maximum and local minimum values have same signs.

   III It has a horizontal point of inflexion.

   A. **I**

   B. **II only**

   C. **III only**

   D. **II or III**

2. If the parametric equations of a curve are $x = \sin t$ and $y = \cos^2 t + 1$, then the Cartesian equation of the curve is

   A. $y = x^2 - 1$

   B. $y = 1 - x^2$

   C. $y = 2 - x^2$

   D. $y = x^2 - 1$

3. Which of the set of 3 numbers could be the roots of the polynomial equation $x^3 + ax^2 - 41x + 42 = 0$?

   A. 2, 3, 7

   B. 1, -6, 7

   C. -1, -2, 21

   D. -1, -3, -14.

4. What is the value of $\int_0^{\frac{\sqrt{3}}{2}} \sqrt{1-y^2}dy$

   A. $\frac{\pi}{6}$

   B. $\frac{\pi}{3}$

   C. $\frac{\pi}{6} + \frac{\sqrt{3}}{8}$

   D. $\frac{\pi}{3} + \frac{\sqrt{3}}{8}$

5. When a polynomial $(3x^2 + 8x - 3)$ is multiplied by $(px - 1)$ and the resulting product is divided by $(x + 1)$ the remainder is 24. What is the value of p?

   A. -4

   B. 2

   C. 4

   D. $\frac{11}{4}$

6. The largest value obtained by $3\cos^2 x + 2\sin x + 1$ equals

   A. $\dfrac{11}{5}$

   B. $\dfrac{13}{3}$

   C. $\dfrac{12}{5}$

   D. $\dfrac{14}{9}$

7. In the range $0 \leq x \leq 2\pi$, the equation $2^{\sin^2 x} + 2^{\cos^2 x} = 2$

   A. has no solutions

   B. has 1 solution

   C. has 2 solutions

   D. holds for all values of x

8. The graph of $f(x) = 0.6\cos^{-1}(x-1)$, defines a curve that, when rotated about the y=axis will produce a solid that is to be the shape and size of a new biscuit. Which integral expression will give the volume of the biscuit?

A. $\pi \displaystyle\int_0^{0.6} \left[\cos\left(\frac{5}{3}y\right)+1\right]^2 dy$

B. $\pi \displaystyle\int_0^{0.6} \left[\cos\left(\frac{3}{5}y\right)+1\right]^2 dy$

C. $\pi \displaystyle\int_0^{0.6\pi} \left[\cos\left(\frac{5}{3}y\right)+1\right]^2 dy$

D. $\pi \displaystyle\int_0^{0.6\pi} \left[\cos\left(\frac{3}{5}y\right)+1\right]^2 dy$

9. Let n be a positive integer. The coefficient of x^3y^5 in the expansion of $(1+xy+y^2)^n$ equals

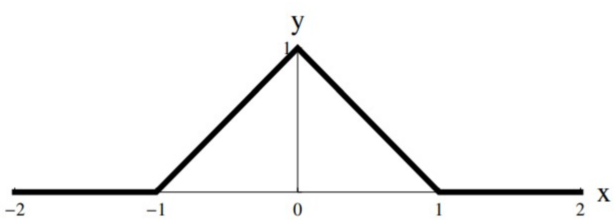
A. n

B. 2^n

C. $\binom{n}{3}\binom{n}{5}$

D. $4\binom{n}{4}$

10. The graph of the function $y = f(x)$ is sketched below. The value of $\displaystyle\int_{-1}^{1} f(x^2-1)dx$ equals



A. 2

B. $\frac{1}{3}$

C. $\sqrt{2}$

D. $\frac{2}{3}$

Section II
60 Marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklets. Extra writing booklets are available.

For questions in section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11. (15 Marks) Use the Question 11 Writing Booklet

(a) (i) Solve $\dfrac{x+5}{x-1} \geq \dfrac{9}{x}$ for $x \neq 1, x \neq 0$ 3

(ii) Hence find the set of values of x which satisfies $\dfrac{e^x+5}{e^x-1} \geq \dfrac{9}{e^x}$ 2

(b) Find the remainder when $x^3 - 5x^2 + 7$ is divided by $(x-1)^2$ 3

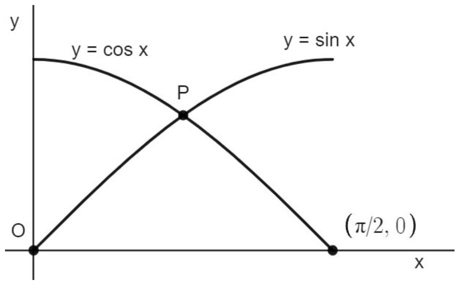
(c) (i) Express $f(\theta) = \sqrt{3}\sin 2\theta + \cos 2\theta$ in the form $P\cos(2\theta - \alpha)$, for $0° \leq \alpha \leq 90°$ 2

(ii) Find the maximum and minimum values of $f(\theta)$ and the values of θ when they occur. 2

(d) Evaluate $\cos\left(\tan^{-1}\frac{4}{3} - \cos^{-1}\frac{5}{13}\right)$. Exact answer required with working. 3

Question 12. (15 Marks) Use the Question 12 Writing Booklet.

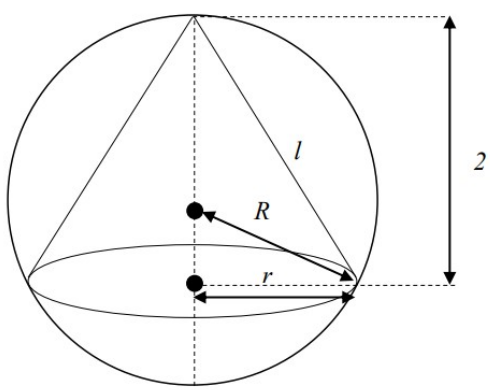
(a)



With origin O, the curves with equations $y = \sin x$ and $y = \cos x$ where $0 \leq x \leq \frac{1}{2}\pi$, meet at the point P with coordinates $\left(\frac{1}{4}\pi, \frac{1}{2}\sqrt{2}\right)$.

Find the exact value of the volume of the solid formed when the area bounded by the curves $y = \sin x$ and $y = \cos x$ and the x - axis is rotated about the x - axis by 2π radians. 4

(b) The diagram shows a right circular cone of height 2 units and radius r and slant height l inscribed in a sphere of radius R



(i) Show that $A = 4\pi\sqrt{R^2 - R}$, where A is the curved surface area of the cone. 2
(Curved surface area of a right circular cone of base radius r and slant height l is $\pi r l$)

(ii) If the volume of the sphere is increasing at the rate of $8\ units^3/sec$, 4

find the exact rate of change of A at the instant when $R = 2$ units.
(Volume of sphere is $\frac{4}{3}\pi R^3$)

(c) If cosec $\theta - \cot\theta = \frac{4}{5}$, find the exact value of $\tan\theta$, without finding the value of θ 3

(d) Prove that $\sin^2 2\theta(\cot^2\theta - \tan^2\theta) = 4\cos 2\theta$ 2

Question 13. (15 Marks) Use the Question 13 Writing Booklet.

(a) (i) Find $\displaystyle\int \frac{1-\ln x}{x\ln x}dx$ 2

(ii) Use the substitution $u = x-8$ to find $\displaystyle\int_8^{8.5} \frac{dx}{\sqrt{(7-x)(x-9)}}$ 3

(b) α, β and γ are the roots of the equation $2x^3+5x^2-4x+8=0$. Given that $\alpha^2+\beta^2+\gamma^2 = \dfrac{41}{4}$, 2

and $\alpha^3+\beta^3+\gamma^3 = \dfrac{-341}{8}$,

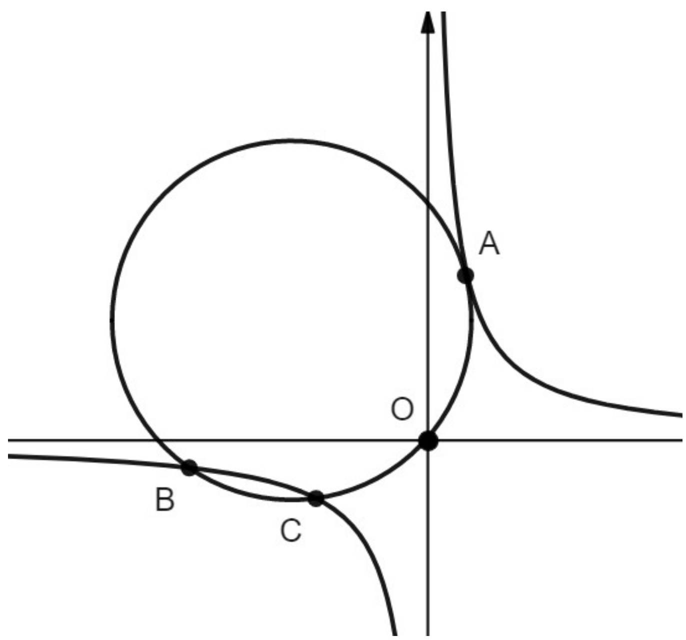
find the value of $\alpha^4+\beta^4+\gamma^4$

(c) Use vector methods to find the point P on the circle $(x-5)^2+(y-4)^2=4$ which is closest to the 3

circle $(x-1)^2+(y-1)^2=1$

You may assume that the point P and the centres of the two circles are collinear.

(d) A circle passing through the origin O is tangent to the hyperbola $xy=1$ at A and intersects the hyperbola again at two distinct points B and C.The co-ordinates of the points A, B and c are $(t, \frac{1}{t}), (t_1, \frac{1}{t_1})$ and $(t_2, \frac{1}{t_2})$ respectively



(i) Find the gradient of OA 1

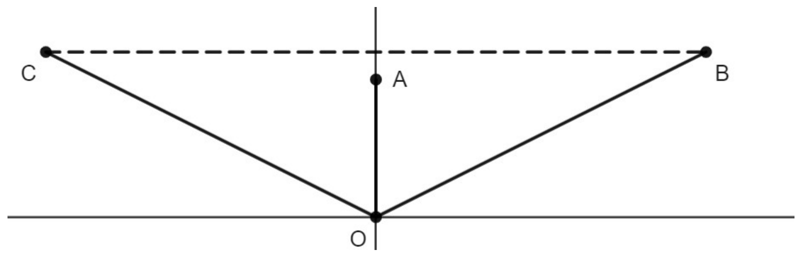
(ii) Show that the gradient of BC is $\dfrac{-1}{t_1t_2}$ 1

(iii) Prove that OA is perpendicular to BC 3

Question 14. (15 Marks) Use the Question 14 Writing Booklet.

(a) With reference to the origin O, the points A and B have position vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively, and O, A and B are non-collinear. The point C, with position vector $\underset{\sim}{c}$, is the reflection of B in the line through O and A. 3

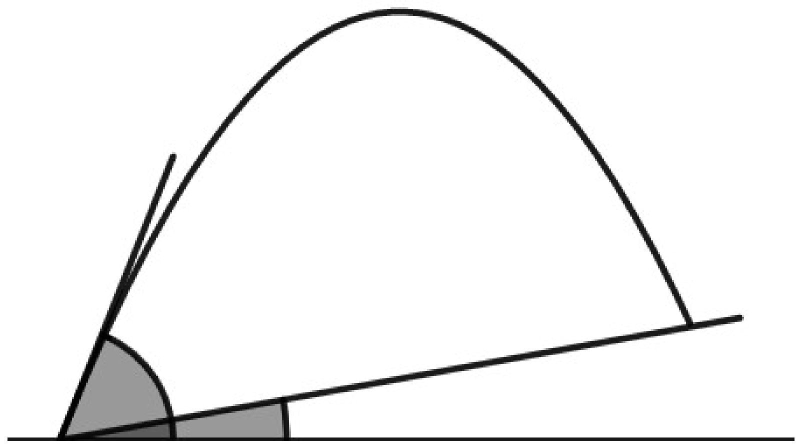
Show that $\underset{\sim}{c}$ can be written in the form $\underset{\sim}{c} = \lambda\, \underset{\sim}{a} - \underset{\sim}{b}$, where $\lambda = \dfrac{2\underset{\sim}{a}.\underset{\sim}{b}}{\underset{\sim}{a}.\underset{\sim}{a}}$



(b) Prove by induction that for all positive integers n 3

$$n^2 - (n-1)^2 + (n-2)^2 - (n-3)^2 \cdots + (-1)^{n-1}(1)^2 = \tfrac{n}{2}(n+1)$$

(c) Part of a golf course is on a hill which slopes at an angle α to the horizontal. The ball is hit straight up the hill with velocity $\underset{\sim}{u} = [(|\underset{\sim}{u}|\cos\theta)\underset{\sim}{i} + (|\underset{\sim}{u}|\sin\theta)\underset{\sim}{j}]m/s$. The position vector$\underset{\sim}{r}$ of the particle at any time t is $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j}$ and the acceleration due to gravity is g ms^{-2}.



(i) At any time t on its trajectory, derive the vector equation $\underset{\sim}{r} = \underset{\sim}{u}t - \dfrac{gt^2}{2}\underset{\sim}{j}$ 3

(ii) Hence show that $\dfrac{|\underset{\sim}{r}|}{\cos\theta} = \dfrac{|\underset{\sim}{u}|t}{\cos\alpha} = \dfrac{gt^2}{2\sin(\theta-\alpha)}$ 2

(iii) Hence show that the ball first lands at a distance of 2

$\dfrac{2|\underset{\sim}{u}|^2\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}$ up the hill.

(iv) For different values of θ show that the greatest distance up the hill the ball can be hit is 2

$\dfrac{|\underset{\sim}{u}|^2}{g(1+\sin\alpha)}$.

End of Paper

Suggested solutions to 12 Maths Ext 1 Trial : NSB 2022

1. D

2. C

3. B

4. C

5. B

6. B

7. A

8. C

9. D

10. D.

11. (a) (i)

$$\frac{x+5}{x-1} \geq \frac{9}{x} \qquad x \neq 0, x \neq 1$$

Multiplying both sides by $x^2(x-1)^2$,

$x^2(x+5)(x-1) \geq 9x(x-1)^2$

$\Rightarrow x(x-1)[9(x-1) - x(x+5)] \leq 0$

$\Rightarrow x(x-1)(-x^2+4x-9) \leq 0$

$\Rightarrow x(x-1)[(x-2)^2+5] \geq 0$

$\Rightarrow x(x-1) \geq 0$

$\Rightarrow \underline{\underline{x < 0}}$ or $\underline{\underline{x > 1}}$

(ii)

$$\frac{e^x+5}{e^x-1} \geq \frac{9}{e^x}$$

$\Rightarrow e^x < 0$ or $e^x > 1$

~~$\Rightarrow$ not true~~

$e^x \not< 0 \quad \therefore e^x > 1$

$\underline{\underline{x > 0}}$

(b) Let $P(x) = x^3 - 5x^2 + 7$

$\Rightarrow P(x) = x^3 - 5x^2 + 7 = (x-1)^2 \cdot Q(x) + Ax + B$

$P(1) = 0 \Rightarrow A + B = 1 - 5 + 7 = 3$ —— (1)

$P'(x) = 3x^2 - 10x = 2(x-1)Q(x) + (x-1)^2 Q'(x) + A$

$P'(1) = 0 \Rightarrow 3 - 10 = A$ —— (2)

From (1) and (2)

$A = -7$ and $B = 10$

$\therefore$ The remainder is $\underline{\underline{(-7x+10)}}$

11 (c) (i) $f(x) = \sqrt{3}\sin 2\theta + \cos 2\theta$

$= 2\left[\frac{\sqrt{3}}{2}\sin 2\theta + \frac{1}{2}\cos 2\theta\right]$

$= 2\left[\cos 2\theta \cos 60 + \sin 2\theta \sin 60^\circ\right]$

$= 2\cos(2\theta - 60^\circ)$

$= R\cos(2\theta - \alpha)$ where $R = 2$ and $\alpha = 60^\circ$

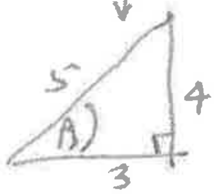
(ii) $f(x) = 2\cos(2\theta - 60^\circ)$

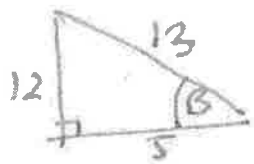
$f(x)_{max} = 2$ when $2\theta - 60 = 0 \Rightarrow \theta = 30^\circ$

$f(x)_{min} = -2$ when $2\theta - 60 = 180 \Rightarrow \theta = 120^\circ$

11 (d) Let $A = \tan^{-1}(\frac{4}{3})$ and $B = \cos^{-1}(\frac{5}{13})$

$\Rightarrow \tan A = \frac{4}{3}$ and $\cos B = \frac{5}{13}$





$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$= \frac{3}{5}\cdot\frac{5}{13} + \frac{4}{5}\times\frac{12}{13}$

$= \frac{63}{65}$

(ii) $\cos\left[\tan^{-1}(\frac{4}{3}) - \cos^{-1}(\frac{5}{13})\right] = \frac{63}{65}$

13. (a) Required volume $= V_1 + V_2$

$$V_1 = \pi \int_0^{\pi/4} \sin^2 x \, dx$$

$$= \frac{\pi}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/4}$$

$$= \frac{\pi}{2}\left[\frac{\pi}{4} - \frac{1}{2}\right] \text{ units}^3.$$

$$V_2 = \pi \int_{\pi/4}^{\pi/2} \cos^2 x \, dx$$

$$= \frac{\pi}{2}\left[x + \frac{1}{2}\sin 2x\right]_{\pi/4}^{\pi/2}$$

$$= \frac{\pi}{2}\left[\frac{\pi}{2} - \left(\frac{\pi}{4} + \frac{1}{2}\right)\right]$$

$$= \frac{\pi}{2}\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$\therefore V = V_1 + V_2$$

$$= \pi\left[\frac{\pi}{4} - \frac{1}{2}\right] \text{ units}^3$$

or

$$= \frac{\pi}{4}\left[\pi - 2\right] \text{ units}^3$$

(b) (i) By Pythagoras' Theorem $l = \sqrt{r^2 + 4}$

$$R^2 = r^2 + (2 - R)^2 \Rightarrow r^2 = 4R - 4$$

and $l = \sqrt{4R - 4 + 4} = \sqrt{4R}$

$$A = \pi r l$$

$$= \pi \sqrt{4R - 4} \cdot \sqrt{4R}$$

$$= 4\pi \sqrt{R^2 - R}$$

(b) (ii)

$$\frac{dA}{dt} = \frac{dA}{dR} \times \frac{dR}{dV} \times \frac{dV}{dt}$$

$$= \frac{2\pi(2R-1)}{\sqrt{R^2-R}} \cdot \frac{1}{4\pi R^2} \times 8$$

when $R = 2$

$$\frac{dA}{dt} = \frac{2\pi(4-1)}{\sqrt{4-2}} \times \frac{1}{4\pi(4)} \times 8$$

$$= \frac{3\sqrt{2}}{2}$$

4(c) $\operatorname{cosec}\theta - \cot\theta = \frac{4}{5}$ ——— (1)

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

$\Rightarrow (\operatorname{cosec}\theta + \cot\theta)\left(\frac{4}{5}\right) = 1$

$\Rightarrow \operatorname{cosec}\theta + \cot\theta = \frac{5}{4}$ ——— (2)

(2) − (1) $\quad 2\cot\theta = \frac{5}{4} - \frac{4}{5}$

$\cot\theta = \frac{9}{40}$

$\Rightarrow \tan\theta = \frac{40}{9}$

(d) LHS $= \sin^2 2\theta(\cot^2\theta - \tan^2\theta)$

$= 4\sin^2\theta\cos^2\theta \dfrac{(\cos^4\theta - \sin^4\theta)}{\sin^2\theta\cos^2\theta}$

$= 4(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= 4\cos 2\theta$

$=$ RHS

Q13 (a)

(i) $\int \frac{1-\ln x}{x\ln x}\,dx$

$= \int \frac{1}{x\ln x} - \frac{1}{x}\,dx$

$= \ln(\ln x) - \ln x + C$

$= \ln\left(\frac{\ln x}{x}\right) + C$

(ii) $\int_{8}^{8.5} \frac{dx}{\sqrt{(7-x)(x-9)}}$

let $u = x-8$

$\frac{du}{dx} = 1$

$x=8,\ u=0$

$x=8.5\ \ u=0.5$

$= \int_{0}^{0.5} \frac{du}{\sqrt{-(u+1)(u-1)}}$

$= \int_{0}^{0.5} \frac{du}{\sqrt{1-u^2}}$

$= \left[\sin^{-1}(u)\right]_{0}^{0.5}$

$= \frac{\pi}{6}$

(iii) $P(x) = 2x^3+5x^2-4x+8=0 \quad \alpha,\beta,\gamma$

$P(0) \neq 0 \Rightarrow x \neq 0$

$\Rightarrow 2x^4+5x^3-4x^2+8x=0$

α is a root

$\therefore 2\alpha^4+5\alpha^3-4\alpha^2+8\alpha=0$ —— ①

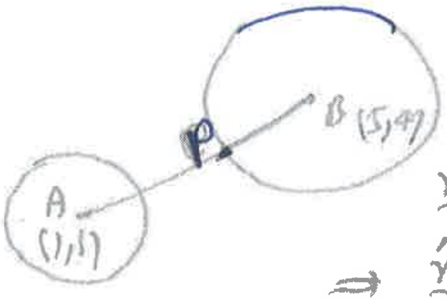
Similarly $2\beta^4+5\beta^3-4\beta^2+8\beta=0$ —— ②

$2\gamma^4+5\gamma^3-4\gamma^2+8\gamma=0$ —— ③

① + ② + ③ $2(\alpha^4+\beta^4+\gamma^4)+5\left(-\frac{341}{8}\right)-4\left(\frac{41}{4}\right)+8\left(-\frac{5}{2}\right)=0$

$\Rightarrow 2(\alpha^4+\beta^4+\gamma^4) = 274\frac{1}{8} \quad \therefore \alpha^4+\beta^4+\gamma^4 = 137\frac{1}{16}$

13(c)



$\underset{\sim}{r} = \overrightarrow{BA} = \begin{pmatrix}-4\\-3\end{pmatrix}$

$\Rightarrow \hat{\underset{\sim}{r}} = \frac{1}{5}\begin{pmatrix}-4\\-3\end{pmatrix}$

$\overrightarrow{BC} = 2\hat{\underset{\sim}{r}} = \frac{2}{5}\begin{pmatrix}-4\\-3\end{pmatrix}$

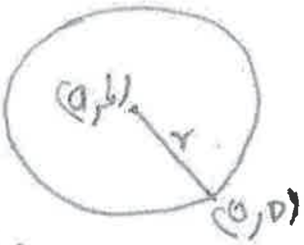
$\therefore \underset{\sim}{p} = \begin{pmatrix}5\\4\end{pmatrix} + \frac{2}{5}\begin{pmatrix}-4\\-3\end{pmatrix}$

$= \begin{pmatrix}3.4\\2.8\end{pmatrix}$ $\therefore P(3.4, 2.8)$

(d) (i) gradient of OA $= m_{OA} = \frac{\frac{1}{t}-0}{t-0} = \frac{1}{t^2}$

~~$m_{BC} = 1/t_2 - 1/t_1$~~

(ii) $m_{BC} = \frac{\frac{1}{t_2}-\frac{1}{t_1}}{t_2-t_1} = -\frac{1}{t_1t_2}$



$r = \sqrt{a^2+b^2}$

$\therefore$ The equation of the circle is

(iii)

$(x-a)^2 + (y-b)^2 = a^2+b^2$

$\Rightarrow x^2 - 2ax + y^2 - 2by = 0$

$A(t, \frac{1}{t})$ is on the circle

$\Rightarrow t^2 - \frac{2a}{t} + \frac{1}{t^2} - \frac{2b}{t} = 0$

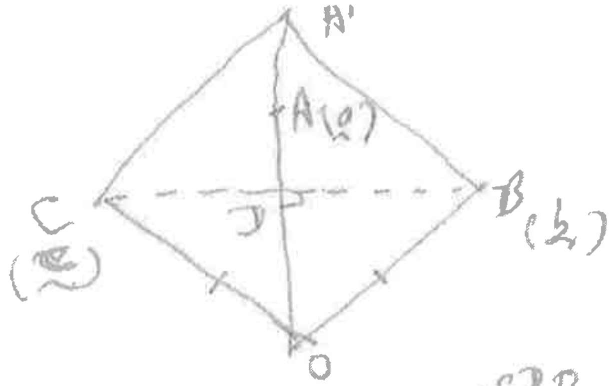
$\Rightarrow t^4 - 2at^3 - 2bt + 1 = 0$

Product of roots $t^2 t_1 t_2 = -1 \Rightarrow \frac{1}{t^2 t_1 t_2} = -1$

$\Rightarrow \frac{1}{t^2} \times \frac{1}{t_1t_2} = -1$

$\Rightarrow m_{OA} \times m_{BC} = -1 \Rightarrow OA \perp BC$

Q14. (a)



~~OB~~ $OB = OC$, $\angle CJO = \angle BJO = 90$

$\Rightarrow$ ~~OJ~~ is OA is the bisector of $\angle BOC$

So that A is on the diagonal OA' of the parallelogram $OBA'C$ (I) is a rhombus, $OB = OC$)

$\therefore \underset{\sim}{b} + \underset{\sim}{c} = \lambda \underset{\sim}{a}$

$BC \perp OA \Rightarrow (\underset{\sim}{b} - \underset{\sim}{c}) \cdot \underset{\sim}{a} = 0$

$\Rightarrow \cancel{[\ldots] \cdot \underset{\sim}{a}} = 0$

$\Rightarrow [\underset{\sim}{b} - (\lambda \underset{\sim}{a} - \underset{\sim}{b})] \cdot \underset{\sim}{a} = 0$

$\Rightarrow [2\underset{\sim}{b} - \lambda \underset{\sim}{a}] \cdot \underset{\sim}{a} = 0$

$\Rightarrow \lambda = \dfrac{2\underset{\sim}{b} \cdot \underset{\sim}{a}}{\underset{\sim}{a} \cdot \underset{\sim}{a}}$ //

Q14 (b) Let P_n be the statement

" $n^2-(n-1)^2+(n-2)^2-(n-3)^2+\cdots+(-1)^{n-1}(1)^2=\frac{n}{2}(n+1)$ ", $n\in\mathbb{Z}^+$,

When $n=1$,

LHS $=(-1)^0(1)^2=1$, RHS $=\frac{1}{2}(1+1)=1 \Rightarrow$ LHS $=$ RHS

$\therefore P_1$ is true.

Assume P_k is true for some $k\in\mathbb{Z}^+$,

(i.e. $k^2-(k-1)^2+(k-2)^2-\cdots+(-1)^{k-1}(1)^2=\frac{k}{2}(k+1)$)

To prove that P_{k+1} is true,

$(k+1)^2-k^2+(k-1)^2-(k-2)^2\cdots+(-1)^{(k+1)-1}(1)^2=\frac{(k+1)}{2}[(k+1)+1]$

(i.e. $(k+1)^2-k^2+(k-1)^2+\cdots+(-1)^k(1)^2=\frac{(k+1)}{2}(k+2)$)

LHS $=(k+1)^2-\frac{k}{2}(k+1)$

$=\frac{(k+1)}{2}[2(k+1)-k]$

$=\frac{(k+1)}{2}[k+2]$

$=$ RHS

$\therefore P_k$ is true $\Rightarrow P_{k+1}$ is true

Since P_1 is true, by the method of mathematical induction,

P_n is true for all $n\in\mathbb{Z}^+$.

14(b)

(i) $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j}$

$\Rightarrow \ddot{\underset{\sim}{r}} = \ddot{x}\underset{\sim}{i} + \ddot{y}\underset{\sim}{j}$

$= 0\underset{\sim}{i} - g\underset{\sim}{j}$

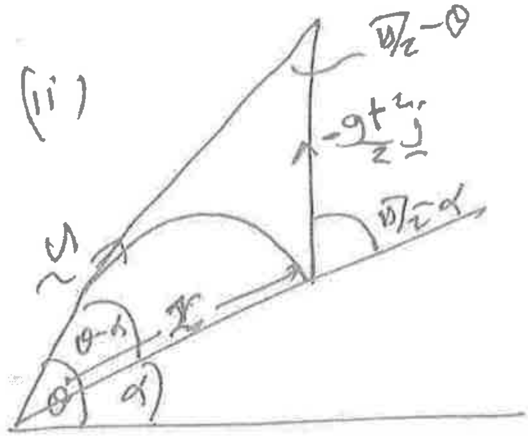
$\Rightarrow \dot{\underset{\sim}{r}} = -gt\underset{\sim}{j} + c$

$t=0, \dot{\underset{\sim}{r}} = u \Rightarrow c = \underset{\sim}{u}$

$\dot{\underset{\sim}{r}} = -gt\underset{\sim}{j} + \underset{\sim}{u}$

$\Rightarrow \underset{\sim}{r} = -\frac{gt^2}{2}\underset{\sim}{j} + \underset{\sim}{u}.t$

(ii)



Applying Sine Rule in ΔABC,

$$\frac{|\underset{\sim}{r}|}{\sin(\frac{\pi}{2}-\theta)} = \frac{|\underset{\sim}{u}|t}{\cos\alpha} = \frac{\frac{1}{2}gt^2}{\sin(\theta-\alpha)}$$

$$\Rightarrow t = \frac{2|\underset{\sim}{u}|\sin(\theta-\alpha)}{g\cos\alpha}$$

(iii)

$$\Rightarrow \frac{|\underset{\sim}{r}|}{\cos\theta} = \frac{|\underset{\sim}{u}|}{\cos\alpha}\,\frac{2|\underset{\sim}{u}|\sin(\theta-\alpha)}{g\cos\alpha}$$

$$\Rightarrow |\underset{\sim}{r}| = \frac{2|\underset{\sim}{u}|^2\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}$$

(iv) $$r = \frac{2u^2\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}$$

$$= \frac{u^2}{g\cos^2\alpha}\left[\sin(2\theta-\alpha) - \sin\alpha\right]$$

$$= \frac{u^2}{g\cos^2\alpha}\left[1-\sin\alpha\right] \quad \text{when } 2\theta-\alpha = \pi/2$$

$$= \frac{u^2\cancel{(1-\sin\alpha)}(1+\sin\alpha)}{\cancel{g\cos^2\alpha}(1+\sin\alpha)}$$

$$= \frac{u^2}{g(1+\sin\alpha)} \;//$$