SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2014

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time – 5 Minutes
- Working time – 2 hours
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question if full marks are to be awarded.
- Answer in simplest exact form unless otherwise instructed.

Total Marks – 70 Marks

Section I – **10 Marks**

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – **60 Marks**

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Examiner: *J. Chen*
R. Boros

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1. If $(7, b)$ divides $(3, -4)$ and $(9, -7)$ internally in the ratio $a: 1$, find the values of a and b.

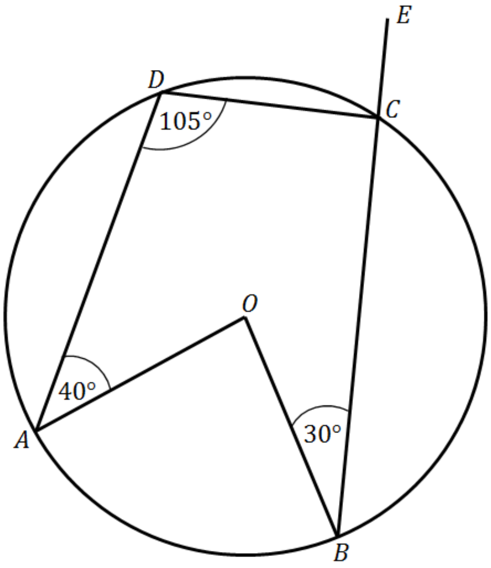
(A) $a = \frac{1}{2},\ b = -\frac{23}{3}$

(B) $a = 2,\ b = -\frac{23}{3}$

(C) $a = \frac{1}{2},\ b = -6$

(D) $a = 2,\ b = -6$

2. In the diagram below, O is the centre of the circle $ABCD$. BCE is a straight line. If $\angle ADC = 105°$, $\angle OBC = 30°$ and $\angle OAD = 40°$, then $\angle DCE =$



(A) 75°

(B) 80°

(C) 85°

(D) 90°

3. $\alpha 3 \beta$ is a 3-digit number, where α and β are integers from 1 to 9 inclusive. Find the probability that the 3-digit number is divisible by 5.

(A) $\frac{1}{10}$

(B) $\frac{9}{50}$

(C) $\frac{1}{9}$

(D) $\frac{1}{5}$

4. Let $b > 1$ and $c > 1$. If $a = \log_c \sqrt{b}$, then $a^{-1} =$

(A) $\log_b c^2$

(B) $2 \log_c b$

(C) $\log_c \frac{1}{\sqrt{b}}$

(D) $\log_{\frac{1}{c}} \frac{1}{\sqrt{b}}$

5.

$$\frac{d}{dx}(x \sin^{-1} x) =$$

(A) $\sin^{-1} x - \frac{x}{\sqrt{1-x^2}}$

(B) $\sin^{-1} x + \frac{x}{\sqrt{1-x^2}}$

(C) $\cos^{-1} x + \frac{x}{\sqrt{1-x^2}}$

(D) $\cos^{-1} x - \frac{x}{\sqrt{1-x^2}}$

6. It is given that α and β are roots of the equation $x^2 + 1 = 6x$, then $\alpha - \beta =$

(A) $-4\sqrt{2}$

(B) $4\sqrt{2}$

(C) $\pm 4\sqrt{2}$

(D) 32

7. If ${}^{n}P_2 = 56$, then

(A) $n = -7$

(B) $n = 8$

(C) $n = 11$

(D) $n = 112$

8. The minimum value of $\frac{1}{\sin^2 x - 2}$ is

(A) $-\frac{1}{2}$

(B) -1

(C) $-\frac{1}{3}$

(D) 0

9.

$$\int \frac{1}{\sqrt{25-4x^2}}.dx =$$

(A) $\frac{1}{4}\sin^{-1}\left(\frac{5x}{2}\right)+C$

(B) $\frac{1}{4}\sin^{-1}\left(\frac{2x}{5}\right)+C$

(C) $\frac{1}{2}\sin^{-1}\left(\frac{5x}{2}\right)+C$

(D) $\frac{1}{2}\sin^{-1}\left(\frac{2x}{5}\right)+C$

10. The coefficient of x^{2n} in the binomial expansion of $(1+x)^{4n}$ is

(A) $\frac{4n!}{2n!2n!}$

(B) $\frac{(4n)!}{2(n!)^2}$

(C) $\frac{(4n)!}{(2n)!}$

(D) None of the above

End of Section A

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a NEW writing booklet. Extra pages are available

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a NEW Writing Booklet

(a) Determine the acute angle, between the line $x - 3y + 2 = 0$ and the line BC where B is $(-1, -1)$ and C is $(1, 3)$. **2**

(b) Evaluate **1**

$$\lim_{x \to 0} \frac{3x}{2 \sin 4x}$$

(c) Solve for x, **2**

$$\frac{(x-2)}{(x-1)(x-3)} \geq 0$$

(d) Write down a general solution to the equation $\cos 2x = -\frac{1}{2}$. Leave your answer in terms of π. **2**

(e)

(i) Express $12 \cos x - 5 \sin x$ in the form $A \cos(x + \alpha)$ where A is positive and $0° \leq \alpha \leq 180°$, correct α to the nearest minute. **2**

(ii) Hence find the maximum value of $12 \cos x - 5 \sin x$ and the smallest positive value of x for which this maximum occurs. **2**

(f) Calculate the number of different arrangements which can be made using all the letters of the word BANANA. **1**

Question 11 continues on page 7

(g)

(i) Differentiate $\cot x$ with respect to x. **1**

(ii) Hence differentiate $x \cot x$ with respect to x. **1**

(iii) Hence find **1**

$$\int x \operatorname{cosec}^2 x \, . \, dx$$

End of Question 11

Question 12 (15 Marks) Start a NEW Writing Booklet

(a) Express $\sin 2\theta$ and $\cos 2\theta$ in terms of $t = \tan\theta$ to show that **2**

$$\frac{1+\sin 2\theta - \cos 2\theta}{1+\sin 2\theta + \cos 2\theta} = \tan\theta$$

(b) In the expansion of $(1+2x)^n(1-x)^2$, the coefficient of x^2 is 9. Find the coefficient of x in the expansion. **3**

(c) If the roots of $x^3 - 6x^2 + 3x + k = 0$ are consecutive terms of an arithmetic series, find k. **2**

(d) Evaluate **2**

$$\int_0^{\frac{3}{4}} x\sqrt{1-x}.\,dx$$

using the substitution $u = 1 - x$, express your answer in simplest exact form.

(e)

(i) Prove by the Principle of Mathematical Induction that **3**

$$1\times 2^1 + 2\times 2^2 + 3\times 2^3 + \cdots + n\times 2^n = (n-1)\times 2^{n+1} + 2$$

for all positive integers n.

(ii) Using the result of (i), simplify **2**

$$\sum_{r=1}^{n}(r+1)\times 2^r$$

(f) Brian is to celebrate his 16^{th} birthday by having a dinner with 11 other family members. At this dinner, Brian will sit at the head of a non-circular table. In how many ways can everyone be seated? **1**

End of Question 12

Question 13 (15 Marks) Start a NEW Writing Booklet

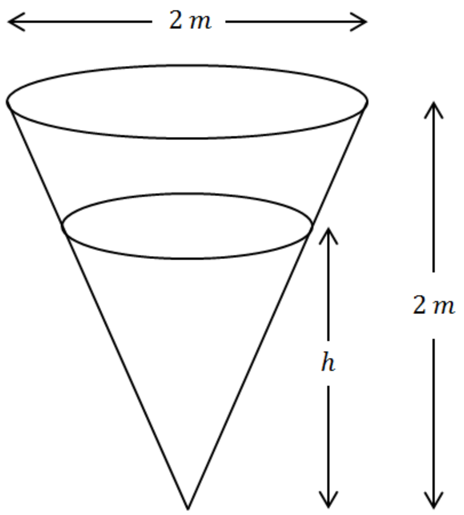
(a) A particle moves up and down so that its vertical displacement, x from a point O, is given by $x = 10 + 8\sin 2t + 6\cos 2t$ where x is in metres and t is in seconds.

(i) Show that the particle moves in Simple Harmonic motion. **1**

(ii) What is the period of the motion? **1**

(iii) What is the amplitude? **1**

(b) A container in the shape of a right cone with both height and diameter $2\ m$ is being filled with water at a rate of $\pi\ m^3/min$.



(i) Show that **2**

$$\frac{dV}{dt} = \frac{\pi h^2}{4} . \frac{dh}{dt}$$

(ii) Find the rate of change of height h of the water when the container is $\frac{1}{8}th$ full by volume. **2**

Question 13 continues on page 10

(c) The rate of change in the number of members of the Sydney Boys High School Old Boys Mathematical Society, M, is given by

$$\frac{dM}{dt} = k(M - 50)$$

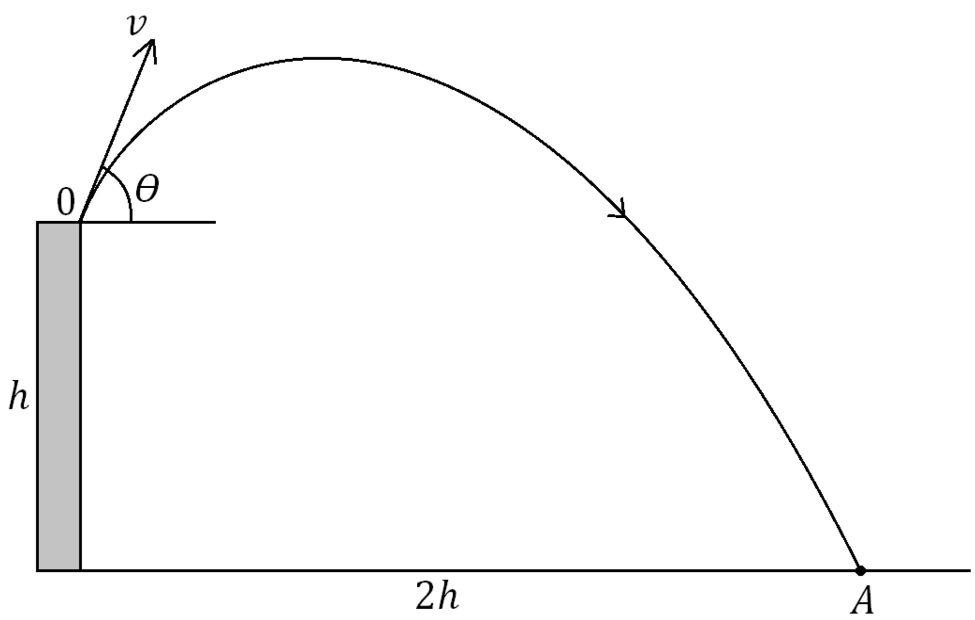
The number of members of this society at the start of 1995 was 70.

(i) Show that $M = 20e^{kt} + 50$ satisfies the differential equation above. **1**

(ii) In 2000, the number of members was 150. Find the number of members in 2005. **1**

(iii) There is a year that this society will eventually become a “ghost society” with no members. Do you agree? Give reasons. **1**

(d)



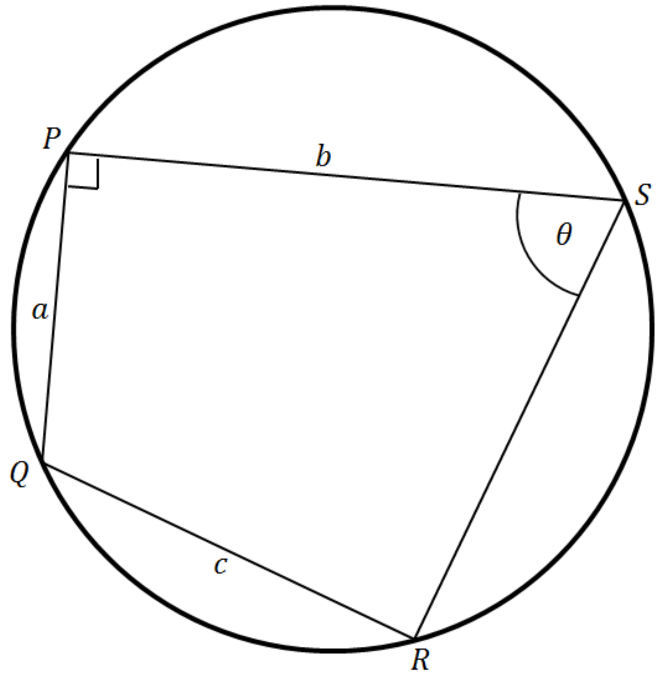
A projectile is fired with speed $\sqrt{\frac{4gh}{3}}$ at an angle θ to the horizontal from the top of a cliff of height h and the projectile strikes the ground a horizontal distance $2h$ from the base of the cliff.
You may assume $y = Vt\sin\theta - \frac{1}{2}gt^2$ and $x = Vt\cos\theta$.

(i) Show that $y = x\tan\theta - \frac{gx^2}{2V^2}(1 + \tan^2\theta)$. **1**

(ii) Find the 2 possible values of $\tan\theta$. **2**

Question 13 continues on page 11

(e) In the diagram below, $PQRS$ is a cyclic quadrilateral, $\angle QPS = 90°$ and $\angle PSR = \theta$, $PQ = a$, $PS = b$ and $QR = c$. **2**



Show that $(a^2 + b^2)\sin^2\theta = a^2 + c^2 + 2ac\cos\theta$.

End of Question 13

Question 14 (15 Marks) Start a NEW Writing Booklet

(a) At an election, 30% of the voters favoured party A. If 5 voters were selected at random, what is the probability (as a decimal) that

(i) exactly 3 favoured party A. **1**

(ii) at most 2 favoured party A. **1**

(b)

(i) Show that **3**

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

(ii) If x satisfies the equation $\tan 3x = \cot 2x$, show that x also satisfies the equation $5\tan^4 x - 10\tan^2 x + 1 = 0$. **2**

(iii) Using the result of (ii), deduce that **4**

$$\tan\frac{\pi}{5} = \sqrt{5 - 2\sqrt{5}}$$

(c) In the expansion of $(1 + x)^n$, let S_1 be the terms containing the coefficients

$${}^nC_0,\ {}^nC_2,\ {}^nC_4, \ldots$$

whilst S_2 be the terms containing the coefficients

$${}^nC_1,\ {}^nC_3,\ {}^nC_5, \ldots$$

Prove that,

(i) $4S_1S_2 = (1 + x)^{2n} - (1 - x)^{2n}$ **2**

(ii) $(S_1)^2 - (S_2)^2 = (1 - x^2)^n$ **2**

End of paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

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