NORTH SYDNEY GIRLS HIGH SCHOOL

2015 TRIAL HSC EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I Pages 2 - 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 7 – 14

60 Marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

NAME:____________________ **TEACHER:**____________________

STUDENT NUMBER:____________________

QUESTION	MARK
1–10	/10
11	/15
12	/15
13	/15
14	/15
TOTAL	/70

Section I

10 marks
Attempt Questions 1−10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 What is the value of $\lim_{x\to\infty}\frac{2x^3-3x^2+5x-7}{4x-x^3}$

(A) $\frac{1}{2}$

(B) 2

(C) $-\frac{1}{2}$

(D) –2

2 Which of the following is equivalent to $\sqrt{3}\sin\theta-\cos\theta$?

(A) $2\sin\left(\theta+\frac{\pi}{6}\right)$

(B) $2\sin\left(\theta-\frac{\pi}{6}\right)$

(C) $2\sin\left(\theta+\frac{\pi}{3}\right)$

(D) $2\sin\left(\theta-\frac{\pi}{3}\right)$

3 A curve is defined by $x = 2t$ and $y = \log_e t$.

Which of the following is the value of $\frac{dy}{dx}$ at the point $(2, 0)$?

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 1

(D) 2

4 What is the value of $\lim_{x \to 0} \frac{2\sin 2x}{3\tan 3x}$?

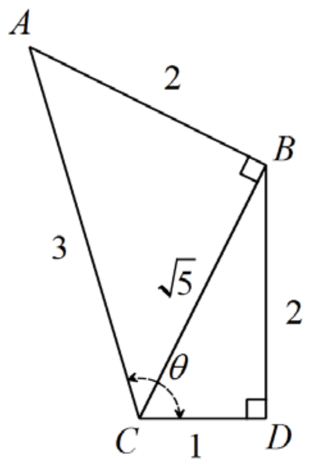
(A) $\frac{2}{3}$

(B) $\frac{3}{2}$

(C) $\frac{4}{9}$

(D) 1

5 What is the value of $\sin\theta$, given that $\angle ACD = \theta$ in the diagram below?



(A) $\dfrac{2}{3\sqrt{5}} + \dfrac{2}{3}$

(B) $\dfrac{2}{\sqrt{5}} + \dfrac{\sqrt{5}}{3}$

(C) $\dfrac{2}{\sqrt{5}} + \dfrac{2}{3}$

(D) $\dfrac{4}{3\sqrt{5}} + \dfrac{1}{3}$

6 What is the correct expression for $\displaystyle\int \frac{dx}{\sqrt{4-x^2}}$?

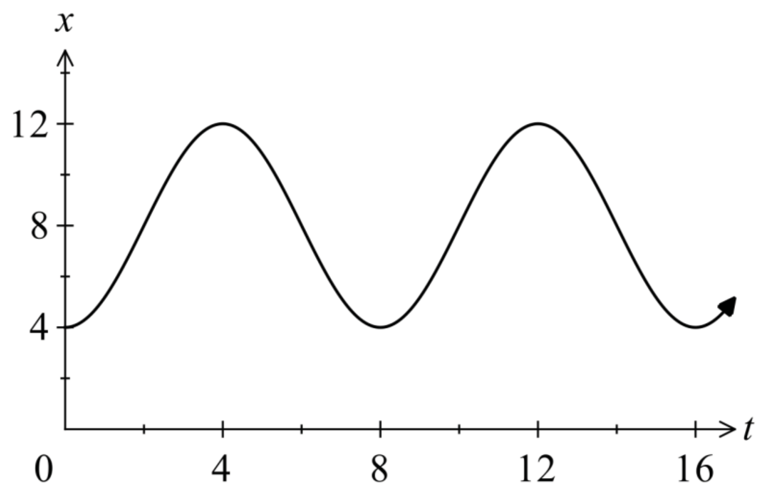
(A) $\sin^{-1}\left(\dfrac{x}{4}\right) + c$

(B) $\sin^{-1}\left(\dfrac{x}{2}\right) + c$

(C) $\dfrac{1}{4}\sin^{-1}\left(\dfrac{x}{4}\right) + c$

(D) $\dfrac{1}{2}\sin^{-1}\left(\dfrac{x}{2}\right) + c$

7 The graph below represents the depth of water in a channel (in metres) as it changes over time (in hours).



Which of the following is NOT true?

(A) The centre of motion is at 8 m

(B) The period of oscillation is 8 hours

(C) The amplitude is 8 m

(D) The rate of change in the depth of water is the fastest when the depth is 8 m

8 Which of the following are the roots of the equation $x^3 + 4x^2 + x - 6 = 0$?

(A) $-1,\ 3,\ 2$

(B) $1,\ -3,\ -2$

(C) $1,\ 1,\ -6$

(D) $-1,\ -1,\ 6$

9 What is the value of $\cos^{-1}(\sin\alpha)$ where $\frac{\pi}{2} < \alpha < \pi$?

(A) $\pi - \alpha$

(B) $\frac{\pi}{2} - \alpha$

(C) $\alpha - \frac{\pi}{2}$

(D) $\frac{\pi}{2} + \alpha$

10 In solving $\dfrac{x-1}{\sqrt{x}} > \dfrac{2}{x-1}$ within the natural domain, three students obtain the following inequalities.

Student I: $(x-1)^2 > 2\sqrt{x}$

Student II: $(x-1)^3 > 2(x-1)\sqrt{x}$

Student III: $(x-1)^3\sqrt{x} > 2x(x-1)$

Which students will obtain the correct solution to the original inequality?

(A) Student I only

(B) Student II only

(C) Student III only

(D) Student II and Student III

Section II

Total marks − 60
Attempt Questions 11−14
Allow about 1 hour 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 to 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Differentiate $x\cos^{-1}(ex)$ with respect to x. **2**

(b) Find $\int \sin^2 3x\,dx$ **2**

(c) The point P divides the interval joining $A(-1,5)$ to $B(2,3)$ externally in the ratio $4:3$. Find the coordinates of P. **2**

(d) Find the size of the acute angle between the line $y=2x$ and the curve $y=x^2$ at the point of intersection $(2,4)$. **3**

Give your answer to the nearest degree.

(e) Use the substitution $u=\sqrt{x}$ to determine $\int_{\frac{1}{3}}^{1} \frac{dx}{(1+x)\sqrt{x}}$. **3**

Give your answer in exact form.

(f) (i) Sketch the graph of $y=\sin\left(\frac{\pi x}{2}\right)$ for the domain $-3 \le x \le 3$. **1**

(ii) Hence, or otherwise, find for what positive values of m, the equation $\sin\left(\frac{\pi x}{2}\right)=mx$, has exactly three solutions. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Angela is preparing food for her baby and needs to use cooled boiled water. The equation $y = Ae^{kt}$ describes how the water cools, where t is the time in minutes, A and k are constants and y is the difference between the water temperature and the room temperature at time t, both measured in degrees Celsius.

The temperature of the water when it boils is 100°C and the room temperature is a constant 23°C.

(i) Find the value of A. **1**

(ii) The water cools to 88°C after 5 minutes. Find the value of k correct to three significant figures. **2**

(iii) Angela can prepare the food when the water has cooled to 50°C. How much longer must she wait? **2**

(b) A particle's displacement satisfies the equation $t = x^2 - 5x + 4$, where x is measured in cm and t is in seconds. Initially, the particle is 4 cm to the right of the origin.

(i) Show that the velocity is given by $v = \dfrac{1}{2x-5}$. **1**

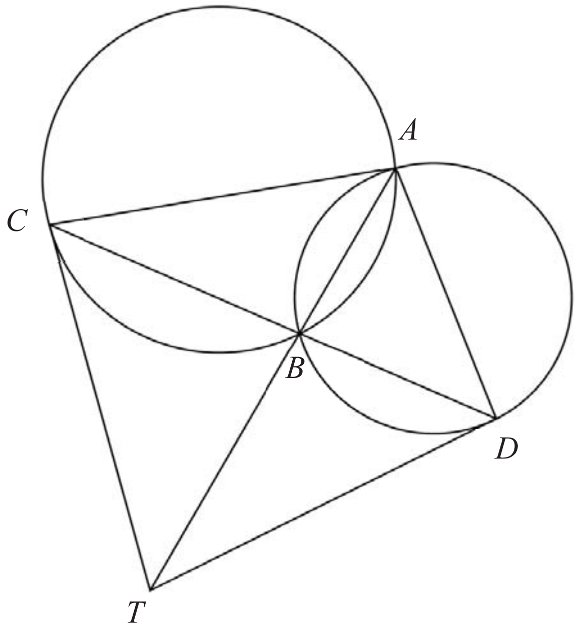
(ii) Find an expression for the acceleration, a in terms of x. **2**

(iii) Find the position of the particle 10 seconds after the start of the motion. **2**

(iv) Briefly describe the motion of the particle. **1**

Question 12 continues on page 9

(c) *BAC* and *BAD* are two circles such that the tangents at *C* and *D* meet at *T* on *AB* produced.



Copy or trace the diagram into your writing booklet.

If *CBD* is a straight line prove that:

(i) *TCAD* is a cyclic quadrilateral **3**

(ii) $TC = TD$ **1**

End of Question 12

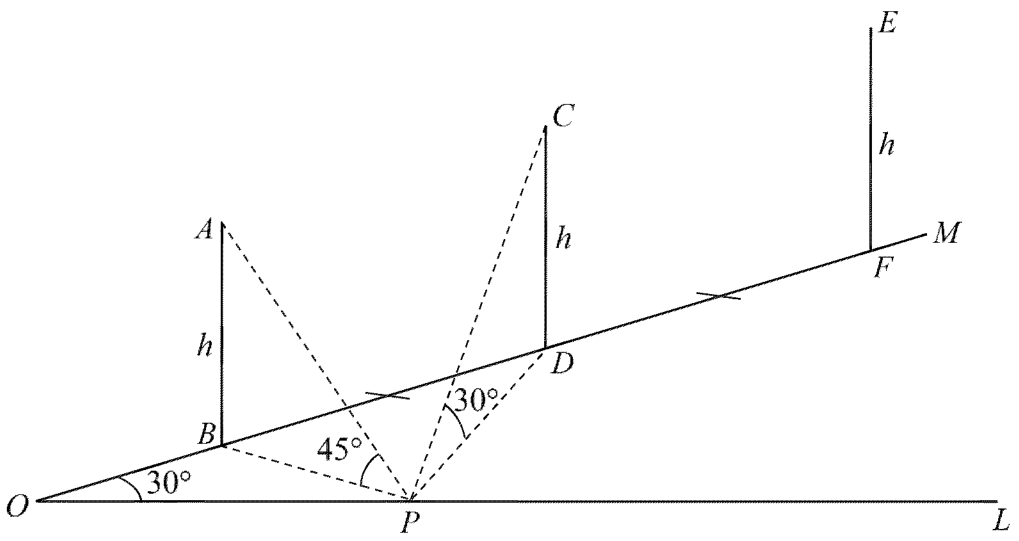
Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Fully factorise $P(x) = x^3 - x^2 - 8x + 12$. **3**

(ii) Hence, find any values of k, such that $Q(x) \geq 0$ for all real x, where $Q(x) = P(x)(2x - k)$. **1**

(b) In the diagram below, OL is a road that runs due east. OM is another road and intersects OL at 30°. Both roads are on flat ground. On OM there are three equally spaced vertical telegraph poles AB, CD and EF of equal height h m. The distance between adjacent poles is twice the height of the poles.

From an observer at P, the bearing of the first pole AB is 300°T. The angles of elevation of A and C from P are 45° and 30° respectively.



(i) Explain why triangle BDP is right angled. **2**

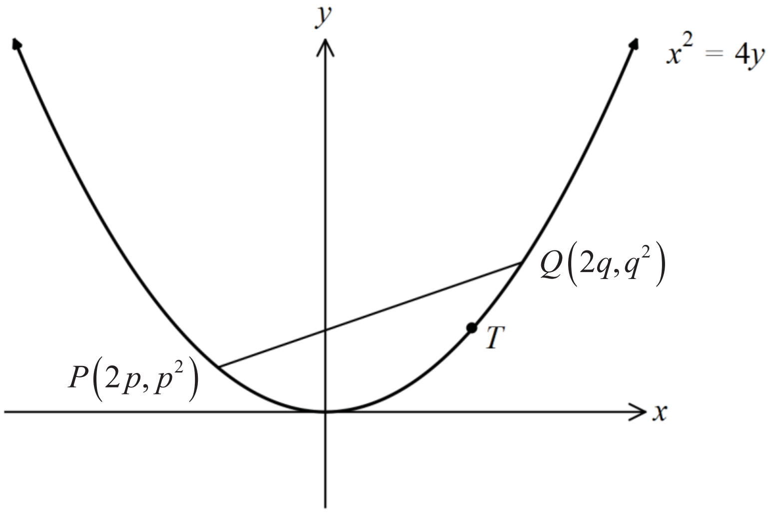
(ii) Deduce that $PF = h\sqrt{13}$. **3**

Question 13 continues on page 11

Question 13 (continued)

(c) Consider the parabola $x^2 = 4y$.

$P\left(2p, p^2\right)$ and $Q\left(2q, q^2\right)$ lie on the parabola.



(i) Find the equation of the chord *PQ*. **2**

(ii) Show that if *PQ* is a focal chord then $pq = -1$. **1**

(iii) $T\left(2t, t^2\right)$, $t > 0$ and $R\left(2r, r^2\right)$ are two other points on the parabola distinct from *P* and *Q*. **3**

If *TR* is also a focal chord and *P*, *T*, *Q* and *R* are concyclic, show that $p^2 + q^2 = t^2 + r^2$.

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) A particle is undergoing simple harmonic motion such that its displacement x centimetres from the origin after t seconds is given by :

$$x + 2 = 4\sin\left(2t + \frac{\pi}{3}\right).$$

(i) Between which two positions is the particle oscillating? **1**

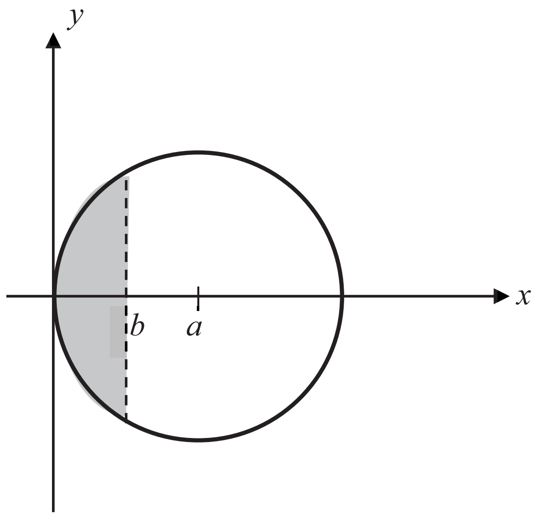
(ii) At what time does the particle first move through the origin in the positive direction? **3**

(b) Use the principle of mathematical induction to prove $3^n + 7 < 4^n$ for all integers $n \geq 3$. **3**

Question 14 continues on page 13

Question 14 (continued)

(c) Consider the region enclosed by the circle $(x-a)^2+y^2=a^2$ and the line $x=b$ shown in the diagram below, where $0<b<2a$.



(i) Show that the volume of the spherical cap formed by rotating this region around the x-axis is given by **2**

$$V=\frac{\pi b^2}{3}(3a-b) \text{ cubic units}$$

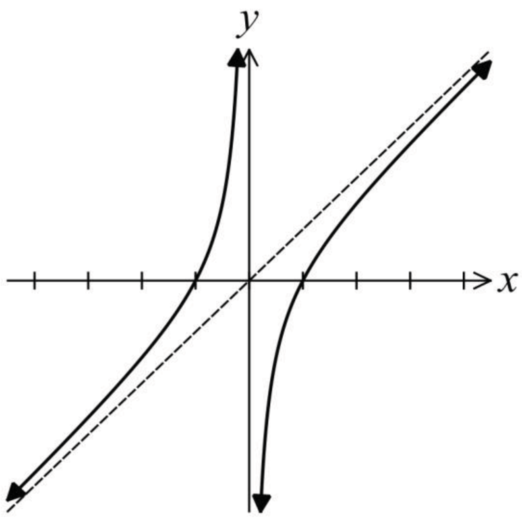
(ii) A spherical goldfish bowl of radius 10 cm is being filled with water at a constant rate of 75 cm^3 per minute. **2**

Using part (i) or otherwise, find the rate at which the water level in the bowl is rising when the bowl is half full of water.

Question 14 continues on page 14

Question 14 (continued)

(d) Consider the function $f(x) = x - \frac{1}{x}$ whose graph is shown below.



(i) By restricting the domain of the original function to $x > 0$, find the equation of $f^{-1}(x)$. **2**

(ii) Hence, without solving directly, find the value(s) of x for which $\frac{x-1}{\sqrt{x}} = 16$. Leave your answer in exact form. **2**

No marks will be awarded for solving the equation directly for x.

End of paper

Mathematics Extension 1 Trial HSC 2015 – Suggested Solutions

Section I

1. D

Degree of numerator and denominator is the same. The limit is the ratio of the leading coefficients ie $\frac{2}{-1}=-2$.

2. B

Using auxiliary angle method, this is o the form $R\sin\left(\theta-\alpha\right)$ where $R=\sqrt{\sqrt{3}^2+(-1)^2}=2$.

$\tan\theta=\frac{1}{\sqrt{3}}\Rightarrow\theta=\frac{\pi}{6}$.

3. B

Use parametric differentiation. $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$.

$\frac{dy}{dt}=\frac{1}{t}$; $\frac{dx}{dt}=2$; $\frac{dy}{dx}=\frac{1}{2t}$. At $\left(2,0\right)$, $t=1$; $\frac{dy}{dx}=\frac{1}{2}$.

4. C

$$\begin{aligned}\lim_{x\to0}\frac{2\sin 2x}{3\tan 3x}&=\frac{2}{3}\times\lim_{x\to0}\frac{\sin 2x}{\tan 3x}\\&=\frac{2}{3}\times\frac{2}{3}\times\lim_{x\to0}\frac{\sin 2x}{2x}\times\lim_{x\to0}\frac{3x}{\tan 3x}\\&=\frac{4}{9}\end{aligned}$$

5. A

$\sin\theta=\sin\left(B\hat{C}D+A\hat{C}B\right)$

$=\frac{2}{\sqrt{5}}.\frac{\sqrt{5}}{3}+\frac{1}{\sqrt{5}}.\frac{2}{3}=\frac{2}{3}+\frac{2}{3\sqrt{5}}$

using compound angle result

6. B using standard integrals table

7. C

The amplitude is the distance from the centre of motion to the extreme of motion which is 12 – 8 = 4.

8. B

Sum of roots = -4 and product of roots is 6.

9. C

$$\begin{aligned}\cos^{-1}\left(\sin\alpha\right)&=\cos^{-1}\left(\sin\left(\pi-\alpha\right)\right);\quad \pi-\alpha\ \text{acute}\\&=\cos^{-1}\left(\cos\left(\tfrac{\pi}{2}-\left(\pi-\alpha\right)\right)\right);\quad \tfrac{\pi}{2}-\left(\pi-\alpha\right)\ \text{acute}\\&=\cos^{-1}\left(\cos\left(\alpha-\tfrac{\pi}{2}\right)\right);\quad \alpha-\tfrac{\pi}{2}\ \text{acute}\\&=\alpha-\tfrac{\pi}{2}\end{aligned}$$

Alternately sub a second quadrant angle into your calculator and verify which option works.

10. D

As $\sqrt{x}>0$ it is not necessary to multiply by the square of $\sqrt{x}$ only by the square of $x-1$ as Student II has done. However by multiplying by the square of $\sqrt{x}$ Student III does not generate extra solutions because $x=0$ is not an admissible solution.

Mathematics Extension 1 Trial HSC 2015 – Suggested Solutions

Section II

Question 11

(a) Using chain rule,

$$\frac{d}{dx}\left(x\cos^{-1}(ex)\right)=1.\cos^{-1}(ex)+x\frac{-1}{\sqrt{1-(ex)^2}}\times e$$

$$=\cos^{-1}(ex)-\frac{ex}{\sqrt{1-e^2x^2}}$$

(b) Using double angle results,

$$\int\sin^2 3x\,dx=\int\left(\frac{1-\cos 6x}{2}\right)dx$$

$$=\frac{1}{2}\left(x-\frac{\sin 6x}{6}\right)+c$$

(c) Using a ratio of $-4:3$

$$P=\left(\frac{-4\times 2+3\times -1}{-4+3},\frac{-4\times 3+3\times 2}{-4+3}\right)=(11,-3)$$

(d) $y=2x; m_1=2$ and $y=x^2; y'=2x; m_2=4$ at $x=2$

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{2-4}{1+8}\right|=\frac{2}{9}$$

$\theta=13°$ (nearest degree)

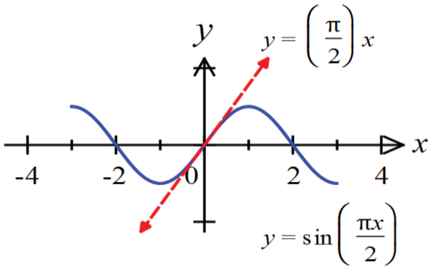
(e) $u=\sqrt{x}\Rightarrow\frac{du}{dx}=\frac{1}{2\sqrt{x}}\Rightarrow 2du=\frac{dx}{\sqrt{x}}$

$x=\frac{1}{3}\Rightarrow u=\frac{1}{\sqrt{3}}; x=1\Rightarrow u=1$

$$\int_{\frac{1}{3}}^{1}\frac{dx}{(1+x)\sqrt{x}}=\int_{\frac{1}{\sqrt{3}}}^{1}\frac{2.du}{1+u^2}=2\left[\tan^{-1}u\right]_{\frac{1}{\sqrt{3}}}^{1}$$

$$=2\left[\frac{\pi}{4}-\frac{\pi}{6}\right]=\frac{\pi}{6}$$

(f) (i)



(ii) The upper bound of *m* to ensure exactly three solutions is found by finding the gradient of the tangent at $x=0$ and is $\frac{\pi}{2}$. As we need positive values of *m*, then the required range of values for *m* is $0<m<\frac{\pi}{2}$.

Question 12

(a) (i) At $t = 0;\ y = 100 - 23 = 77$

$$77 = Ae^0 \Rightarrow A = 77$$

(ii) $t = 5;\ T = 88 - 23 = 65$

$$65 = 77e^{5k}$$

$$e^{5k} = \frac{65}{77}$$

$$5k = \ln\left(\frac{65}{77}\right)$$

$$k = \frac{1}{5}\ln\left(\frac{65}{77}\right) = -0.0339 \text{ (4dp)}$$

(iii) $50 - 23 = 77e^{kt}$

$$e^{kt} = \frac{27}{77}$$

$$kt = \ln\left(\frac{27}{77}\right)$$

$$t = \frac{1}{k}\ln\left(\frac{27}{77}\right)$$

$$t = 30.928 \approx 30\text{m } 55\text{s}$$

Therefore, she must wait another 25 min 55 sec.

(b) (i) $t = x^2 - 5x + 4$

$$\frac{dt}{dx} = 2x - 5$$

$$v = \frac{dx}{dt} = \frac{1}{2x-5}$$

(ii) $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$$a = \frac{d}{dx}\left(\frac{1}{2} \times \frac{1}{(2x-5)^2}\right)$$

$$= \frac{1}{\cancel{2}} \times \frac{-2}{(2x-5)^3} \times \cancel{2}$$

$$= \frac{-2}{(2x-5)^3}$$

(iii) When $t = 10$

$10 = x^2 - 5x + 4$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x = -1, 6$

Initially, $x = 4$ so $v = \dfrac{1}{2(4)-5} = \dfrac{1}{3} > 0$

So the particle is moving to the right. v can never be zero so the particle never turns around. So it can never be at $x = -1$. $\therefore x = 6$.

(iv) Initially the particle is 4 units to the right and moving to the right. the acceleration at this time is negative, so the particle is slowing down.

(c) (i)

Let $\angle TCB = \alpha$ and $\angle TDB = \beta$

$\angle TCB = \angle CAB = \alpha$ (Angle between tangent and chord equal to angle in alternate segment)

$\angle TDB = \angle DAB = \beta$ (Angle between tangent and chord equal to angle in alternate segment)

$\therefore \angle CAD = \angle CAB + \angle DAB = \alpha + \beta$ (1)

Now, $\angle CTD = 180 - (\alpha + \beta)$ (Angle sum of ΔCTD) (2)

$\therefore \angle CAD + \angle CTD = 180$ (adding (1) and (2))

$\therefore TCAD$ is a cyclic quadrilateral. (opposite angles are supplementary)

(ii)

$\angle CAT = CDT$ (angles in the same segment in circle $TCAD$)

$\therefore \alpha = \beta$

This means that $\angle TCB = \angle TDB$

$TC = TD$ (equal sides opposite equal angles in ΔTCD)

Question 13

(a) (i) $P(2) = 0 \Rightarrow (x-2)$ is a factor

$P(x) = (x-2)(x^2 + x - 6)$ by inspection. [Alternately use long division].

$P(x) = (x-2)(x+3)(x-2)$

$P(x) = (x-2)^2(x+3)$

(ii) $Q(x) = P(x)(2x-k)$

$Q(x) = (x-2)^2(x+3)(2x-k)$

$= 2(x-2)^2(x+3)\left(x - \dfrac{k}{2}\right)$

If $Q(x) \geq 0$ for all x, then $\left(x - \dfrac{k}{2}\right) \equiv (x+3)$

Or $k = -6$

(b) (i) $BP = h\cot 45° = h$ and

$DP = h\cot 30° = h\sqrt{3}$

$BP^2 + DP^2 = h^2 + 3h^2 = 4h^2$

$= (2h)^2 = BD^2$

$\therefore \Delta BDP$ is right angled at P (converse of Pythagoras Theorem)

(ii) $\angle DBP = \tan^{-1}\left(\frac{\sqrt{3}\cancel{h}}{\cancel{h}}\right) = 60°$

In ΔBPF, $BF = 2h + 2h = 4h$

Using the cosine rule,

$PF^2 = BP^2 + BD^2 - 2.BP.BD.\cos P\hat{B}F$

$= h^2 + (4h)^2 - \cancel{2}.h.4h.\frac{1}{\cancel{2}}$

$= 17h^2 - 4h^2 = 13h^2$

$\therefore PF = \sqrt{13}h$ as required

(c) (i) $m_{PQ} = \frac{q^2 - p^2}{2q - 2p} = \frac{\cancel{(q-p)}(q+p)}{2\cancel{(q-p)}} = \frac{p+q}{2}$

Equation of PQ is:

$y - p^2 = \frac{p+q}{2}(x - 2p)$

$2y - \cancel{2p^2} = (p+q)x - \cancel{2p^2} - 2pq$

$y = \frac{p+q}{2}x - pq$

(ii) Focus is $(0,1)$. Sub into eqn of chord PQ

$1 = 0 - pq$ or $pq = -1$

(iii) PQ and TR are chords of circle $PTQR$ and intersect at the focus S.

$PS \times SQ = TS \times SR$

(product of intercepts of intersecting chords)

But $PS = p^2 + 1$ using the locus definition; distance from focus = distance from directrix

Similarly, $QS = q^2 + 1$ etc

$\therefore (p^2+1)(q^2+1) = (t^2+1)(r^2+1)$

$p^2q^2 + p^2 + q^2 + 1 = t^2r^2 + t^2 + r^2 + 1$

But $pq = -1 \Rightarrow p^2q^2 = 1$ and $tr = -1 \Rightarrow t^2r^2 = 1$

$\therefore p^2 + q^2 = t^2 + r^2$

Question 14

(a) (i) Centre of motion is – 2 . Amplitude is 4. Hence, oscillates between – 6 and + 2.

(ii) Solving for $x = 0$

$$4\sin\left(2t + \frac{\pi}{3}\right) = 2$$

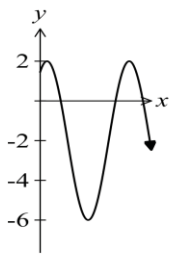
$$\sin\left(2t + \frac{\pi}{3}\right) = \frac{1}{2}$$

$$2t + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \ldots$$

$$2t = \cancel{-\frac{\pi}{6}}, \frac{\pi}{2}, \frac{11\pi}{6}, \ldots \text{ as } t > 0$$

$$t = \frac{\pi}{4}, \frac{11\pi}{12}, \ldots$$

Graph of displacement is as below:



Hence, crosses the origin in a positive direction the second time ie at $t = \frac{11\pi}{12}$. Alternately, use $v > 0$ to find when it crosses in a positive direction.

(b) To prove $3^n + 7 < 4^n$ for $n \geq 3$

Test if true for $n = 3$:
LHS = $3^3 + 7 = 27 + 7 = 34$ and RHS = $4^3 = 64$
LHS < RHS, hence true for $n = 3$.

Assume the result is true for some $n = k$ where $k \geq 1; k \in \mathbb{Z}^+$
ie assume that $3^k + 7 < 4^k$

Prove true for $n = k + 1$ ie prove that $3^{k+1} + 7 < 4^{k+1}$
$3^k + 7 < 4^k$ by assumption. Multiply both sides by 3.

$$3^{k+1} + 21 < 3.4^k$$

$$3^{k+1} + 7 + 14 < 3.4^k$$

$$3^{k+1} + 7 < 3.4^k$$

$$3^{k+1} + 7 < 4.4^k$$

$$3^{k+1} + 7 < 4^{k+1}$$

Hence, the proposition is true for all $n \geq 3$ by Mathematical Induction.

(c) (i) $y^2 = a^2 - (x-a)^2$

$$V = \pi\int_0^b y^2\,dx$$

$$= \pi\int_0^b \left(a^2 - (x-a)^2\right)dx$$

$$\pi\int_0^b \left(\cancel{a^2} - x^2 + 2ax - \cancel{a^2}\right)dx$$

$$= \pi\left[\frac{-x^3}{3} + \cancel{2}\frac{ax^2}{\cancel{2}}\right]_0^b$$

$$= \pi\left[\left(\frac{-b^3}{3} + ab^2\right) - (0)\right]$$

$$= \pi\left[\frac{-b^3}{3} + \frac{3ab^2}{3}\right]$$

$$V = \frac{\pi b^2}{3}(3a-b)$$

(ii) $a = 10$ and $\frac{dV}{dt} = 75$

$$V = \frac{\pi b^2}{3}(30-b) = 10\pi b^2 - \frac{\pi}{3}b^3$$

$$\frac{dV}{db} = 20\pi b - \frac{\pi}{\cancel{3}} \times \cancel{3}b^2$$

$$\frac{dV}{dt} = \frac{dV}{db} \times \frac{db}{dt} \text{ (Chain Rule)}$$

$$\frac{db}{dt} = \frac{dV}{dt} \div \frac{dV}{db}$$

$$\frac{db}{dt} = 75 \div \left(20\pi b - \pi b^2\right)$$

When the bowl is half full, $b = 10$

$$\frac{db}{dt} = 75 \div (200\pi - 100\pi) = \frac{75}{100\pi} = \frac{3}{4\pi}$$

(d) (i) $y = x - \frac{1}{x}$

For the inverse: $x = y - \frac{1}{y}$

Multiply by y

$$xy = y^2 - 1$$

$$y^2 - xy - 1 = 0$$

$$y = \frac{x \pm \sqrt{x^2+4}}{2}$$

As $y > 0$ for the inverse, then

$$y = \frac{x + \sqrt{x^2+4}}{2}$$

(ii) $\frac{x-1}{\sqrt{x}}=16 \Rightarrow \sqrt{x}-\frac{1}{\sqrt{x}}=16$

$f\left(\sqrt{x}\right)=16$

$\sqrt{x}=f^{-1}(16)=\frac{16+\sqrt{16^2+4}}{2}=8+\sqrt{65}$

$x=\left(8+\sqrt{65}\right)^2=129+16\sqrt{65}$

End of solutions