PENRITH HIGH SCHOOL
2006
TRIAL HSC
MATHEMATICS EXTENSION 1

Question(1) 12 marks

a) Find the coordinates of the point P that divides the interval joining $(-6, 8)$ and $(10, 12)$ internally in the ratio $3 : 1$.

b) Solve $\frac{x}{x+4} \leq 2$.

c) Evaluate $\lim\limits_{x \to \infty} \frac{x(2x-4)}{4x^2}$.

d) Give the domain and range of the function $f(x) = 5e^{2-x}$.

e) Use the substitution $u = 9 - x$ to evaluate $\int_0^9 \frac{x}{\sqrt{9-x}}\, dx$.

Question(2) 12 marks

a) Sketch the graph of $y = \sin^{-1} 2x$.

b) Find $\frac{d}{dx}(2x \cos^{-1} x)$.

c) Evaluate $\int_0^{\sqrt{3}} \frac{1}{9+x^2}\, dx$.

d) Find $\int \cos^2 4x\, dx$.

e) Solve for x in the interval $0 \leq x \leq 2\pi$, the equation $\sin 2x = \tan x$.

Question(3) **12 marks**

a) Find the acute angle between the two lines $y = 2x - 4$ and $x - 3y = 7$. Give your answer to the nearest degree.

b)

i. Use mathematical induction to prove that $\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots + \frac{1}{2^{n+2}} = \frac{1}{4} - \frac{1}{2^{n+2}}$ for all positive integers n.

ii. Hence state value of $\lim_{n \to \infty}\left(\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots + \frac{1}{2^{n+2}}\right)$.

c) A particle moves in a straight line and its position at time t is given by $x = 5\sin(3t + \frac{\pi}{6})$.

i. Show that the particle is undergoing simple harmonic motion.
ii. Find the period of the motion.
iii. When does the particle first reach a maximum speed?

Question(4) **12 marks**

a) The function $f(x) = \log_e(x+2) - x^2$ has a zero near $x = 1$. Taking $x = 1$ as a first approximation, use one application of Newton's method to find a second approximation to the zero. Give your answer correct to three decimal places.

b) The cubic polynomial $P(x) = 2x^3 - x^2 - 13x - 6$.

i. Use the Factor Theorem to show that $2x + 1$ is a factor of $P(x)$.
ii. Hence find all the factors of $P(x)$.
iii. Hence solve: $P(x) > 0$.

c) The equation $2x^3 + 2x^2 - 11x - 12 = 0$ has roots α, β and γ. Find the value of:

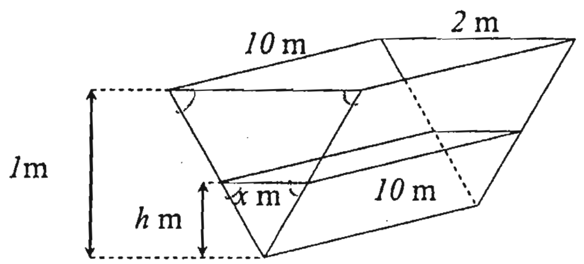
i. $\alpha + \beta + \gamma$.

ii. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.

d) It is known that the two roots of the equation $x^2 + bx + c$ differ by 2. Show that $b^2 = 4(1 + c)$.

Question(5) 12 marks

a)

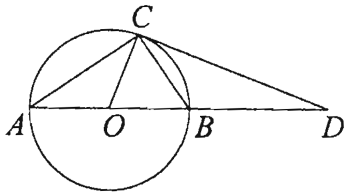


A water through is in the shape of a triangular prism which is open at the top. Water is being poured into the trough at a constant rate of 1.5 cubic metres per second. Its top measures 10 metres by 2 metres and its triangular end has a vertical height of 1 metre. When the water depth is h metres, the water surface measures x metres by 10 metres.

i. Show that when the water depth is h metres the volume $V\,\text{m}^3$ of water in the trough is given $V = 10h^2$.

ii. Find the rate at which the depth of water is changing when $h = 0.5$.

b)



AB is a diameter of a circle with centre O. DC is a tangent with point of contact C.

i. Explain why $\angle CAO = \angle BCD$.

ii. Prove $\angle BOC = 2\angle BCD$.

c) Room temperature is 26°C. A cup of coffee at 96°C is left to stand in the room. The rate at which heat is lost is proportional to the difference between the coffee's temperature and room temperature i.e. $\frac{dT}{dt} = k(T - 26)$ where T is the temperature of the coffee after t minutes.
Twenty minutes later the temperature of the coffee is 66°C .

i. $T = 26 + Ae^{kt}$ satisfies the above equation. Find the values of k and A to three significant figures if necessary.

ii. Find the temperature of the coffee in a **further** 10 minutes (nearest degree).

Question(6) **12 marks**

a) The velocity of a particle moving on the x – axis is given by $v = \frac{e^x}{x}$ m/s. Initially the particle is at $x = \frac{1}{2}$ m.

i. Show that the acceleration, $a = \frac{e^{2x}(x-1)}{x^3}$.

ii. In what direction is the particle moving when $x = \frac{1}{2}$ m and explain whether it is speeding up or slowing down.

iii. Where is the acceleration equal to 0?

iv. Find the slowest speed attained by the particle.

b) The normal at any point $P(2at, at^2)$ on the parabola $x^2 = 4ay$ cuts the y-axis at Q. P is the midpoint of interval RQ.

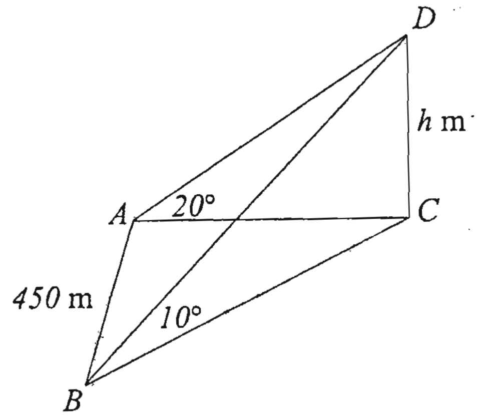
i. Given that the slope of the tangent at P is t, derive the equation of the normal at P.

ii. Find the coordinates of Q and R.

iii. Show also that the locus of R is another parabola and state the co-ordinates of the vertex and focus of this second parabola.

Question(7) 12 marks

a)



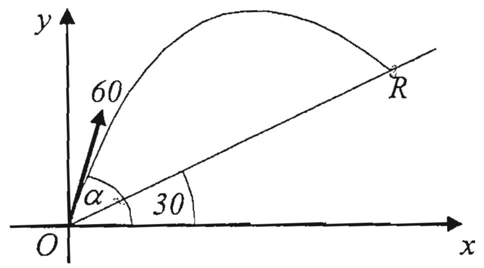
A vertical flagpole CD of height h metres stands with its base C on horizontal ground. A is a point on the ground due West of C and B is a point on the ground 450 metres due South of A. From A and B the angles of elevation of the top D of the flagpole are $20°$ and $10°$ respectively.

i. Show that $AC = h \tan 70°$.

ii. Find the height of the flagpole correct to the nearest metre.

iii. Find the perpendicular distance from BC to the point A. Answer to the nearest metre.

b)



A stone is projected from O with velocity $60\ m/s$ at an angle α^0 above the horizontal. A straight road goes through O at an angle 30^0 above the horizontal, (note that $\alpha > 30$. The stone strikes the road at R. Air resistance is to be ignored, and the acceleration due to gravity, $g = 10\ m/s^2$.

i. If the stone is at the point (x, y) at time t, find expressions for x and y in terms of t. Hence show that the equation of the path of the stone is $y = x \tan \alpha - \dfrac{x^2 \sec^2 \alpha}{720}$.

ii. If R is the point (X, Y), express X and Y in terms of OR.

iii. Hence show that the range OR of the stone up the road is given by $OR = 960 \cos\alpha \sin(\alpha - 30^0)$.

iv. Hence show that the maximum value of OR is 240 metres.