

Sydney Girls High School

2007

TRIAL HIGHER SCHOOL
CERTIFICATE EXAMINATION

Mathematics

Extension 2

This is a trial paper ONLY.
It does not necessarily
reflect the format or the
contents of the 2007 HSC
Examination Paper in this
subject.

General Instructions

- Reading Time – 5 minutes
- Working time – 3 hours
- Attempt ALL questions
- ALL questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied
- Board-approved calculators may be used
- Diagrams are not to scale
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

Question One (15 marks)**Marks**

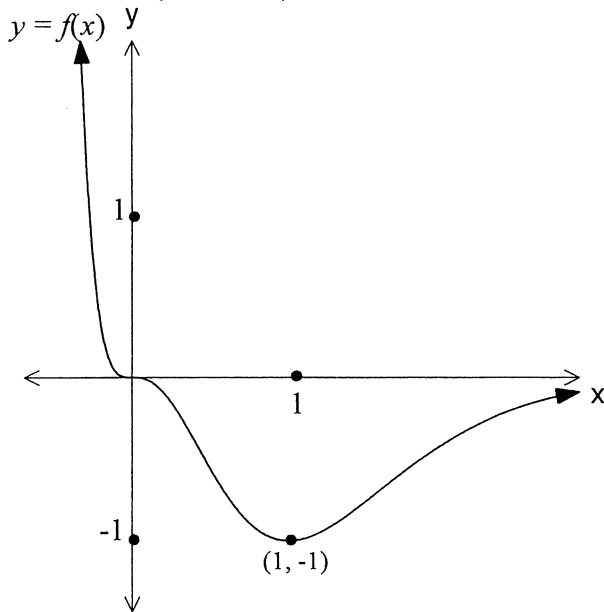
- a) i) Show that $\sin(A - B) + \sin(A + B) = 2 \sin A \cos B$ 1
 ii) Hence or otherwise find $\int \sin 3x \cos x dx$ 1
- b) Find $\int x\sqrt{x^2 + 7} dx$ 2
- c) By completing the square find $\int \frac{dx}{\sqrt{x^2 - 4x + 8}}$ 2
- d) i) Given that $\frac{4x^2 + 3x + 33}{(x^2 + 16)(x + 1)} \equiv \frac{Ax + B}{x^2 + 16} + \frac{C}{x + 1}$ where A, B and C are real numbers, find A, B and C 3
 ii) Hence find $\int \frac{4x^2 + 3x + 33}{(x^2 + 16)(x + 1)} dx$ 3
- e) Find $\int \frac{4dx}{x^2\sqrt{x^2 - 4}}$ 3

Question Two (15 marks)**Marks**

- a) Given $z_1 = 2 - i$ and $z_2 = 3 + 4i$ express $z_1 z_2$ in the form $a + bi$ 1
- b) i) Express $\frac{1 - 3i}{1 + 2i}$ in modulus argument form. 3
 ii) Hence find $\left(\frac{1 - 3i}{1 + 2i}\right)^7$ in simplest modulus argument form 1
- c) i) Find the square roots of $-15 - 8i$ in the form $a + bi$ 2
 ii) Hence solve $z^2 + (2i - 3)z + (5 - i) = 0$ 2
- d) Sketch the region represented by $0 \leq \text{Re}(z^2) \leq 4$ on an Argand Diagram 3
- e) The equation $|z + 2| + |z - 6| = 10$ represents an ellipse in the Argand diagram. Sketch the ellipse, and clearly showing the centre and the lengths of the minor and major axes 3

Question Three (15 marks)

Marks

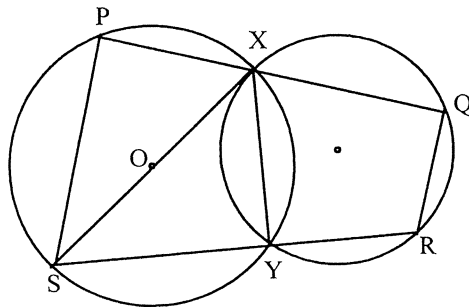


- a) The diagram above shows the graph of $y = f(x)$. The graph has a minimum turning point at $(1, -1)$, a horizontal point of inflexion at the origin and is asymptotic to the positive X - axis

Draw separate 1/3 page sketches of the graphs of the following showing relevant features:

- | | |
|-----------------------------|---|
| i) $y = f(-x)$ | 1 |
| ii) $y = f(x) $ | 2 |
| iii) $y^2 = f(x)$ | 2 |
| iv) $y = \frac{f(x)}{x}$ | 2 |
| v) $y = \frac{d}{dx}[f(x)]$ | 2 |

- b) In the diagram below, SX is a diameter of the circle centre O



- | | |
|-------------------------------------|---|
| i) Prove that $PQ \perp QR$ | 2 |
| ii) Prove $\angle PYQ = \angle SXR$ | 4 |

Question Four (15 marks)

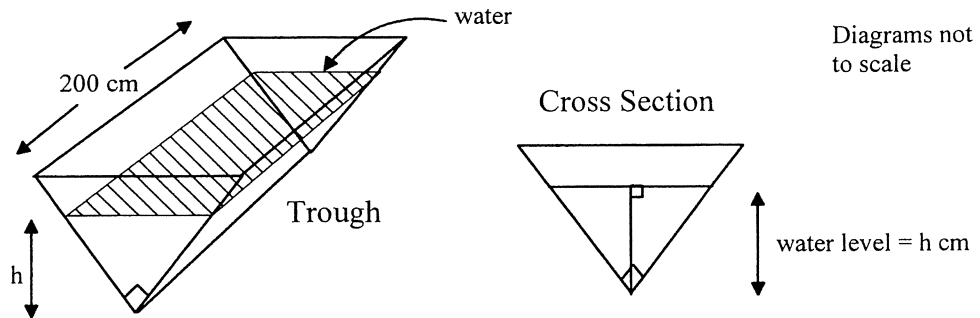
Marks

- a) Given the hyperbola $\frac{y^2}{16} - \frac{x^2}{9} = 1$
- i) Determine the eccentricity 1
 - ii) Find the co ordinates of the foci 1
 - iii) Determine the equations of the directrices 1
 - iv) Determine the equations of the asymptotes 1
 - v) Sketch the hyperbola 1
- b) i) Show that the point P with co ordinates $(2 \sec \theta, \sqrt{5} \tan \theta)$ lies 1
- on the hyperbola $\frac{x^2}{4} - \frac{y^2}{5} = 1$
- ii) From the equation of the hyperbola $\frac{x^2}{4} - \frac{y^2}{5} = 1$ derive the equation 3
- of the tangent at the point P $(2 \sec \theta, \sqrt{5} \tan \theta)$
- iii) The tangent cuts the asymptotes at the points A and B. Find the 4
- co ordinates of A and B
- iv) Show that P is the midpoint of AB 2

Question Five (15 marks)

Mark

- a) Without using calculus draw a 1/3 page sketch of the graph of $y = \frac{1}{x^2 + x - 6}$ 3
- b) Find the equation of the tangent to the curve $x^2 + y^2 + xy - 4 = 0$ at the point $(0, 2)$ 3
- c) The polynomial $P(x) = x^4 - 4x^3 + 11x^2 - 14x + 10$ has roots $a + ib$, $a + 2ib$ where a and b are real. Find the values of a and b 3
- d) The cross section of a water trough is in the shape of a right isosceles triangle. The trough is 200 centimetres long. 3



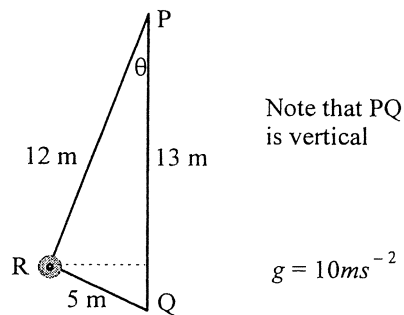
Water is flowing into the trough at the rate of $12\text{cm}^3\text{s}^{-1}$.
 Find the rate of change of the upper surface area
 of the water when the height of the water is 12cm

6

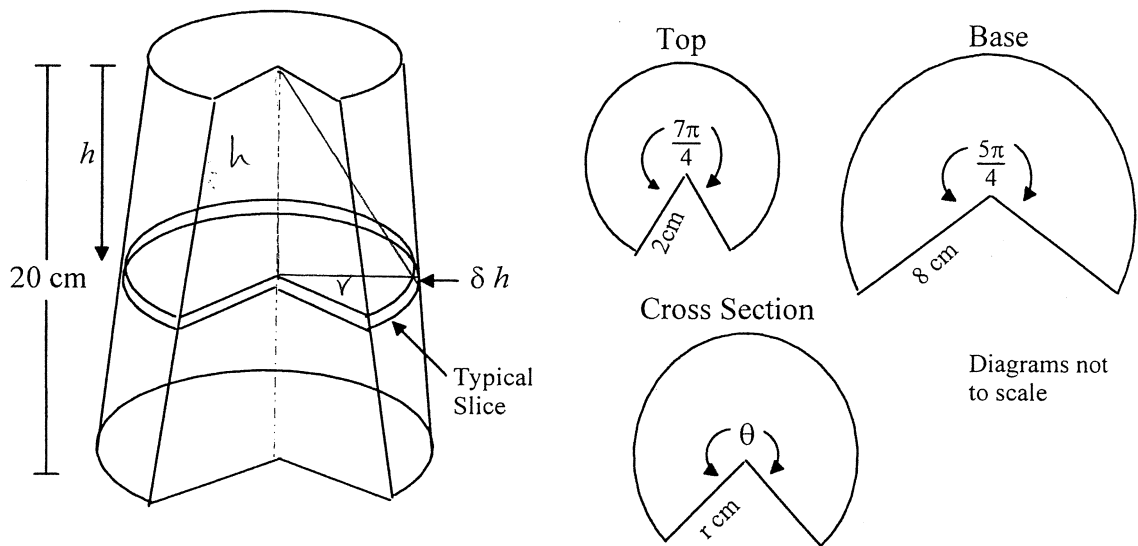
Question Six (15 marks)

Marks

- a) If α, β, γ are the roots of the equation $x^3 + 6x^2 + 5x + 5 = 0$ find the equation with roots:
- i) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ 2
 - ii) $\frac{1}{\alpha\beta}, \frac{1}{\beta\gamma}, \frac{1}{\alpha\gamma}$ 2
- b) A 4kg mass at R rotates about PQ. How fast must the mass rotate (in metres per second) if the tension T_1 in the string PR is to be equal to the tension T_2 in the string QR? 5



- c) A solid has a base and top in the shape of a sector of a circle as shown below. The height of the solid is 20 cm. All other dimensions are shown on the right below. Note that angles are given in radians



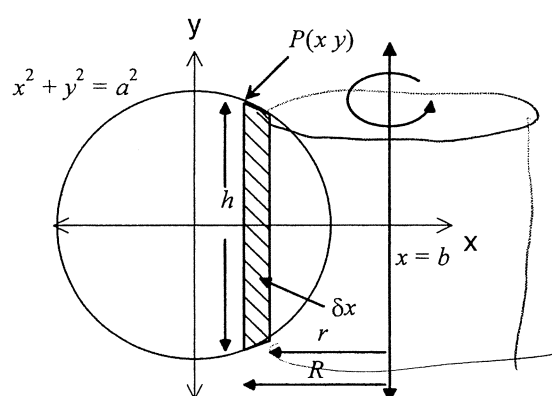
Cross sections are taken perpendicular to the height. A typical slice is shown h centimetres from the top of the solid. A linear relationship exists between r and h , θ and h ,

- i) Find r in terms of h and θ in terms of h 3
- ii) Hence find the volume of the solid 3

Question Seven (15 marks)

Marks

- a) Given that $P(x) = 3x^3 - 11x^2 + 8x + 4$ has a double root, fully factorise $P(x)$ 3
- b) i) Use De Moivre's show $\sin 5\theta = 16\sin^5 \theta - 20\sin^3 \theta + 5\sin \theta$ 3
 ii) Hence solve $\sin 5x = \sin x$ for $0 \leq x \leq 2\pi$ 3
- c) The diagram below show the graphs of $x^2 + y^2 = a^2$ and the line $x = b$ where $b > a$

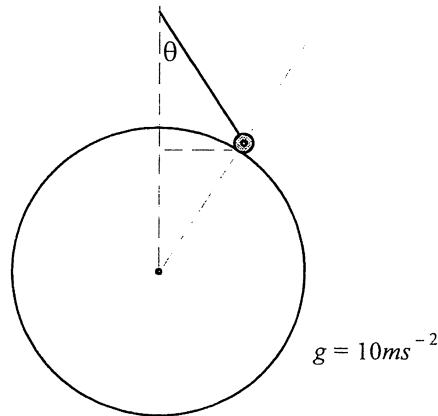


- i) The shaded strip of width δx and height h is rotated about the line $x = b$ to form a cylindrical shell. Find expressions for R , r and h and hence the volume of the shell formed. 3
- ii) Hence find the volume of the solid formed when $x^2 + y^2 = a^2$ is rotated about the line $x = b$ 3

Question Eight (15 marks)

Marks

- a) Use Mathematical Induction to prove the following 4
 $\cos(x + n\pi) = (-1)^n \cos x$ where $n > 0$
- b) From a point 1.3 metres above a sphere of radius 1.7 metres a mass of 6 kilograms is on the end of a string of length 1.7 metres. The mass moves in a horizontal circle of radius 0.8 metres with an angular velocity of 2 rad.s^{-1}



- i) Copy the diagram and show all forces 2
- ii) Find the tension in the string 3
- iii) Find the normal force exerted by the sphere on the mass 1
- iv) Explain what would happen to the mass if the tension in the string and the normal force were equal. 2
- c) The equation $x^3 + px^2 + qx + r = 0$ has one root equal to the sum of the other two. Show that $p^3 - 4pq + 8r = 0$ 3

$\alpha\beta\chi\delta\epsilon\phi\eta\theta\iota\kappa\lambda\mu\nu\pi\rho\sigma\tau\upsilon\omega\xi\psi$

