



NORTH SYDNEY BOYS HIGH SCHOOL

1998

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Question 1 (start a new page)

Differentiate $\ln\sqrt{1-x^2}$.

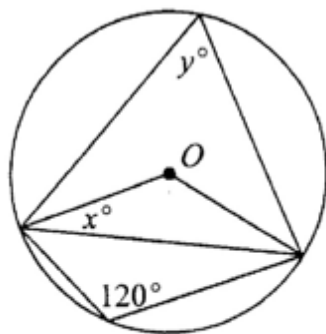
Find the coordinates of the point P that divides the interval AB externally in the ratio $3:1$, where A and B are the points $(-4, -6)$ and $(6, -1)$ respectively.

Show that the line $5x - 12y - 6 = 0$ is a tangent to the circle $(x+2)^2 + (y-3)^2 = 16$.

Use the table of standard integrals to find $\int \frac{dx}{\sqrt{x^2+16}}$.

Use the substitution $u = 1+x$ to evaluate $\int_{-1}^3 x\sqrt{1+x} dx$.

Question 2 (start a new page)



O is the centre of the circle. Find the values of x and y .

Solve

(i) $|2x - 1| \leq 5$;

(ii) $\frac{x+2}{x} < 3$.

Evaluate $\int_0^{\frac{\pi}{4}} \sin^2 \theta d\theta$.

(i) Show that $\frac{d}{dx}(\tan^3 x) = 3 \sec^4 x - 3 \sec^2 x$.

(ii) Hence evaluate $\int_0^{\frac{\pi}{3}} \sec^4 x dx$.

Question 3 (start a new page)

Consider the function $y = 1 - 3 \cos 2x$.

- (i) Write down the period of the function.
- (ii) Write down the amplitude of the function.
- (iii) Sketch the curve $y = 1 - 3 \cos 2x$ where $-\pi \leq x \leq \pi$.
- (iv) How many solutions exist in the domain $-\pi \leq x \leq \pi$ to the equation $1 - 3 \cos 2x = 5$? Justify your answer.

A bowl of water heated to 100°C is placed in a coolroom where the temperature is maintained at -5°C . After t minutes, the temperature T° of the water is changing so that $\frac{dT}{dt} = -k(T + 5)$.

- (i) Prove that $T = Ae^{-kt} - 5$ satisfies this equation, and find the value of A .
- (ii) After 20 minutes, the temperature of the water has fallen to 40°C . How long, to the nearest minute will the water need to be in the coolroom before ice begins to form? [ie. the temperature falls to 0°C]

State the domain and range of the function $y = 2 \sin^{-1}\left(\frac{x}{3}\right)$.

Hence sketch the curve.

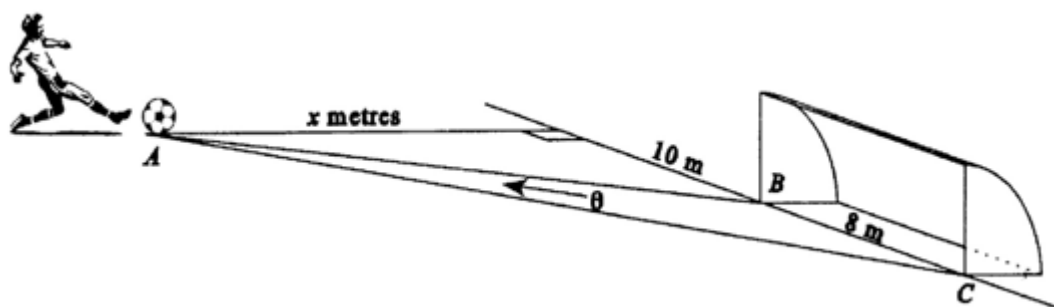
Question 4 (start a new page)

Solve for n : $2 \times {}^nC_4 = 5 \times {}^nC_2$.

The area between the curve $y = \ln x$, the x -axis, and the lines $x = 2$ and $x = 4$ is rotated about the x -axis. Use Simpson's rule with 3 function values to estimate the volume of the solid so formed. Give your answer correct to two decimal places.

- (i) Show that the equation $\ln x + x^2 - 4x = 0$ has a root lying between $x = 3$ and $x = 4$.
- (ii) By taking $x = 4$ as a first approximation, use one application of Newton's method to obtain a better approximation for the root.

Question 5 (start a new page)



A soccer player A is x metres from a goal line of a soccer field. He takes a shot at the goal BC , with the ball not leaving the ground.

- (i) Show that the angle θ within which he must shoot is given by

$$\theta = \tan^{-1}\left(\frac{8x}{180 + x^2}\right)$$

when he is 10 metres to one side of the near goalpost and 18 metres to the same side of the far post.

- (ii) Find the value of x that makes this angle a maximum.

A particle moves in a straight line such that its velocity V m/s is given by

$$v = 2\sqrt{2x - 1} \text{ when it is } x \text{ metres from the origin. If } x = \frac{1}{2} \text{ when } t = 0, \text{ find}$$

- (i) the acceleration;
 (ii) an expression for x in terms of t ;

A monic polynomial of degree 4 has exactly two zeros, $x = 2$ and $x = -2$, and is *even* [ie. $P(x) = P(-x)$]. This polynomial has a value of 55 when $x = 3$. Find $P(x)$.

Question 6 (start a new page)

How many four-digit numbers can be formed from the digits 1, 2, 3, 4

- (i) without repetition;
 (ii) with repetition ?

The tangent to the parabola $x^2 = 4ay$ at the variable point $P(2ap, ap^2)$ cuts the x -axis at Q and the y -axis at R .

- (i) Show that the equation of the tangent at P is $y = px - ap^2$.
 (ii) Find the coordinates of Q and R .

Show that $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{4}{7}\right)$.

The displacement x metres of a particle from the origin is given by $x = 5 \cos\left(3t - \frac{\pi}{6}\right)$ where t is the time elapsed in seconds. Show that $\ddot{x} = -9x$, and describe the motion

Question 7 (start a new page)

Prove that $\frac{\cos A - \sin A}{\cos A + \sin A} = \frac{\cos 2A}{1 + \sin 2A}$.

Solve for $0 \leq x \leq 2\pi$:

$$4 \cos x - 3 \sin x = -1.$$

Give your solutions correct to two decimal places.

Use the remainder theorem to find one factor of $P(x) = x(x - 1) - a(a - 1)$.
Hence factorize $P(x)$.

A stone is thrown from the top of a cliff, 70 metres above sea level, with a speed of 65 m/s, at an angle of elevation of $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$ to the horizontal. Find

- (i) the distance from the base of the cliff to the point where the stone hits the sea.
- (ii) the velocity at the instant of hitting the water.

End of Exam