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CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL



2014 Trial Examination

FORM VI

MATHEMATICS 2 UNIT

Friday 1st August 2014

General Instructions

- Reading time — 5 minutes
- Writing time — 3 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.
- A list of standard integrals is provided at the end of the examination paper.

Total — 100 Marks

- All questions may be attempted.

Section I – 10 Marks

- Questions 1–10 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

Section II – 90 Marks

- Questions 11–16 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your candidate number on each booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place your multiple choice answer sheet inside the answer booklet for Question Eleven.
- Write your candidate number on this question paper and submit it with your answers.

Checklist

- SGS booklets — 6 per boy
- Multiple choice answer sheet
- Candidature — 91 boys

Examiner

MLS

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

What is the gradient of the line $6x + 3y - 2 = 0$?

- (A) 2
- (B) -2
- (C) $\frac{1}{2}$
- (D) $-\frac{1}{2}$

QUESTION TWO

What is 5.29784 correct to three significant figures?

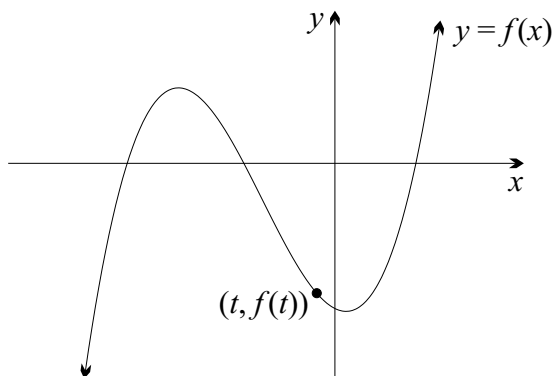
- (A) 5.29
- (B) 5.297
- (C) 5.30
- (D) 5.298

QUESTION THREE

Which of the following is equal to $\frac{1}{\sqrt{5} + 2\sqrt{3}}$?

- (A) $\frac{\sqrt{5} - 2\sqrt{3}}{7}$
- (B) $\frac{2\sqrt{3} + \sqrt{5}}{7}$
- (C) $\frac{2\sqrt{3} - \sqrt{5}}{7}$
- (D) $\frac{\sqrt{5} + 2\sqrt{3}}{-7}$

QUESTION FOUR



The diagram shows the graph of $y = f(x)$. Which of the following statements is true?

- (A) $f'(t) > 0$ and $f''(t) < 0$
- (B) $f'(t) > 0$ and $f''(t) > 0$
- (C) $f'(t) < 0$ and $f''(t) < 0$
- (D) $f'(t) < 0$ and $f''(t) > 0$

QUESTION FIVE

The acceleration of a particle is given by $\ddot{x} = 4 \cos 2t$ where x is the displacement in metres and t is time in seconds. Which of the following is a possible expression for its displacement?

- (A) $-2 \sin 2t$
- (B) $2 \sin 2t$
- (C) $\cos 2t$
- (D) $-\cos 2t$

QUESTION SIX

Which of the following is the derivative of $y = \frac{e^{7x}}{e^{3x}}$?

- (A) $4e^{4x}$
- (B) e^{4x}
- (C) $\frac{7e^{3x}e^{7x} + 3e^{3x}e^{7x}}{e^{9x}}$
- (D) $\frac{3e^{3x}e^{7x} - 7e^{7x}e^{3x}}{e^{9x}}$

QUESTION SEVEN

A particle moves so that its displacement in metres from the origin at time t seconds is given by $x = 20t - 5t^2$. At what time is it stationary?

- (A) 0 seconds
- (B) 2 seconds
- (C) 4 seconds
- (D) 6 seconds

QUESTION EIGHT

How many terms are in the series $31 + 44 + 57 + \dots + 226$?

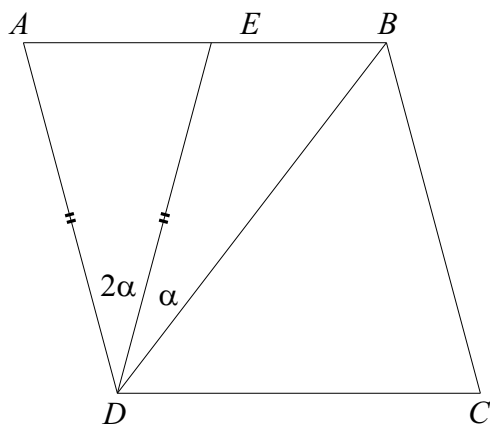
- (A) 4
- (B) 13
- (C) 15
- (D) 16

QUESTION NINE

Given that $\int_0^4 (x + k) dx = 12$ and k is a constant, what is the value of k ?

- (A) 1
- (B) -1
- (C) 0
- (D) 8

QUESTION TEN



The point E lies on the side AB of the rhombus $ABCD$ such that $AD = DE$. The angle ADE is 2α and the angle EDB is α . Find the value of α .

- (A) 45°
- (B) 30°
- (C) 18°
- (D) 15°

————— End of Section I —————

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

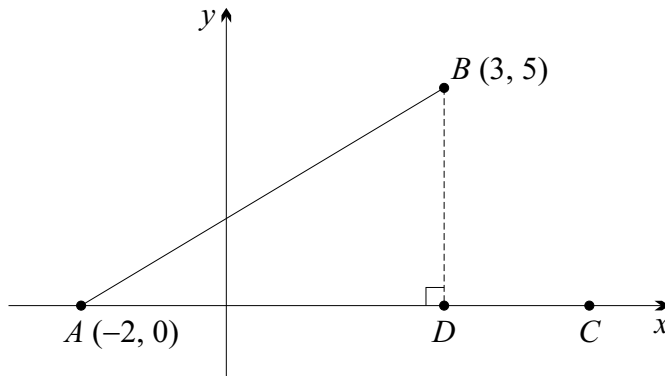
Start a new booklet for each question.

QUESTION ELEVEN	(15 marks)	Use a separate writing booklet.	Marks
(a)	Integrate $\frac{3}{x}$ with respect to x .		1
(b)	Factorise $3x^2 - 7x + 2$.		2
(c)	Solve $\frac{5x - 8}{x} = 1$.		2
(d)	Find the equation of the tangent to the curve $y = x^3 + 4$ at the point $(1, 5)$.		2
(e)	Differentiate $y = \cos(6x + 5)$.		2
(f)	Find the exact value of θ such that $\sin 2\theta = 1$, where $0 \leq \theta \leq \pi$.		2
(g)	A sector with radius 5 cm has an arc length of 20 cm. Find the area of the sector.		2
(h)	Find the limiting sum of the series $\frac{17}{3} + \frac{17}{9} + \frac{17}{27} + \dots$.		2

QUESTION TWELVE (15 marks) Use a separate writing booklet.

Marks

(a)



The diagram shows the points $A(-2, 0)$, $B(3, 5)$ and the point C which lies on the x -axis. The point D also lies on the x -axis such that BD is perpendicular to AC .

- (i) Show that the gradient of AB is 1. 1
- (ii) Find the equation of the line AB . 1
- (iii) What is the size of $\angle BAC$? 1
- (iv) The length of BC is 13 units. Find the length of DC . 1
- (v) Calculate the area of $\triangle ABC$. 1
- (vi) Calculate the size of $\angle ABC$, to the nearest degree. 2

(b) A particle moves on a horizontal line so that its displacement x cm to the right of the origin at time t seconds is given by the function

$$x = \frac{1}{3}t^3 - 6t^2 + 27t - 18.$$

- (i) Find the velocity function. 1
- (ii) When is the particle stationary? 1
- (iii) Find the acceleration function. 1
- (iv) When is the acceleration zero? 1
- (v) Where is the particle when the acceleration is zero? 1

(c) A company starts with 60 employees. At the beginning of each subsequent year the number of employees increases by 15%.

- (i) Find a formula for the number of employees at the beginning of the n th year. 1
- (ii) In which year did the number of employees first exceed 120? 2

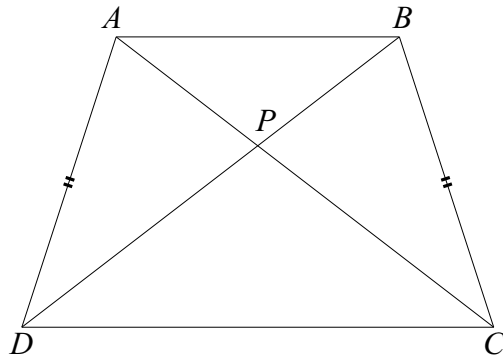
QUESTION THIRTEEN (15 marks) Use a separate writing booklet. **Marks**

- (a) (i) Find the values of m for which the equation $mx^2 - 4x + m = 0$ has real roots. **2**
- (ii) (α) For what values of m does the equation $mx^2 - 4x + m = 0$ have one root only? **1**
- (β) Find this root for each value of m in (α). **1**
- (b) The rate of increase in the number of bacteria N in a culture after t hours is proportional to the number present. This can be represented by the differential equation $\frac{dN}{dt} = kN$. Initially there are 1000 bacteria present and two hours later there are 1080.
- (i) Show that $N = 1000e^{kt}$, where k is a constant, is a solution to the differential equation $\frac{dN}{dt} = kN$. **1**
- (ii) Find the exact value of k . **1**
- (iii) Find the number of bacteria present after a further two hours. **2**
- (iv) At what time will the culture have doubled its initial size? **2**
- (c) Suppose that $f'(x) = \sin 2x$ and $f(\pi) = 1$.
- (i) Find the function $f(x)$. **3**
- (ii) Find the exact value of $f(\frac{\pi}{3})$. **2**

QUESTION FOURTEEN (15 marks) Use a separate writing booklet.

Marks

(a)



The quadrilateral $ABCD$ has diagonals AC and BD which intersect at P . It is known that $AD = BC$ and $AC = BD$. Copy the diagram into your answer booklet.

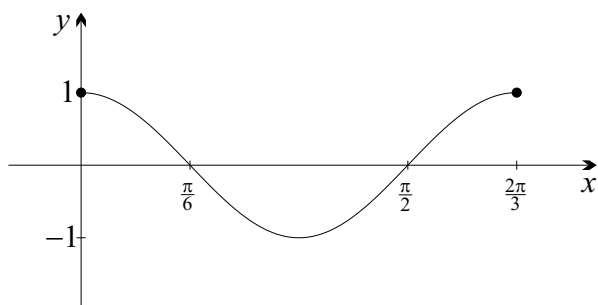
- (i) Prove that the triangles ABC and BAD are congruent. 3
 - (ii) Show that triangle ABP is isosceles. 2
 - (iii) Hence show that triangle CDP is isosceles. 2
 - (iv) Show that AB is parallel to CD . 3
- (b)
- (i) Find the gradient of the tangent to $y = \sin x$ at the origin. 1
 - (ii) Draw the graphs of $y = \sin x$, $y = \frac{2}{3}x$ and the tangent in part (i). Draw your three graphs on the same set of axes for $0 \leq x \leq \pi$. 3
 - (iii) For what values of m does the equation $\sin x = mx$ have a solution in the domain $0 < x < \pi$? 1

QUESTION FIFTEEN (15 marks) Use a separate writing booklet.

Marks

3

(a)



The diagram above shows the graph of the function $y = \cos 3x$. Find the total area bounded by $y = \cos 3x$ and the x -axis from $x = 0$ to $x = \frac{\pi}{3}$.

(b) If α and β are the roots of the quadratic equation $5x^2 - x - 3 = 0$, find the value of:

(i) $\alpha + \beta$

1

(ii) $\alpha\beta$

1

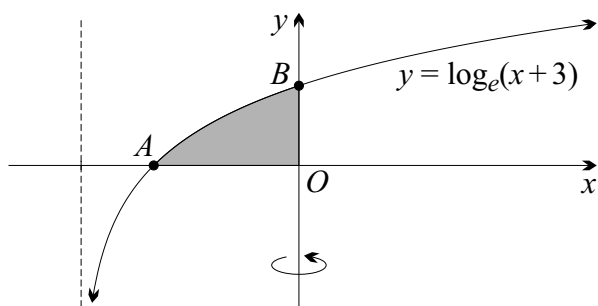
(iii) $\alpha^2 + \beta^2$

1

(iv) $\frac{1}{\alpha} + \frac{1}{\beta}$

1

(c)



The diagram shows the graph of the function $y = \log_e(x + 3)$. The graph crosses the axes at A and B as shown.

(i) Write down the coordinates of B .

1

(ii) Write x as a function of y .

1

(iii) Find the exact value of the volume generated when the shaded region AOB is rotated about the y -axis.

3

QUESTION FIFTEEN (Continued)

(d) Consider the function given by $y = \sin^2 x$.

(i) Copy and complete the following table in your answer booklet.

1

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y					

(ii) Use Simpson's rule with five function values to find an approximation to

2

$$\int_0^{\pi} \sin^2 x \, dx.$$

The Exam continues on the next page

QUESTION SIXTEEN (15 marks) Use a separate writing booklet.

Marks

(a) A company borrows \$800 000 to update its car fleet. The interest rate is 12% p.a. compounded monthly. It pays off the loan by 24 equal monthly instalments. The first instalment is paid one month after the loan is taken out.

Let A_n be the amount owing after n instalments are paid. Let M be the amount of each instalment.

(i) Show that the amount owing after two months is $A_2 = 816\,080 - M(2.01)$. 2

(ii) Show that $M = \frac{8000 \times 1.01^{24}}{1.01^{24} - 1}$. 2

(iii) Hence calculate M to the nearest dollar. 1

(iv) After paying ten instalments, the company decides to increase its repayments to \$60 000 each month. Find the total number of months it takes the company to pay off its debt. 3

(b) A van is to travel 1000 kilometres at a constant speed of v km/h.

When travelling at v km/h, the van uses fuel at a rate of $(6 + \frac{v^2}{50})$ litres per hour.

The truck company pays \$1.50 per litre for fuel and pays each of the two drivers \$30 per hour while the van is travelling.

(i) Let the total cost of fuel and the drivers' wages for the trip be C dollars. Show that 3

$$C = \frac{69\,000}{v} + 30v.$$

(ii) The van must take no longer than 12 hours to complete the trip, and speed limits require that $v \leq 110$. 4

At what speed v should the van travel to minimise the cost C ?

————— End of Section II —————

END OF EXAMINATION

B L A N K P A G E

The following list of standard integrals may be used:

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$



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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A B C D

Question Two

A B C D

Question Three

A B C D

Question Four

A B C D

Question Five

A B C D

Question Six

A B C D

Question Seven

A B C D

Question Eight

A B C D

Question Nine

A B C D

Question Ten

A B C D