Mr Antonio
Mrs Collett
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Name: .....

Teacher:.....

Pymble Ladies' College

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION
2014

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Use pencil for Questions 1-10.
- Write using black or blue pen for Questions 11-16. Black pen is preferred.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- In Questions 11-16, show relevant mathematical reasoning and/or calculations.

Total Marks – 100

Section I Pages 1-4

10 marks

- Attempt Questions 1-10
- Allow about 15 mins for this section

Section II Pages 5-15

90 marks

- Attempt Questions 11-16
- Allow about 2 hours 45 minutes for this section

Mark	**/100**
Highest Mark	**/100**
Rank	

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

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1 What is the period of $y = 5\sin 2x$?

(A) π

(B) 2π

(C) $\frac{\pi}{2}$

(D) 5

2 What is the value of x if $8(x-3)^3 - 1 = 0$?

(A) $3 - \frac{1}{2}$

(B) $3 + \frac{1}{2}$

(C) $3 \pm \frac{1}{2}$

(D) 5

3 What are the values of m that will give the equation $mx^2 + 6x - 3 = 0$ two real and different roots?

(A) $m \leq -3$

(B) $m \geq -3$

(C) $m > -3$

(D) $m < -3$

Multiple Choice (continued).

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4 What is the gradient of the normal to the curve $y = 2x^2 - 5x + 1$ at the point $(2, -1)$?

(A) $\frac{1}{3}$

(B) $-\frac{1}{3}$

(C) 3

(D) -3

5 What is the derivative of $f(x) = \ln(\cos x)$?

(A) $f'(x) = -\tan x$

(B) $f'(x) = \tan x$

(C) $f'(x) = \frac{1}{\cos x}$

(D) $f'(x) = -\frac{1}{\sin x}$

6 What are the solutions of $2\sin x + \sqrt{3} = 0$ in the domain $0 \le x \le 2\pi$?

(A) $\frac{\pi}{3}, \frac{2\pi}{3}$

(B) $\frac{2\pi}{3}, \frac{5\pi}{3}$

(C) $\frac{\pi}{3}, \frac{4\pi}{3}$

(D) $\frac{4\pi}{3}, \frac{5\pi}{3}$

Multiple Choice (continued).

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7 Given $\log_{10} y = 2 - \log_{10} x$, which expression is equivalent to y?

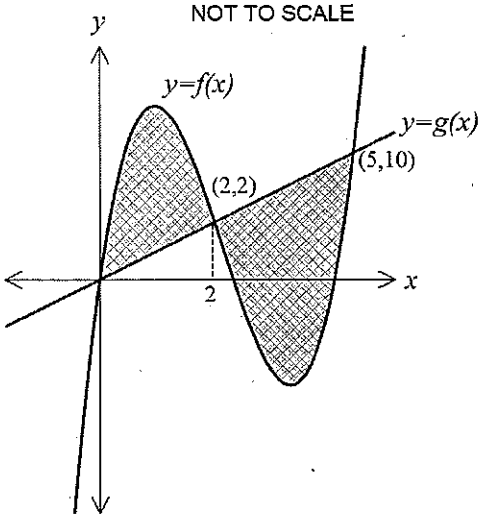
(A) $y = \log_{10}(2) - x$

(B) $y = 2 - x$

(C) $y = \dfrac{100}{x}$

(D) $y = 100 - x$

8 Which expression using integral notation is equivalent to the area of the shaded regions?



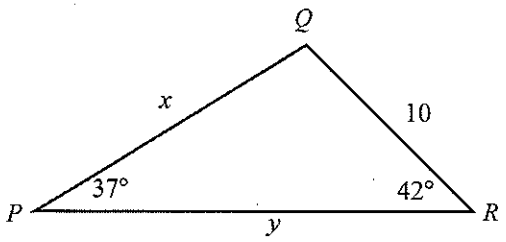
(A) $\displaystyle\int_0^5 \{f(x) - g(x)\}\,dx$

(B) $\displaystyle\int_0^2 \{g(x) - f(x)\}\,dx + \int_2^5 \{f(x) - g(x)\}\,dx$

(C) $\displaystyle\int_0^2 \{f(x) - g(x)\}\,dx + \int_2^5 \{f(x) - g(x)\}\,dx$

(D) $\displaystyle\int_0^2 \{f(x) - g(x)\}\,dx + \int_2^5 \{g(x) - f(x)\}\,dx$

9 PQR is a triangle with side lengths x, 10 and y, as shown below.
In this triangle, angle $RPQ = 37°$ and angle $QRP = 42°$.



Which one of the following expressions is correct for triangle PQR?

(A) $x = \dfrac{10}{\sin 37°}$

(B) $x = 10 \times \dfrac{\sin 42°}{\sin 37°}$

(C) $y = 10 \times \dfrac{\sin 37°}{\sin 101°}$

(D) $10^2 = x^2 + y^2 - 2xy\cos 42°$

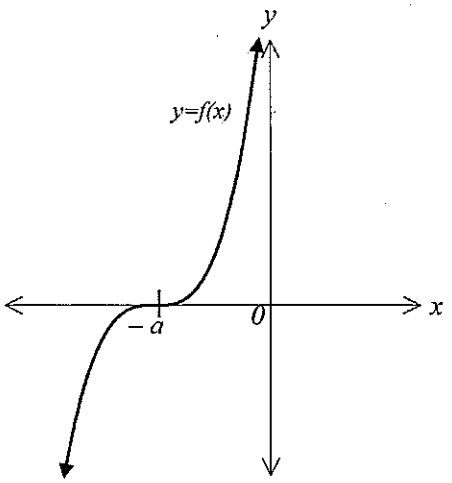
10 At $x = -a$, which of the following correctly describes the graph of $y = f(x)$?

(A) $f(-a) = 0,\ f'(-a) > 0$

(B) $f'(-a) = 0,\ f''(-a) = 0$

(C) $f(0) = -a,\ f'(-a) > 0$

(D) $f'(-a) > 0,\ f''(-a) = 0$



Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Solve the equation $|3+5x|=2$. **2**

(b) Show that $3\sqrt{5}-2\sqrt{2}$ is a square root of $53-12\sqrt{10}$. **2**

(c) Differentiate $(5-\cos 2x)^4$. **2**

(d) Find a primitive of $\sec^2 x-3$. **2**

Question 11 continues on page 6.

(e) α and β are the roots of $x^2 - 4x + 1 = 0$.

(i) Find $\alpha\beta$. **1**

(ii) Hence, prove $\alpha + \dfrac{1}{\alpha} = 4$. **1**

(f) Find the coordinates of the focus of the parabola $x^2 = -32(y-2)$. **2**

(g) A circle is divided into n sectors in such a way that the angles of the sectors are in arithmetic progression. The smallest two angles are 3° and 5°. Find the value of n. **3**

End of Question 11

Question 12 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Given that $y = \dfrac{x^2}{\tan 4x}$, find $\dfrac{dy}{dx}$. **2**

(b) Find $\int \sqrt{7x-2}\ dx$. **2**

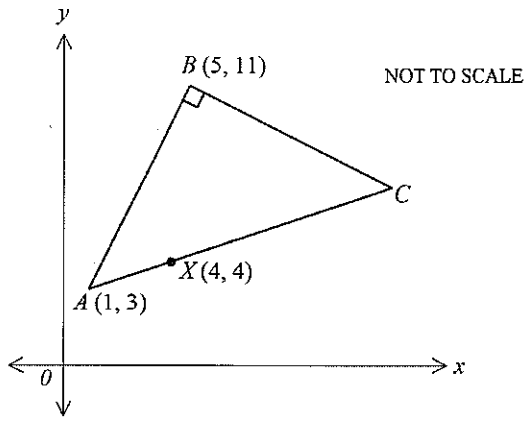
(c) In a geometric series, all the terms are positive, the second term is 24 and the fourth term is $13\frac{1}{2}$. Find

(i) the first term, **2**

(ii) the sum to infinity of the series. **1**

Question 12 continues on page 8.

(d)



The diagram above shows a triangle ABC in which A has coordinates $(1, 3)$, B has coordinates $(5, 11)$ and angle ABC is 90°. The point X $(4, 4)$ lies on AC. Find

(i) the gradient of AB. **1**

(ii) the equation of BC. **2**

(iii) the coordinates of C. **3**

(e) It is given that $f(x) = \frac{1}{x^3} - x^3$. Show that $f(x)$ is a decreasing function. **2**

End of Question 12

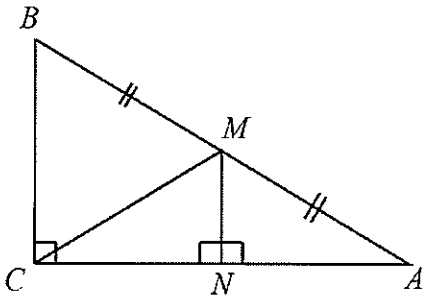
Question 13 (15 marks) Use a **separate** writing booklet. **Marks**

(a) A function is given by $f(x) = x^3 - 3x^2 - 9x + 11$.

(i) Find the coordinates of the stationary points of $f(x)$ and determine their nature. **3**

(ii) Hence, sketch the graph $y = f(x)$ showing all stationary points and the y - intercept. **2**

(b) In the diagram, M is the midpoint of AB. $\angle ACB = \angle MNA = 90°$.
Copy the diagram into your booklet.



Prove that

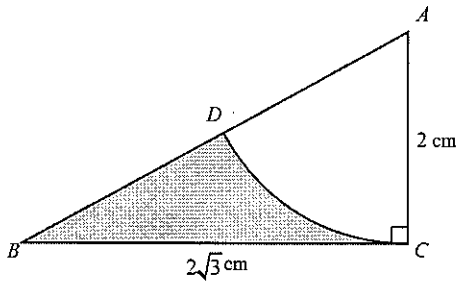
(i) $MN = \frac{1}{2}BC$. **2**

(ii) ΔAMN and ΔCMN are congruent. **2**

(iii) $CM = \frac{1}{2}AB$ **1**

Question 13 continues on page 10.

(c)



In the diagram, D lies on the side AB of the triangle ABC and CD is an arc of a circle with centre A and radius 2 cm. The line BC is of length $2\sqrt{3}$ cm and is perpendicular to AC. Find the area of the shaded region BDC, giving your answer in terms of π and $\sqrt{3}$. **2**

(d) A curve is such that $\frac{d^2y}{dx^2} = 4e^{-2x}$. Given that $\frac{dy}{dx} = 3$ when $x = 0$ and that the curve passes through the point $\left(2, e^{-4}\right)$, find the equation of the curve. **3**

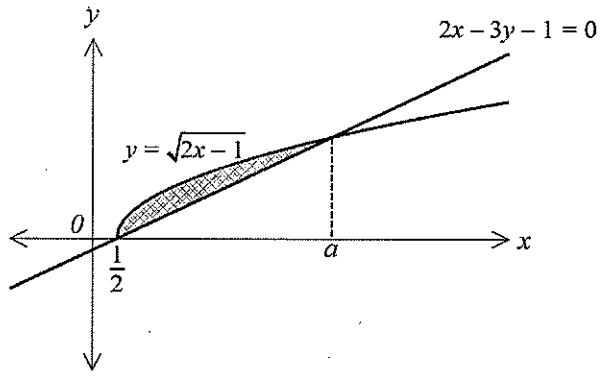
End of Question 13

Question 14 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Prove that $\dfrac{\sin A}{1+\cos A}+\dfrac{1+\cos A}{\sin A}=2\operatorname{cosec}A$ **3**

(b) The curves $y=\sqrt{2x-1}$ and $2x-3y-1=0$ are drawn below.

They intersect at $x=\dfrac{1}{2}$ and $x=a$ as indicated on the diagram.



(i) Show that $a=5$. **2**

(ii) Find, showing all necessary working, the area of the shaded region. **3**

(c) The temperature $T°C$ of an object in a room, after t minutes, satisfies the differential equation

$$\frac{dT}{dt}=k\left(T-22\right),\ \text{where } k \text{ is a constant.}$$

(i) Show that $T=Ae^{kt}+22$, satisfies the differential equation. **1**

(ii) When $t=0,\ T=100,$ and when $t=15,\ T=70$.

(α) Use this information to find the values of A and k. **3**

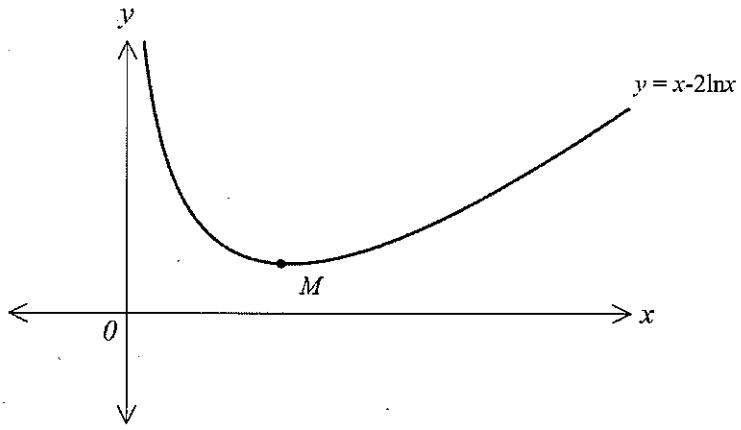
(β) Hence find the value of t when $T=40$, correct to 1 decimal place. **3**

End of Question 14

Question 15 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Evaluate $\int_2^3 \frac{x^2}{x^3-2}\,dx$. **2**

(b)



The above diagram shows the curve $y = x - 2\ln x$ and its minimum point M.

(i) Find the x coordinate of M. **2**

(ii) Use 2 applications of the trapezoidal rule to estimate the value of **3**

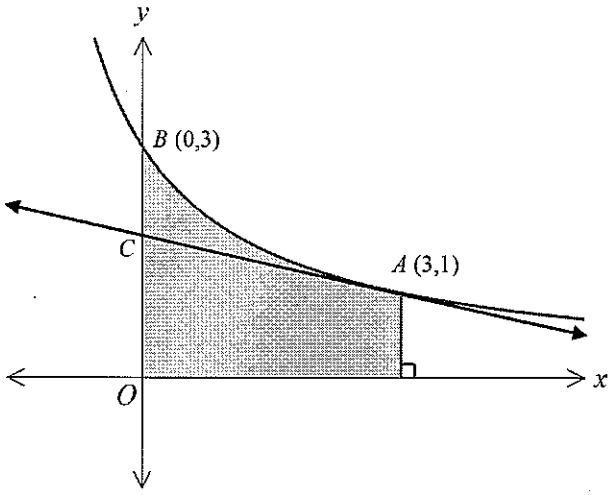
$$\int_2^4 (x - 2\ln x)\,dx.$$

Give your answer to correct to 2 decimal places.

(iii) State, with a reason, whether the trapezoidal rule gives an under-estimate or an over-estimate of the true value of the integral in part (ii). **1**

Question 15 continues on page 13.

(c)



The above diagram shows part of the curve $y = \dfrac{9}{2x+3}$, crossing the y-axis at the point B (0,3).

The point A on the curve has coordinates (3,1).

The tangent to the curve at A crosses the y-axis at C.

(i) Find the equation of the tangent to the curve at A. **3**

(ii) Determine, showing all necessary working, whether C is nearer to the point B or to the point O. **1**

(iii) Find the exact volume obtained when the shaded region is rotated through 360° about the x-axis. Show all necessary working. **3**

End of Question 15

Question 16 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Solve $4e^{2x} - e^{x} = 0$. **2**

(b) A particle moves in a straight line and at time t it has velocity v, where

$$v = 3t^2 - 2\sin 3t + 6$$

(i) Find an expression for the acceleration of the particle at time t. **1**

(ii) When $t = \frac{\pi}{3}$, show that the acceleration of the particle is $2\pi + 6$. **1**

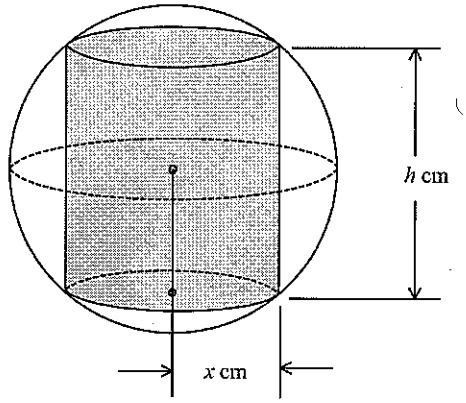
(iii) When $t = 0$, the particle is at the origin.
Find an expression for the displacement of the particle from the origin at time t. **2**

(c) (i) Find $\frac{d}{dx}\left(e^{\cos 2x}\right)$. **1**

(ii) Hence, find $\int x + \sin 2x \, e^{\cos 2x} dx$. **2**

Question 16 continues on page 15.

(d) A machinist has a spherical ball of brass with diameter 10 cm. The ball is placed in a lathe and machined into a cylinder.



(i) If the cylinder has radius x cm, show that the cylinder's volume is given by **2**

$$V(x) = \pi x^2 \sqrt{100 - 4x^2} \text{ cm}^3.$$

(ii) Hence, find the dimensions of the cylinder of largest volume which can be made. **4**

End of paper