Mr Antonio
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Name: ....

Teacher:..

Pymble Ladies' College

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

2014

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes.

- Working time – 2 hours.

- Write using pencil for Questions 1-10.

- Write using black or blue pen for Questions 11-14. Black pen is preferred.

- Board approved calculators may be used.

- A table of standard integrals is provided at the back of this paper.

- In Questions 11-14, show relevant mathematical reasoning and/or calculations.

Total Marks – 70

| Section I | Pages 1-4

10 marks
- Attempt all Questions 1-10
- Allow about 15 mins for this section

| Section II | Pages 5-11

60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

Mark	/70
Highest Mark	/70
Rank	

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

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1   The roots of the equation $x^3 - 5x^2 + 4 = 0$ are α, β and γ.

The value of $\alpha + \beta + \gamma$ and the value of $\alpha\beta\gamma$ are respectively.

(A)   5 and 4

(B)   5 and -4

(C) -5 and 4

(D) -5 and -4

2   Evaluate $\sin^{-1}\left(\sin\dfrac{4\pi}{3}\right)$.

(A) $\dfrac{4\pi}{3}$

(B) $\dfrac{\pi}{3}$

(C) $\dfrac{-2\pi}{3}$

(D) $\dfrac{-\pi}{3}$

3   When the polynomial $P(x) = x^4 + ax + 2$ is divided by $x^2 + 1$ the remainder is $2x + 3$.

The value of a is

(A)   1

(B)   2

(C)   0

(D)   3

4 Given the points A (7, 14) and B (1, 2), C is a point on AB produced such that $AB : BC = 2 : 1$.

Find the coordinates of C.

(A) $\left(-5, -10\right)$

(B) $\left(-2, -4\right)$

(C) $\left(3, 6\right)$

(D) $\left(5, 10\right)$

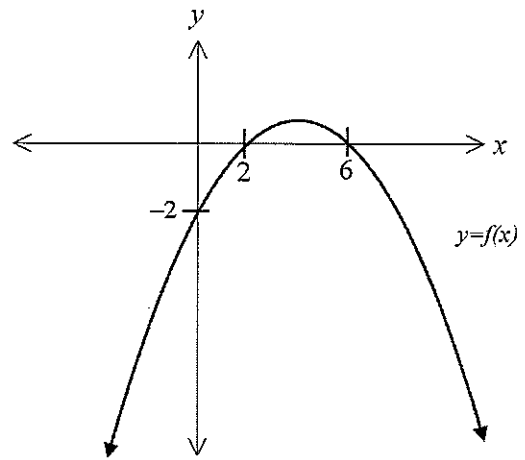**5** Find $\displaystyle\int \frac{1}{\sqrt{1-3x^2}}\, dx$.

(A) $3\sin^{-1}(3x) + C$

(B) $\dfrac{1}{3}\sin^{-1}(3x) + C$

(C) $\sqrt{3}\sin^{-1}\left(\sqrt{3}\, x\right) + C$

(D) $\dfrac{1}{\sqrt{3}}\sin^{-1}\left(\sqrt{3}\, x\right) + C$

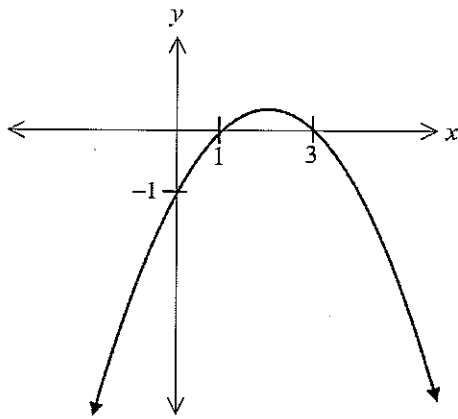**6** Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} \sin^2\theta\, d\theta$.

(A) $\dfrac{\pi}{12} - \dfrac{\sqrt{3}}{8}$

(B) $\dfrac{\pi}{6} - \dfrac{\sqrt{3}}{4}$

(C) $\dfrac{1}{24}$

(D) 1

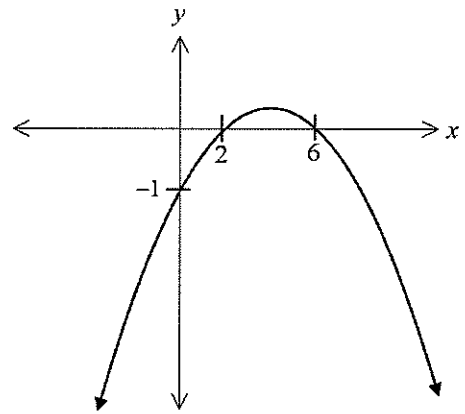7   The figure on the right shows the graph of
    $y = f(x)$.

    If $2f(x) = g(x)$, which of the following
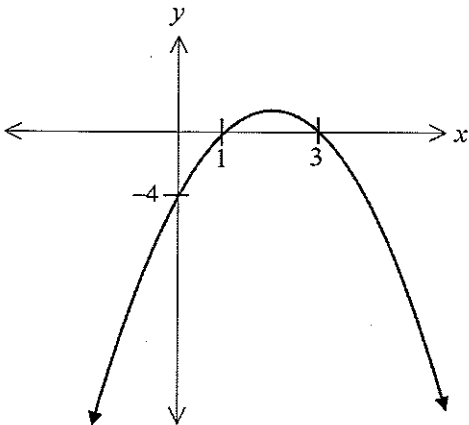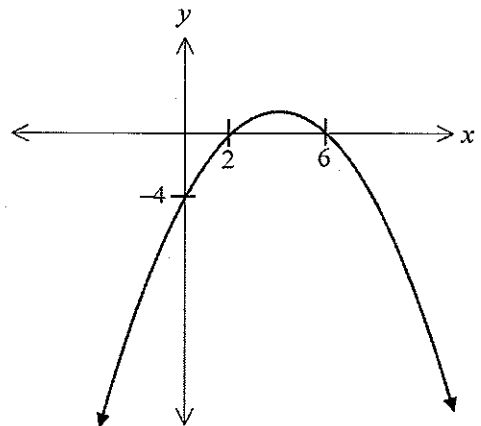   may represent the graph of $y = g(x)$?

(A)

(B)

(C)

(D)

8 If $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$, then which one of the following statements is false?

(A) $f(x)$ is an odd function

(B) $\displaystyle\int_{0}^{a} f(x)\,dx = \int_{-a}^{0} f(-x)\,dx$

(C) $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

(D) The area bounded by the curve $y = f(x)$, the x axis and the lines $x = a$ and $x = -a$ is twice the area bounded by the curve $y = f(x)$, the x axis and the lines $x = 0$ and $x = a$.

9 For $0° \leq \theta \leq 90°$, the least value of $\dfrac{30}{3\sin^2\theta + 2\sin^2(90° - \theta)}$ is

(A) 5

(B) 6

(C) 10

(D) 15

10 Given n is an integer, the general solution of $\tan\left(2x + \dfrac{\pi}{4}\right) = \sqrt{3}$ is

(A) $x = \dfrac{(12n+1)\pi}{24}$

(B) $x = \dfrac{(3n+1)\pi}{6}$

(C) $x = \dfrac{(12n-1)\pi}{24}$

(D) $x = \dfrac{(6n+1)\pi}{6}$

4

Section II

60 marks
Attempt Questions 11-14
Allow about 1 hours and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11. (15 marks). Use a **Separate** Booklet.	**Marks**

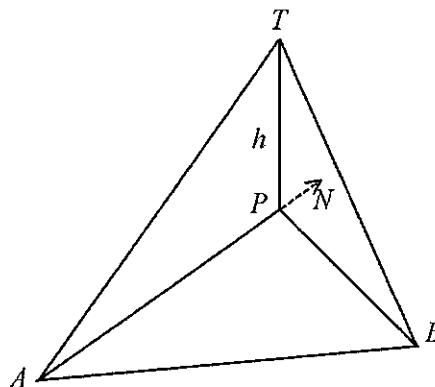**(a)**   Given $f(x) = x^4 + x^2 - 80$.   **2**

Assume there is a zero near $x = 3$. Use Newton's method once to find a better approximation to the root correct to 2 significant figures.

(b)   From a point A due south of a tower, TP, the angle of elevation of the top of the tower, T is $25°$ and from a point B due east of the tower is $32°$.

The distance from A to B is 50 metres.

Let the height of tower TP be h metres.

(i)   Copy the diagram in your answer booklet and complete with all given information.   **1**

(ii)   Find an expression for PA in terms of h.   **1**

(iii)   Find the height of the tower, h, correct to 1 decimal place.   **3**

(c) The function $f(x)$ is defined as $f(x) = \dfrac{3x-4}{x+2}$, where $x \neq -2$.

    (i)   Find an expression for $f^{-1}(x)$.   **2**

    (ii) Write down the domain of $f^{-1}(x)$.   **1**

(d) Solve $\dfrac{4}{(x-1)^2} > 1$.   **3**

(e) Find $\displaystyle\int \dfrac{\ln x}{2x}\, dx$ using the substitution $u = \ln x$.   **2**

End of Question 11

6

Question 12. (15 marks) Use a **Separate** Booklet.   **Marks**

(a)   Find the term independent of x in the expansion of $\left(2x+\dfrac{1}{x^2}\right)^6$.   **2**

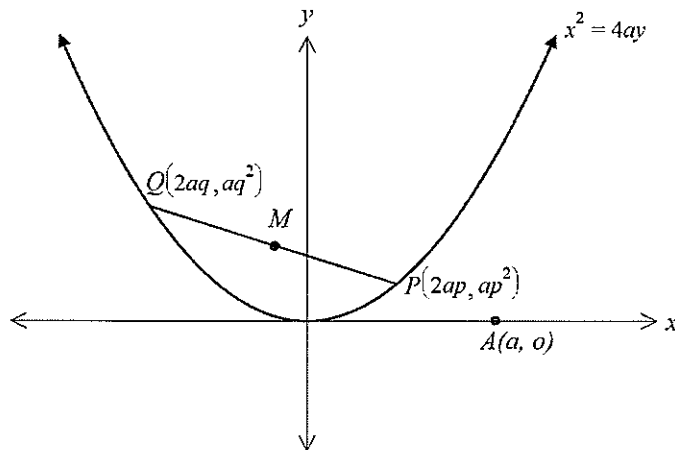**(b)**   (i)   Show that $\tan x=\dfrac{\sin 2x}{1+\cos 2x}$.   **2**

      (ii)   Hence evaluate $\tan\dfrac{\pi}{12}$ in simplest form.   **2**

(c)   Prove by mathematical induction that 8^n-3^n is divisible by 5,   **3**
where n is a positive integer.

(d)

In the diagram above, the points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ lie on the parabola with equation $x^2 = 4ay$.

(i)   Write down the coordinates of the midpoint M of the chord PQ.   **1**

(ii)   Show that the equation of the chord PQ is $y = \dfrac{(p+q)x}{2} - apq$.   **2**

(iii)   Show that the condition for the chord PQ produced to pass through the   **1**
           point $A(a, 0)$ is $p + q = 2pq$.

(iv)   Find the cartesian equation of the locus of M, as the points P and Q move on the   **2**
           parabola subject to the constraint that PQ pass through $A(a, 0)$.

End of Question 12

(a) Find the acute angle between the tangents on the curve $y = \tan^{-1} x$ at the points where $x = 0$ and $x = 1$. Answer correct to the nearest degree.

2

(b) During a chemical reaction, the amount, R kg, of chemical formed at time t hours is modelled by the differential equation

$$\frac{dR}{dt} = 4 - \frac{R}{15}.$$

(i) Show that $R = 60 - 50\,e^{\frac{-t}{15}}$ is a solution to $\dfrac{dR}{dt} = 4 - \dfrac{R}{15}$.

2

(ii) How long will it take for 20 kg of the chemical to form? Give your answer correct to 2 significant figures.

2

(c) In the figure below, BD is a diameter of the circle $ABCD$. If $AB=AC$ and $\angle BDC = 36°$, find $\angle ABD$.
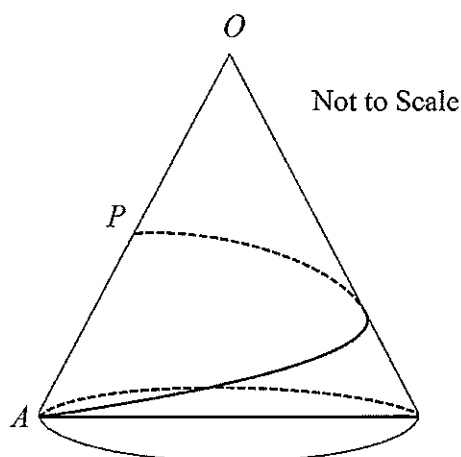
3

Not to Scale
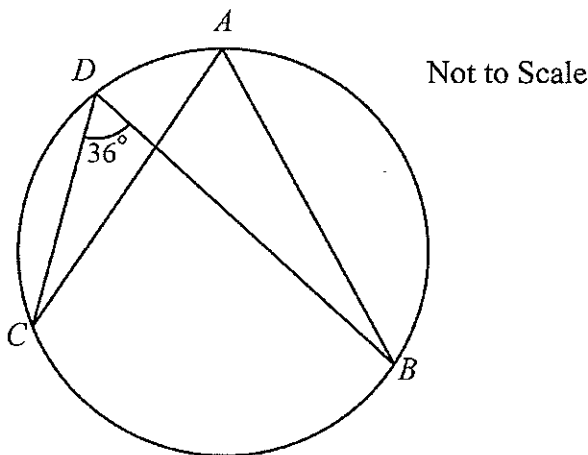
(d)　A thin sheet of smooth metal is in the shape of a sector of a circle with OA, OB as bounding radii each of length 10 cm, and the angle AOB is 60°.

　　(i)　Find the length of the arc AB.　　　　　　　　　　　　　　　　　　1

　　(ii)　The sheet is now bent to form a right circular cone by welding the radii OA and OB together (and inserting a circular disc to close in the cone at the base).

　　　　(α)　Find the volume of the cone in terms of π.　　　　　　　　　　　　3

　　　　　　(Note: The volume of a right circular cone is, $\dfrac{1}{3}\pi r^2 h$.)

　　　　(β)　On the surface of this cone a thin string is pulled tight starting with one end　　2
　　　　　　fixed at the point A and passing once round the cone to the other end P which
　　　　　　is at the midpoint of OA (as shown in diagram).

　　　　　　Find the exact length of this string.

End of Question 13

(a)   Solve $\sin x - 3\cos x = 3$ for $0° \le x \le 360°$.   **3**

(b)   A projectile is fired from a point O with initial speed of V m/s at an angle of elevation θ.
If x and y are the horizontal and vertical displacements of the projectile in metres from O
at time t seconds later then

$x = Vt \ \cos\theta$ and $y = Vt \ \sin\theta - \dfrac{1}{2}gt^2$ where g m/s^2 is the acceleration due to gravity.

The projectile falls to a point P below the level of O such that $PM = OM$.
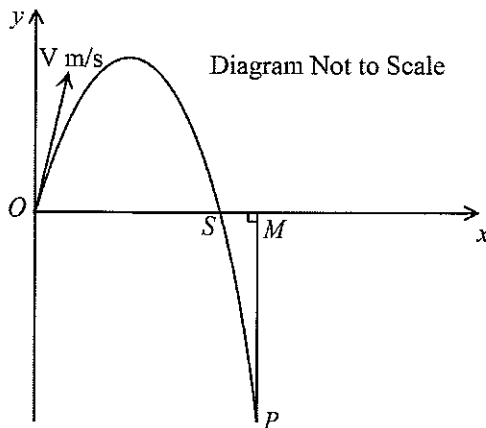
(i)   Prove that the time taken to reach P is $2V\dfrac{(\sin\theta + \cos\theta)}{g}$ seconds.   **1**

(ii)   Show that the distance OM is   **2**

$\dfrac{V^2}{g}(\sin 2\theta + \cos 2\theta + 1)$ metres.

(iii)   If $OS = r$, $OM = \dfrac{4r}{3}$ and $r > 0$, prove that $\sin 2\theta - 3\cos 2\theta = 3$.   **3**

(iv)   Hence, by using Question 14 part (a), find the value of θ.   **2**

(v)   Find an expression for the horizontal and vertical components of the velocity.   **1**

(vi)   If the magnitude of the velocity of the projectile at P is kV m/s, find the exact value of k.   **3**

End of Paper