NORTH SYDNEY BOYS HIGH SCHOOL

2020 YEAR 12 ASSESSMENT TASK 3

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/ or calculations
- **Section I – 10 marks** (pages 2–6)
- Attempt Questions 1–10 • Allow about 15 minutes for this section
- **Section II – 60 marks** (pages 7–13)
- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

O Class Teacher: Please colour

O Mr Hwang
O Dr Jomaa
O Ms Lee
O Mr Lin
O Mr Uma
O Ms Ziaziaris

Total Marks 70

Student Number:____________________________

Question No	1-10	11	12	13	14	Total	Total
Mark	/10	/15	/15	/15	/15	/70	/100

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

1. The domain and range of $y = 4cos^{-1}\left(\frac{3x}{2}\right)$ is

A. Domain : $\frac{-2}{3} \leq x \leq \frac{2}{3}$
Range : $-2\pi \leq y \leq 2\pi$

B. Domain : $\frac{-2}{3} \leq x \leq \frac{2}{3}$
Range : $0 \leq y \leq 4\pi$

C. Domain : $\frac{-2}{3} \leq x \leq \frac{2}{3}$
Range : $0 \leq y \leq \frac{\pi}{4}$

D. Domain : $\frac{-3}{2} \leq x \leq \frac{3}{2}$
Range : $0 \leq y \leq 4\pi$

2. The coefficient of x^3 in the expansion of $(a + x)^6$ is 1280, what is the value of a?

A. 64
B. 4
C. 40
D. $\sqrt[3]{32}$

3. What is the value of y, if $\binom{-6}{4}$ and $\binom{2y}{3}$ are parallel?

A. -1

B. $\frac{9}{4}$

C. $-\frac{9}{4}$

D. 1

4. What is the derivative of $xsin^{-1}3x$?

A. $sin^{-1}3x + \frac{3x}{\sqrt{1-9x^2}}$

B. $sin^{-1}3x + \frac{x}{\sqrt{1-9x^2}}$

C. $\frac{x}{\sqrt{1-9x^2}}$

D. $\frac{3x}{\sqrt{1-9x^2}}$

5. $\overrightarrow{OM} = 2i + 5j$ and $\overrightarrow{OT} = -3i + 7j$

The value of $|\overrightarrow{MT}|$ is

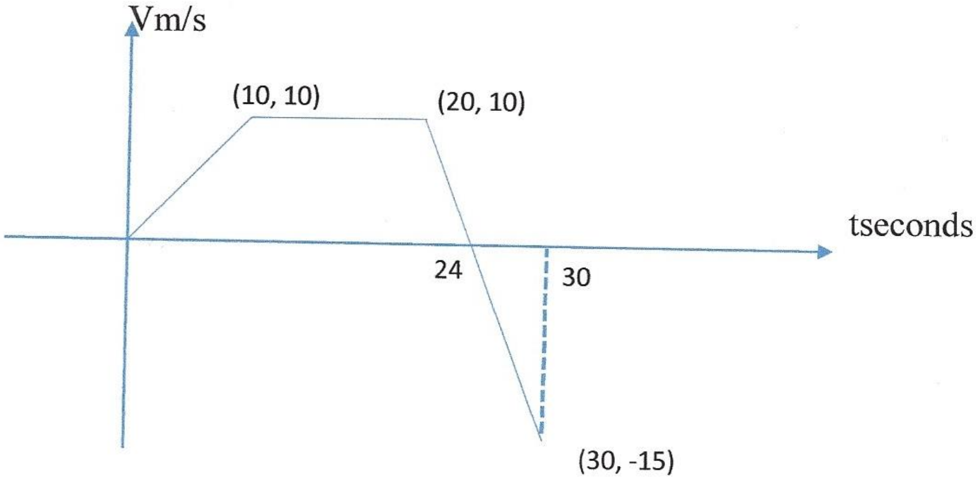
A. 14

B. $\sqrt{98}$

C. $\sqrt{29}$

D. $\sqrt{145}$

6. Given the velocity time graph below find the acceleration for the first 10 seconds and the total distance travelled in the first 30 seconds.

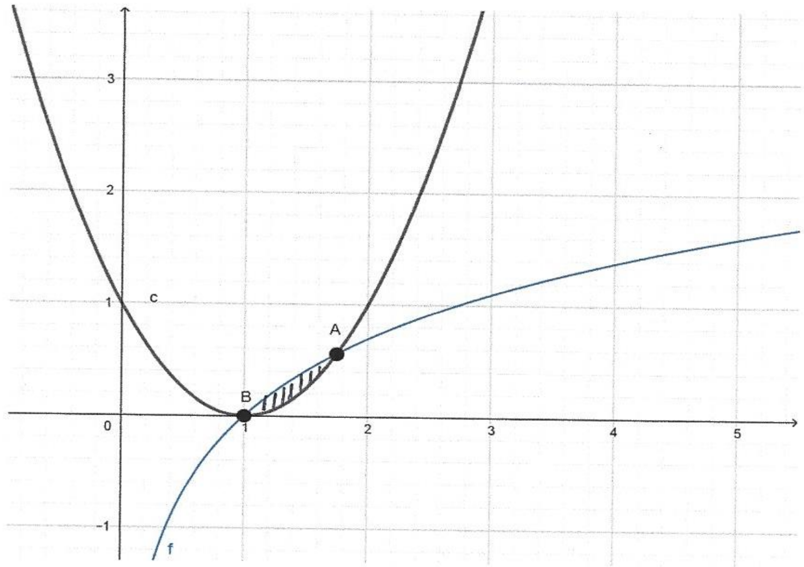


A. acceleration = $1m/s^2$, distance travelled = 215m
B. acceleration = $10m/s^2$, distance travelled = 215m
C. acceleration = $1m/s^2$, distance travelled = 125m
D. acceleration = $10m/s^2$, distance travelled = 125m

7. If two of the roots of $2x^3 + bx^2 + cx + 8 = 0$ are -1 and 2, find the values of b and c.

A. $b = -1, c = 2$
B. $b = -6, c = 0$
C. $b = -6, c = 16$
D. $b = 6, c = 0$

8. The graph below shows the graph of $y = (x-1)^2$ and $y = log_e x$. They intersect at the point $B(1,0)$ and $A\ (a, log_e a)$. The shaded area between these two curves is rotated about the y axis. Which integral represents the volume of the solid of revolution formed?



A. $V = \pi \int_0^{lna} \left(\sqrt{y} + 1 - e^y\right)^2 dy$

B. $V = \pi \int_0^{lna} (y + 2\sqrt{y} + 1 - e^{2y}) dy$

C. $V = \pi \int_1^{a} (lnx)^2 - (x-1)^4 dx$

D. $V = \pi \int_0^{lna} \left(\sqrt{y} + 1 - e^y\right)\ dy$

9. Evaluate $\lim\limits_{\theta \to 0} \frac{2\sin\left(\pi + \frac{\theta}{2}\right)}{\theta}$

A. $\frac{1}{2}$
B. 2
C. 1
D. -1

10. Given the parametric equations

$$\begin{cases} x = sin^{-1}t + 1 \\ y = \dfrac{1}{\sqrt{1 - t^2)}} \end{cases}$$

What is the Cartesian equation?

A. $y = \frac{1}{1-\sin(x-1)}$

B. $y = \frac{1}{\sqrt{2sinx-\sin^2 x}}$

C. $y = \frac{1}{\cos(x-1)}$

D. $y = \pm\frac{1}{\cos(x-1)}$

Section II

60 marks

Attempt Questions 11–14

Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet.

Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) $\frac{3}{3x-1} \leq 5$ 3

(b) Find the term independent of x in the expansion $(x^2 - \frac{1}{x^2})^{10}$ 3

(c) Factorise and solve $2x^3 + x^2 - 7x - 6 = 0$ 3

(d) $\int_0^{\pi} sin^2 \frac{x}{2} dx$ 3

(e) Find the exact value of $\sin(cos^{-1}\left(\frac{2}{3}\right) + tan^{-1}\left(\frac{-3}{4}\right))$ 3

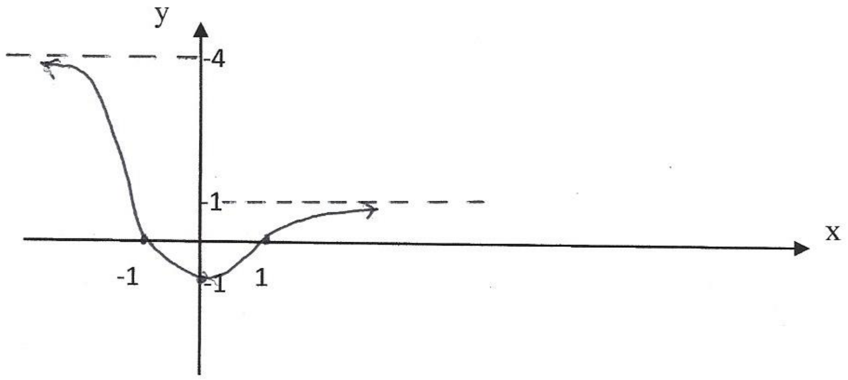
Examination continues on the next page ………………………………………

Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a) (i) Given $\underset{\sim}{a} = \binom{1}{3}$ and $\underset{\sim}{b} = \binom{2}{-1}$ find the vector projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$. **2**

(ii) Calculate the angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **1**

(b) Given the graph of $y = f(x)$ below

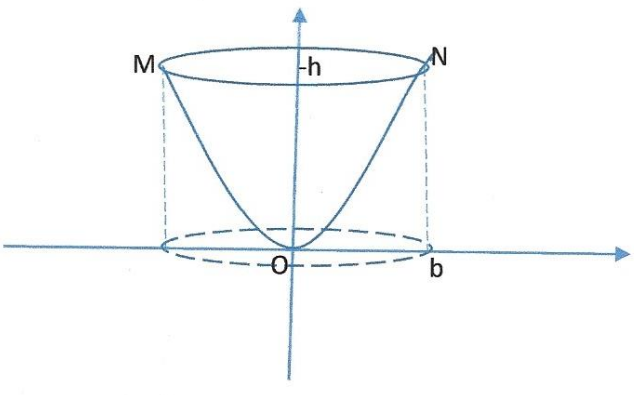


Sketch

(i) $y = \sqrt{f(x)}$ **2**

(ii) $y = \frac{1}{f(|x|)}$ **2**

(c) A parabola with equation of the form $y = ax^2$ is rotated about the y axis to form a paraboloid below. Show that the volume of the paraboloid OMN is half the volume of the circumscribing cylinder. The height of both the paraboloid and cylinder is h units and the radius of the cylinder is b units. **3**



Examination continues on the next page ....

(d) $\int \frac{1}{1+4t^2} dt$ 2

(e) $\int_0^3 \frac{(x^2+2x)}{\sqrt{1+x}} dx$ using $u^2 = 1 + x$ 3

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a) (i) Express $cos\theta - \sqrt{3}sin\theta$ in the form $Rsin(\theta + \alpha)$ where $R > 0$ 2

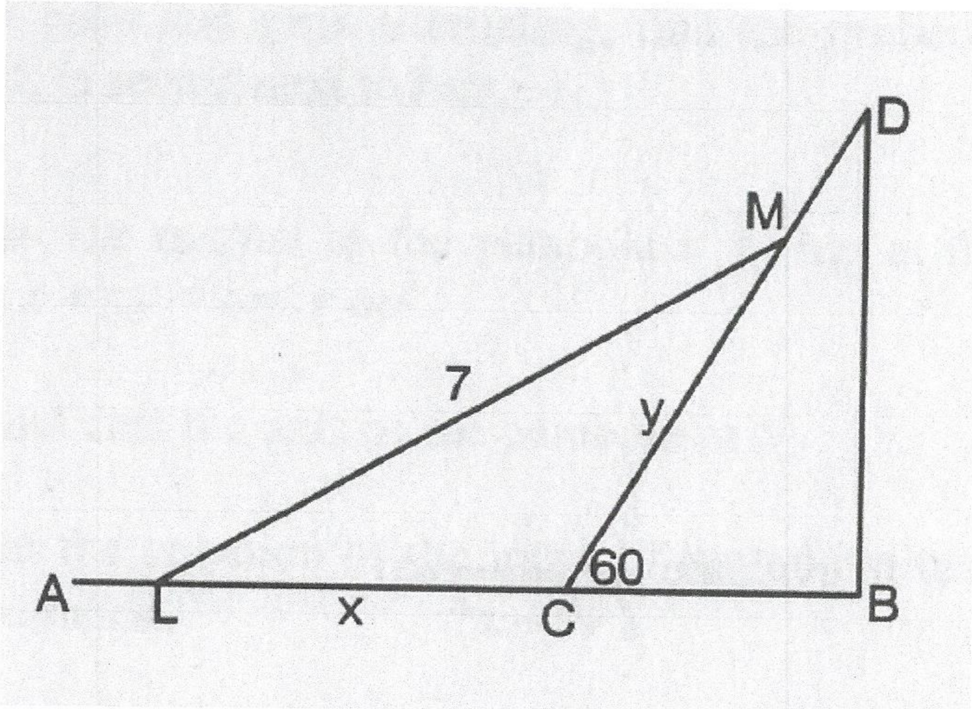
(ii) Hence, solve $cos\theta - \sqrt{3}sin\theta = 1$ for $0 \leq \theta \leq 2\pi$ 2

(b) Given $\underset{\sim}{a} = 3\underset{\sim}{i} - m\underset{\sim}{j}$ and $\underset{\sim}{b} = n\underset{\sim}{i} - 6\underset{\sim}{j}$ for $m, n \in R$

$|\underset{\sim}{a}| = 5$ and $\underset{\sim}{b}$ is perpendicular to $\underset{\sim}{a}$ determine all values of m and n. 2

Examination continues on the next page ....

(c) The diagram shows a wall CD inclined at an angle of 60°to the horizontal floor AB. A plank LM of length 7 metres is slipping down the wall with end L moving along CA at a constant speed of 11m/s.



(i) If $LC = x$ and $MC = y$, show that $x^2 + xy + y^2 = 49$ 1

(ii) Given $\frac{dy}{dx} = -\frac{2x+y}{x+2y}$, find the speed at which end M moves along the incline at the instant when $x = 5$ metres. 2

(iii) Find an expression for $sin\theta$ where $\angle MLC = \theta$ 1

(iv) Calculate the value of $\frac{d\theta}{dt}$ in degrees per second at the instant when $x = 5$ metres. 2

(d) Prove by the Principle of Mathematical Induction for all integers $n \geq 1$ 3

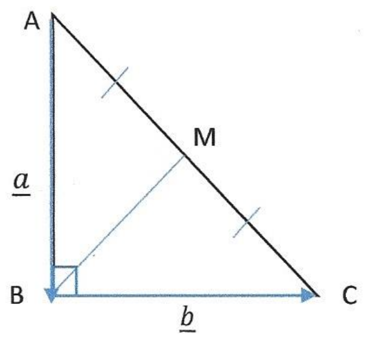
$$a + a^3 + a^5 + \dots\dots\dots + a^{2n-1} = \frac{a(a^{2n} - 1)}{(a+1)(a-1)}$$

Examination continues on the next page ....

Question 14 (15 marks) Use the Question 14 Writing Booklet.

(a) Show that $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots\ldots\ldots\ldots\ldots + \binom{n}{n} = 2^n$ 2

(b) Consider the right angled triangle ABC below.



You are given $|\underset{\sim}{b} + \underset{\sim}{a}|^2 = |\underset{\sim}{b}|^2 + 2\underset{\sim}{a}.\underset{\sim}{b} + |\underset{\sim}{a}|^2$ and $|\underset{\sim}{b} - \underset{\sim}{a}|^2 = |\underset{\sim}{b}|^2 - 2\underset{\sim}{a}.\underset{\sim}{b} + |\underset{\sim}{a}|^2$

You are not required to prove these.

Hence, or otherwise, use vector methods to prove that the midpoint M of the hypotenuse of the right angled triangle is equidistant from all three vertices. 3

Examination continues on the next page ....

(c) A golf ball is hit at a velocity of $110\ ms^{-1}$ at an angle θ to the horizontal.

The position vector *s(t)*, from the starting point, of the ball after *t* seconds is given by

$s = 110t\cos\theta\,\underset{\sim}{\mathbf{i}} + (110t\sin\theta - 4{\cdot}9\,t^2)\underset{\sim}{\mathbf{j}}$

(i) Using gravity of $9{\cdot}8\ ms^{-2}$ show that the maximum horizontal range of the ball is

$$\frac{12100 \sin 2\theta}{9{\cdot}8} \text{ metres.}$$ 2

(ii) To ensure that the ball lands on the green, it must travel between 400 and 450 metres.

What values of θ, correct to the nearest minute, would allow this to happen? 2

(iii) The golfer aims accurately and hits the ball directly towards the green.

After 3.4 seconds of flight, at a point 8 metres above the ground, the ball hits a low flying TV drone. If it had not hit the drone or any other obstacles, would the ball have landed on the green? 3

(d) Given $\sqrt{\frac{1-\sin 2x}{1+\sin 2x}} = \frac{1-\tan x}{1+\tan x}$

show that the exact value of $\tan\frac{\pi}{8} = \sqrt{2} - 1$ 3

END OF EXAMINATION

① D: $-1 \le \frac{3x}{2} \le 1$

$-2 \le 3x \le 2$

$\frac{-2}{3} \le x \le \frac{2}{3}$

R: $0 \le \frac{y}{4} \le \pi$

$0 \le y \le 4\pi$ (B)

B
B
A
C
C
A
B
B
D
C

② $T_{k+1} = {}^6C_3 a^3 x^3$

$1280 = {}^6C_3 a^3$

$\sqrt[3]{\frac{1280}{{}^6C_3}} = a$

$a = 4$ (B)

③ $\sin^{-1}3x \times 1 + x \cdot \frac{1}{\sqrt{1-9x^2}} \times 3$

$= \sin^{-1}3x + \frac{3x}{\sqrt{1-9x^2}}$ (A)

③ $-6\lambda = 2y$ —①

$4\lambda = 3$

$\lambda = \frac{3}{4}$

Sub λ into —①

$-6 \times \frac{3}{4} = 2y$

$-\frac{18}{8} = y$

$y = -\frac{9}{4}$ (C)

⑤ $\vec{MT} = \vec{MO} + \vec{OT}$

$= \begin{pmatrix} -2 \\ -5 \end{pmatrix} + \begin{pmatrix} -3 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} \cancel{6} -5 \\ 2 \end{pmatrix}$

$|\vec{MT}| = \sqrt{25+4}$

$= \sqrt{29}$

Ⓒ

⑥ $a = 1$

$d = \frac{1}{2}(10)(10+24) + \frac{1}{2} \times 6 \times 15$

$= 5 \times 34 + 45$

$= 170 + 45$

$= 215.$

Ⓐ

⑦

$(-1)(2)\alpha = -4$

$\alpha = \frac{-4}{-2}$

$\alpha = 2$

$\therefore 2+2-1 = \frac{-b}{2}.$

$6 = -b$

$b = -6.$

$-2 \times 2 + 4 = \frac{c}{2}$

$c = 0.$ Ⓑ

(8) $y=(x-1)^2$

$\sqrt{y}=(x-1)$

$\sqrt{y}+1=x$

$(\sqrt{y}+1)^2=x^2$

$y=\log_e x$

$e^y=x$

$x^2=e^{2y}$.

$$V=\pi\int_0^{\ln a}(\sqrt{y}+1)^2-e^{2y}\ dy$$

$$=\pi\int_0^{\ln a}y+2\sqrt{y}+1-e^{2y}\ dy$$

(B)

(9) $\lim_{\theta\to 0}\dfrac{\sin(\pi+\frac{\theta}{2})}{\frac{\theta}{2}}$

$=\lim_{\theta\to 0}\dfrac{-\sin\frac{\theta}{2}}{\frac{\theta}{2}}$

$=-1$.

(D)

(10) $x-1=\sin^{-1}t$.

$\sin(x-1)=t$.

$y=\dfrac{1}{\sqrt{1-(\sin(x-1))^2}}$

$=\dfrac{1}{\cos(x-1)}$

(C)

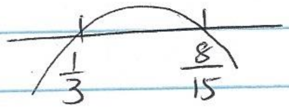
(11) a) $\frac{3}{(3x-1)}(3x-1)^2 \leq 5(3x-1)^2$

$3(3x-1) - 5(3x-1)^2 \leq 0.$

$(3x-1)[3 - 5(3x-1)] \leq 0$

$(3x-1)[3 - 15x + 5] \leq 0$

$(3x-1)(8 - 15x) \leq 0.$



$\boxed{x < \frac{1}{3} \text{ OR } x \geqslant \frac{8}{15}}$

b) $T_{k+1} = {}^{10}C_k (x^2)^{10-k} \left(-\frac{1}{x^2}\right)^k.$

$= {}^{10}C_k \, x^{20-2k} (-1)^k x^{-2k}$

$= {}^{10}C_k \, x^{20-4k} (-1)^k$

Term indep. of x $\Rightarrow 20 - 4k = 0.$

$4k = 20$

$k = 5.$

$\therefore T_6 = {}^{10}C_5 (-1)^5$

$= -{}^{10}C_5$

$= -252$

c) Let $P(x) = 2x^3 + x^2 - 7x - 6$

$P(1) = 2 + 1 - 7 - 6.$

$\neq 0$

$P(-1) = -2 + 1 + 7 - 6.$

$= 0$

$\therefore (x+1)$ is a factor

$$
\begin{array}{r}
2x^2 - x - 6 \\
x+1 \overline{)\, 2x^3 + x^2 - 7x - 6} \\
\underline{2x^3 + 2x^2 \quad\downarrow} \\
-x^2 - 7x \\
\underline{-x^2 - x} \\
-6x - 6. \\
\underline{-6x - 6} \\
0.
\end{array}
$$

$\therefore P(x) = (x+1)(2x^2-x-6)$ $\qquad$ $\begin{matrix} 2x & 3 \\ x & -2 \end{matrix}$

$P(x) = (x+1)(2x+3)(x-2)$

$\therefore \boxed{x = -1, -\frac{3}{2}\ \ 2.}$

d) $\displaystyle\int_0^{\pi} \sin^2 \frac{x}{2}\, dx.$ $\qquad$ $\cos 2x = 1 - 2\sin^2 x.$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$= \displaystyle\frac{1}{2}\int_0^{\pi} (1 - \cos x)\, dx$

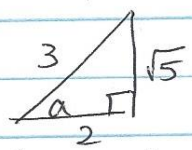
$= \frac{1}{2}\left[x - \sin x\right]_0^{\pi}$

$= \frac{1}{2}(\pi - \sin\pi)$

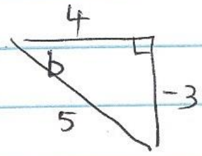
$= \boxed{\frac{\pi}{2}}$

e) $\sin\left(\cos^{-1}\left(\frac{2}{3}\right) + \tan^{-1}\left(-\frac{3}{4}\right)\right) = \sin(a+b)$

Let $\cos^{-1}\frac{2}{3} = a$ $\qquad$ $\tan^{-1}\left(\frac{-3}{4}\right) = b$



$\cos a = \frac{2}{3}$ $\qquad$ $\tan b = -\frac{3}{4}$



(1st quad) $\qquad$ (4th quad)

Now $\sin(a+b) = \sin a \cos b + \cos a \sin b.$

$= \frac{\sqrt{5}}{3} \cdot \frac{4}{5} + \frac{2}{3} \cdot \frac{-3}{5} = \boxed{\frac{4\sqrt{5} - 6}{15}}$

(12) a) (i) $proj_{\underset{\sim}{b}} \underset{\sim}{a} = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|b|^2} \times \underset{\sim}{b}$

$= \dfrac{\binom{1}{3} \cdot \binom{2}{-1}}{\sqrt{4+1}} \cdot (2i - j)$

$= - \dfrac{(2i - j)}{\sqrt{5}}$

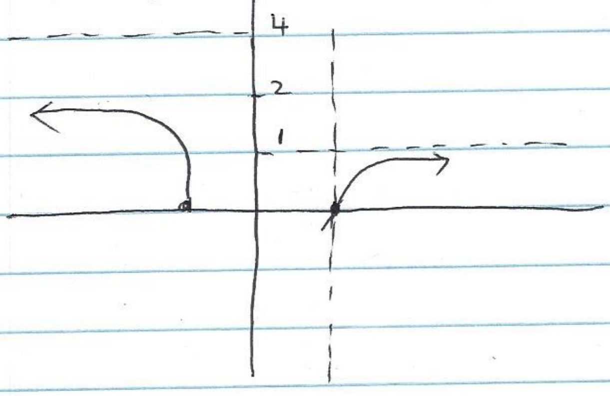
$= \boxed{\dfrac{-1}{\sqrt{5}}(2\underset{\sim}{i} - \underset{\sim}{j})}$

(ii) $\underset{\sim}{a} \cdot \underset{\sim}{b} = |a||b| \cos\theta$

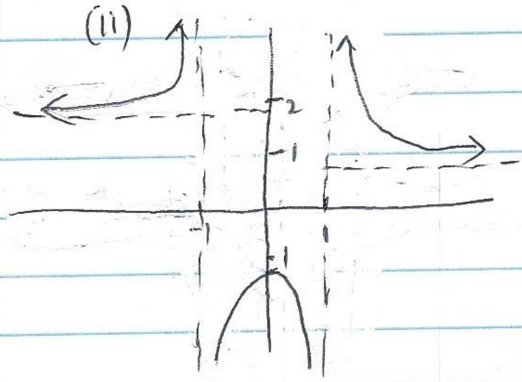
$\dfrac{-1}{\sqrt{10}\sqrt{5}} = \cos\theta$

$\boxed{\theta = 98°8'}$

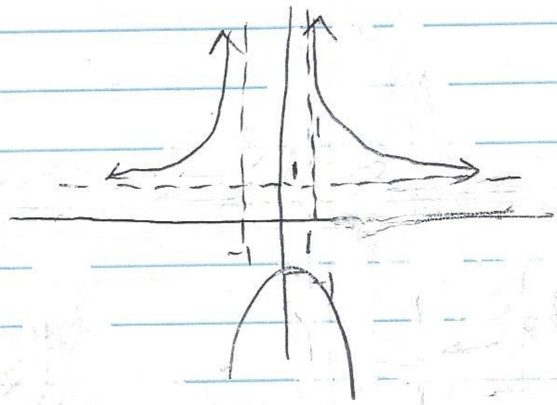
b) (i) $y = \sqrt{f(x)}$



(ii)



$y = \dfrac{1}{f(x)}$



$y = \dfrac{1}{f(|x|)}$

(c) Equation of parabola.

$$y = ax^2$$

$$h = ab^2$$

$$a = \frac{h}{b^2}$$

$$\therefore y = \frac{h}{b^2} x^2. \implies x^2 = \frac{b^2 y}{h}$$

$$V_{PARAB.} = \pi \int_0^h \frac{b^2 y}{h} \, dy$$

$$= \frac{\pi b^2}{h} \left[\frac{y^2}{2} \right]_0^h$$

$$= \frac{\pi b^2}{h} \left[\frac{h^2}{2} \right]$$

$$= \frac{\pi b^2 h}{2}.$$

$$V_{CYL} = \pi r^2 h.$$

$$= \pi b^2 h.$$

$$\therefore V_{PARAB} = \frac{1}{2} V_{CYL}.$$

d) $$\int \frac{1}{1+4t^2} \, dt.$$

$$= \int \frac{1}{4(\frac{1}{4}+t^2)} \, dt$$

$$= \frac{1}{4} \int \frac{1}{\frac{1}{4}+t^2} \, dt$$

$$= \frac{1 \times 2}{4} \tan^{-1} \frac{t}{\frac{1}{2}}$$

$$= \frac{1}{2} \tan^{-1} 2t + C.$$

e) $\int_0^3 \frac{(x^2+2x)}{\sqrt{1+x}}\,dx$ $\quad u^2 = 1+x.$ $\quad x=0, u=1$

$u^2-1 = x$ $\quad x=3, u=2$

$dx = 2u\,du$

$\int_0^3 \frac{x^2+2x+1^2-1}{\sqrt{1+x}}\,dx$

$\int_0^3 \frac{(x+1)^2-1}{\sqrt{1+x}}\,dx.$

$\int_1^2 \frac{u^4-1}{\cancel{u}}\,(2\cancel{u}du)$

$2\int_1^2 (u^4-1)\,du$

$= 2\left[\frac{u^5}{5} - u\right]_1^2$

$= 2\left[\frac{32}{5} - 2 - \frac{1}{5} + 1\right]$

$= \frac{26}{5} \times 2$

$= \frac{52}{5}$

(13) a) (i) $\cos\theta - \sqrt{3}\sin\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$R\sin\alpha = 1$ — ①

$R\cos\alpha = -\sqrt{3}$ — ②

$R^2 = 1+3$

$R = 2 \quad R > 0.$

$\tan\alpha = \frac{1}{-\sqrt{3}}$

α in 2nd quad

$\therefore \alpha = \pi - \pi/6$

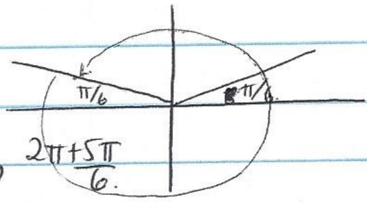
$\alpha = 5\pi/6$

$\therefore \cos\theta - \sqrt{3}\sin\theta = 2\sin(\theta + \pi/6)$

(8) (ii) $2\sin(\theta + 5\pi/6) = 1$

$\sin(\theta + 5\pi/6) = \frac{1}{2}$

$(\theta + 5\pi/6) = 5\pi/6,\ 2\pi + \frac{\pi}{6},\ 2\pi + \frac{5\pi}{6}$



$\theta = 0,\ \frac{8\pi}{6},\ \frac{12\pi}{6}$

$0 \leq \theta \leq 2\pi$

$\frac{5\pi}{6} \quad \theta + \frac{5\pi}{6} \leq 2\pi + \frac{5\pi}{6}$

$\boxed{\theta = 0,\ \frac{4\pi}{3},\ 2\pi}$

b) $\underset{\sim}{a} = 3\underline{i} - m\underline{j} \qquad \underset{\sim}{b} = n\underset{\sim}{i} - 6\underline{j}$

If $\perp$ $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

$\begin{pmatrix} 3 \\ -m \end{pmatrix} \cdot \begin{pmatrix} n \\ -6 \end{pmatrix} = 0$

$3n + 6m = 0$

$n + 2m = 0$, — (1)

$|a| = \sqrt{9 + m^2} = 5$

$9 + m^2 = 25$

$m^2 = 16$

$m = \pm 4$

$\therefore n + 8 = 0$

$\boxed{n = -8,\ m = 4}$

$n - 8 = 0$

OR $\boxed{n = 8,\ m = -4}$

(c) $\frac{dx}{dt} = 11$

(i) $\angle MCL = 120$.

$\therefore$ In ΔLCM, using cosine rule.

$7^2 = x^2 + y^2 - 2xy\cos 120$

$49 = x^2 + y^2 - 2xy(-\cos 60)$

$49 = x^2 + y^2 + 2xy \cdot \frac{1}{2}$

$49 = x^2 + y^2 + xy$.

(ii) Find $\frac{dy}{dt}$ when $x = 5$; $y = 3$.

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= -\frac{(2x+y)}{x+2y} \times 11$

$= -\left(\frac{10+3}{\cancel{5+6}}\right) \cancel{\times 11}$

$= \boxed{-13 \text{ m/s}}$.

When $x = 5$.

$49 = 25 + y^2 + 5y$.

$24 = y^2 + 5y$

$y^2 + 5y - 24 = 0$

$(y+8)(y-3) = 0$

$\therefore y = 3$ as $y > 0$

(iii) $\frac{\sin\theta}{y} = \frac{\sin 120}{7}$

$\sin\theta = \frac{y\sin 60}{7}$

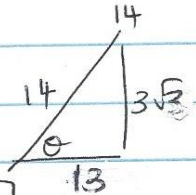
$\boxed{\sin\theta = \frac{y\sqrt{3}}{14}}$

$y = \frac{14\sin\theta}{\sqrt{3}}$

$\frac{dy}{d\theta} = \frac{14}{\sqrt{3}}\cos\theta$.

When $x = 5, y = 3$.

$\sin\theta = \frac{\sqrt{3} \cdot 3}{14}$



(iv) $\frac{d\theta}{dt} = \frac{d\theta}{dy} \times \frac{dy}{dt}$

$= \frac{\sqrt{3}}{14\cos\theta} \times -13$.

? $\frac{d\theta}{dt} = \frac{\sqrt{3}}{\cancel{14} \times \frac{13}{\cancel{14}}} \times -13 = \boxed{99°/s}$.

d) Step1 : Show true for $n=1$

$LHS = a$

$RHS = \frac{a(\cancel{a^2-1})}{\cancel{(a+1)(a-1)}}$

$= a$

$LHS = RHS \therefore$ True for $n=1$

Step2 : Assume true for $n=k$

ie. $a + a^3 + a^5 + \dots + a^{2k-1} = \frac{a(a^{2k}-1)}{(a+1)(a-1)}$

Step3 : Prove true for $n=k+1$

ie. $a + a^3 + a^5 + \dots + a^{2k-1} + a^{2k+1} = \frac{a(a^{2k+2}-1)}{(a+1)(a-1)}$

$LHS = \frac{a(a^{2k}-1)}{(a+1)(a-1)} + a^{2k+1}$

$= \frac{a(a^{2k}-1) + (a^2-1)(a^{2k+1})}{(a+1)(a-1)}$

$= \frac{a^{2k+1} - a + a^{2k+3} - a^{2k+1}}{(a+1)(a-1)}$

$= \frac{a^{2k+3} - a}{(a+1)(a-1)}$

$= \frac{a(a^{2k+2}-1)}{(a+1)(a-1)}$ $\therefore$ True for $n=k+1$

$= RHS$

Step4 : By principle of mathematical induction true for all $n \geq 1$ $n \in Z$

(14) a) Consider $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$

Let $x = 1$.

$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n$.

d) Let $x = \pi/4$

$$\sqrt{\frac{1-\sin \pi/4}{1+\sin \pi/4}} = \frac{1-\tan \pi/8}{1+\tan \pi/8}$$

$$\sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}} = \frac{\tan \pi/4 - \tan \pi/8}{1+\tan \pi/4 \tan \pi/8}.$$

$$\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \tan(\pi/4 - \pi/8)$$

$$\sqrt{\frac{(\sqrt{2}-1)^2}{2-1}} = \tan \pi/8$$

$$\sqrt{2}-1 = \tan \pi/8.$$

c) $\overrightarrow{AC} = \underline{a} + \underline{b}$

$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AC}$

$= \frac{1}{2}(\underline{a}+\underline{b})$

Sim, $\overrightarrow{MC} = \frac{1}{2}(\underline{a}+\underline{b})$

$\overrightarrow{BM} = \overrightarrow{BC} + \overrightarrow{CM}$

$= \underline{b} - \frac{1}{2}(\underline{a}+\underline{b})$

$= \frac{1}{2}(\underline{b}-\underline{a})$

$|\overrightarrow{AM}|^2 = \frac{1}{2}(\underline{a}+\underline{b}) \cdot \frac{1}{2}(\underline{a}+\underline{b})$

$= \frac{1}{4}(\underline{a}\cdot\underline{a} + 2\,\underline{a}\cdot\underline{b} + \underline{b}\cdot\underline{b})$

$= \frac{1}{4}|\underline{a}+\underline{b}|^2$

Sim, $|\overrightarrow{MC}| = \frac{1}{4}|\underline{a}+\underline{b}|^2$

$= \frac{1}{4}|\overrightarrow{AC}|^2$

$|BM|^2 = \frac{1}{4}[(\underline{b}-\underline{a})\cdot(\underline{b}-\underline{a})]$

$= \frac{1}{4}(\underline{b}\cdot\underline{b} - 2\underline{a}\cdot\underline{b} + \underline{a}\cdot\underline{a})$

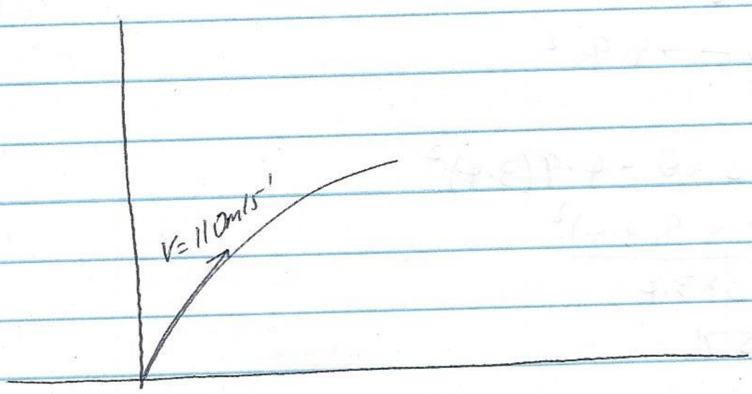
$= \frac{1}{4}|\underline{b}-\underline{a}|^2$

$= \frac{1}{4}|\overrightarrow{AC}|^2$ $\quad |\underline{b}+\underline{a}|^2 = |\underline{b}-\underline{a}|^2$ as $2\underline{a}\cdot\underline{b} = 0$

as $\underline{a} \perp \underline{b}$

$\therefore |BM| = |AM| = |MC|$.

d)



$s = (110t\cos\theta)\underline{i} + (110t\sin\theta - 4.9t^2)\underline{j}$

(i) $x = 110t\cos\theta \qquad y = 110t\sin\theta - 4.9t^2$

Horiz - Range $y=0$.

$0 = 110t\sin\theta - 4.9t^2$

$0 = t(110\sin\theta - 4.9t)$

$t=0, \quad t = \frac{110\sin\theta}{4.9}$.

Subt into x.

$x = 110\left(\frac{110\sin\theta}{4.9}\right)\cos\theta$

$x = 110\times 55 \times \frac{2\sin\theta\cos\theta}{4.9}$.

$x = 110\times\frac{1}{2}\times 110 \times \frac{\sin 2\theta}{4.9}$

$= \boxed{\frac{12100\sin 2\theta}{9.8}}$ show

(ii) $400 < \frac{12100\sin 2\theta}{9.8} < 450$.

$\sin 2\theta < 0.36 \qquad \sin 2\theta > 0.32$.

Consider $\sin 2\theta < 0.36$. $\qquad 2\theta > 18.9$

$2\theta < 21°22'$, $\qquad \theta > 9°27'$

$\theta < 10°41'$

$\therefore \boxed{9°27' < \theta < 10°41'}$

(iii) $t = 3.4,\ y = 8$

$y = 110\sin\theta\, t - 4.9t^2$

$8 = 110 \times 3.4 \sin\theta - 4.9(3.4)^2$

$\sin\theta = \dfrac{8 + 4.9(3.4)^2}{110 \times 3.4}$

$\theta = 9°57'$

Time of flight from (a)

$t = \dfrac{110\sin\theta}{4.9}$

Sub $\theta = 9°57'$

$t = \dfrac{110\sin 9°57'}{4.9}$

Sub into $x = 110t\cos\theta$

$= 110\left(\dfrac{110\sin 9°57'}{4.9}\right)\cos 9°57'$

$= 420.397$

which is in range of $400 \sim 450$.

$\therefore$ If the ball hadn't hit the drone, it would have made the green.