

Question 1 (15 marks) Use a separate page/booklet

Marks

(a) Find: $\int \sqrt{3x-1} dx$

3

(b) By using the substitution $t = \tan \frac{\theta}{2}$, evaluate

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{2 + \sin \theta}$$

3

(c) (i) Split into partial fractions: $\frac{8}{(x+2)(x^2+4)}$

2

(ii) Hence evaluate: $\int_0^2 \frac{8 dx}{(x+2)(x^2+4)}$

3

(d) If $I_n = \int_0^{\frac{\pi}{2}} \cos^n x dx$, ($n \geq 2$)

(i) Show that $I_n = (n-1)I_{n-2} - (n-1)I_n$

2

(ii) Hence evaluate $\int_0^{\frac{\pi}{2}} \cos^6 x dx$

2

Question 2 (15 marks) Use a separate page/booklet

Marks

(a) If $z = 3 + 2i$, plot on an Argand diagram

1

(i) z and \bar{z}

1

(ii) iz

1

(iii) $z(1+i)$

(b) (i) Find all pairs of integers a and b such that $(a+ib)^2 = 8+6i$

1

(ii) Hence solve: $z^2 + 2z(1+2i) - (1+2i) = 0$

2

(c) (i) If $z = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$, find z^6

2

(ii) Plot on an argand diagram, all complex numbers that are the solutions of $z^6 = 1$

2

(d) Sketch the locus of the following. Draw separate diagrams.

(i) $\arg(z-1-2i) = \frac{\pi}{4}$

1

(ii) $\bar{z}z - 3(z+\bar{z}) \leq 0$

2

(iii) $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$

2

Question 3 (15 marks) Use a separate page/booklet

Marks

- (a) For the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$
- (i) Find the eccentricity. 1
 - (ii) Find the coordinates of the foci S and S'. 1
 - (iii) Find the equations of the directrices. 1
 - (iv) Sketch the curve $\frac{x^2}{25} + \frac{y^2}{16} = 1$ 1
 - (v) Show that the coordinates of any point P can be represented by $(5 \cos \theta, 4 \sin \theta)$ 2
 - (vi) Show that $PS + PS'$ is independent of the position of P on the curve. 3
 - (vii) Show that the equation of the normal at the point P on the ellipse is $5x \sin \theta - 4y \cos \theta - 9 \sin \theta \cos \theta = 0$ 3
 - (viii) If the normal meets the major axis at L and the minor axis at M, prove that $\frac{PL}{PM} = \frac{16}{25}$ 3

Question 4 (15 marks) Use a separate page/booklet

Marks

- (a) The depth of water in a harbour on a particular day is 8.2 m at low tide and 14.6 m at high tide. Low tide is at 1:05 pm and high tide is at 7:20 pm. 5
- The captain of a ship drawing 13.3 m water wants to leave the harbour on that afternoon. Find between what times he can leave. (Assume that the tide changes in SHM.)
- (b) If $a > 0, b > 0$ and $c > 0$, show that
- (i) $a^2 + b^2 + c^2 - ab - bc - ca \geq 0$ 2
 - (ii) $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$ 2
 - (iii) $(a+b+d)(b+c+d)(c+a+d)(a+b+c) \geq 8abcd$ 2
- (c) Using mathematical induction prove that $(1+x)^n - nx - 1$ is divisible by x^2 for $n \geq 2, n$ integer. 4

Question 5 (15 marks) Use a separate page/booklet

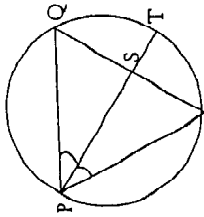
Marks

(a) A concrete beam of length $15m$ has plane sides. Cross-sections parallel to the ends are rectangular. The beam measures $4m$ by $3m$ at one end and $8m$ by $6m$ at the other end.

- (i) Find an expression for the area of a cross-section at a distance x metres from the smaller end. 3
- (ii) Find the volume of the beam. 2

(b) Find the volume of the solid generated by rotating the area bounded by the curve $y = \log_e x$, the x -axis and the line $x = 4$. Use the method of cylindrical shells. Rotate the area about the y -axis and give your answer correct to 1 decimal place. 4

(c)



In the diagram, the bisector of the angle RPQ meets RQ in S and the circum-circle of the triangle PQR in T .

- (i) Prove that the triangles PSQ and PRT are similar. 2
- (ii) Show that $PQ \times PR = PS \times PT$ 2
- (iii) Prove that $PS^2 = PQ \times PR - RS \times SQ$ 2

Question 6 (15 marks) Use a separate page/booklet

Marks

(a) A point is moving in a circular path about O .

- (i) Define the angular velocity of the point with respect to O , at any time t . 1
- (ii) Derive expressions for the tangential and normal accelerations of the point at any time t . 4

(b) A light inextensible string OP is fixed at the end O and is attached at the other end P to a particle of mass m which is moving uniformly in a horizontal circle whose centre is vertically below and distant x from O .

- (i) Prove that the period of this motion is $2\pi \sqrt{\frac{x}{g}}$, where g is the acceleration due to gravity. 3
- (ii) If the number of revolutions per second is increased from 2 to 3, find the change in x . (Take $g = 10 \text{ m/s}^2$)
Give your answer correct to the nearest millimetre. 3

(c) The tangent at $P(a \sec \theta, b \tan \theta)$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets a directrix at Q . S is the corresponding focus.

Given that the equation of the tangent at P is $bx - ay \sin \theta = ab \cos \theta$:

- (i) Find the coordinates of Q . 2
- (ii) Show that PQ subtends a right angle at S . 2

Question 7 (15 marks) Use a separate page/booklet

Marks

- (a) Given $y = \frac{x^3}{x^2 - 4}$
- (i) Find the coordinates of all stationary points. **2**
- (ii) Find the points of intersection with the coordinate axes and the position of all asymptotes. **2**
- (iii) Hence sketch the curve $y = \frac{x^3}{x^2 - 4}$ **2**
- (b) Use the graph $y = \frac{x^3}{x^2 - 4}$ to find the number of roots of the equation $x^3 - k(x^2 - 4) = 0$ for varying values of k . **2**
- (c) Sketch the following curves:
- (i) $y = \log_e(x+1)$ **2**
- (ii) $y = \log_e|x+1|$ **1**
- (iii) $y = |\log_e(x+1)|$ **1**
- (iv) $y = \frac{1}{\log_e(x+1)}$ **3**

Question 8 (15 marks) Use a separate page/booklet

Marks

- (a) Find a polynomial $p(x)$ with real coefficients having 3i and $1+2i$ as zeros. **3**
- (b) A body is projected vertically upwards from the surface of the Earth with initial speed u . The acceleration due to gravity at any point on its path is inversely proportional to the square of its distance from the centre of the Earth.
- (i) Prove that the speed v at any position x is given by $v^2 = u^2 + 2gR^2\left(\frac{1}{x} - \frac{1}{R}\right)$ **3**
- (ii) Prove that the greatest height H above the Earth's surface is given by $H = \frac{u^2 R}{2gR - u^2}$ **3**
- (iii) Show that the body will escape from the Earth if $u \geq \sqrt{2gR}$ **1**
- (iv) Find the minimum speed in km/s with which the body must be initially projected from the surface of the Earth so as to never return. (Take $R = 6400 \text{ km}$, $g = 10 \text{ m/s}^2$) **1**
- (v) If $u = \sqrt{2gR}$, prove that the time taken to reach a height $3R$ above the surface of the Earth is $\frac{14}{3} \sqrt{\frac{R}{2g}}$. **4**