



SAINT IGNATIUS' COLLEGE

Trial Higher School Certificate

2013

MATHEMATICS

Directions to Students

<ul style="list-style-type: none">• Reading Time : 5 minutes	<ul style="list-style-type: none">• Total Marks 100
<ul style="list-style-type: none">• Working Time : 3 hours	
<ul style="list-style-type: none">• Write using blue or black pen. (sketches in pencil).	<ul style="list-style-type: none">• This paper contains two sections. Section 1 contains ten objective response questions. Section 2 contains six free response questions. All questions may be attempted.
<ul style="list-style-type: none">• Board approved calculators may be used	<ul style="list-style-type: none">• Section 1-all questions 1 mark each• Section 2- Q11-16, 15 marks each
<ul style="list-style-type: none">• A table of standard integrals is provided at the back of this paper.	
<ul style="list-style-type: none">• All necessary working should be shown in every question.	
<ul style="list-style-type: none">• Answer each question in the booklets provided and clearly label your name and teacher's name.	

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Section 1 10 Marks

Answer on sheet provided.

1. What is the exact value of $\tan 330^\circ$?

(A) $-\sqrt{3}$

(B) $\sqrt{3}$

(C) $\frac{1}{\sqrt{3}}$

(D) $-\frac{1}{\sqrt{3}}$

2. What is the equation of the normal to the curve $y = x^2 - 4x$ at $(1, -3)$?

(A) $x + 2y - 7 = 0$

(B) $x - 2y - 7 = 0$

(C) $2x - y - 5 = 0$

(D) $2x + y + 5 = 0$

3. What is the value of $\sum_{n=1}^4 n^2$?

(A) 576

(B) 120

(C) 30

(D) 16

4. What is the size of each interior angle in a regular octagon?

(A) $22\frac{1}{2}^\circ$

(B) 80°

(C) 135°

(D) 180°

5. Which of the following is the point of intersection of the two lines $3x - 4y + 6 = 0$ and $x - y - 1 = 0$?

- (A) (0,0)
 - (B) (-2, -3)
 - (C) (10,9)
 - (D) (11,10)
-

6. What are the solutions of the equation $4^x - 5 \times 2^x + 4 = 0$?

- (A) $x = 0, 2$
 - (B) $x = 1, 2$
 - (C) $x = 1, 4$
 - (D) $x = 4, 5$
-

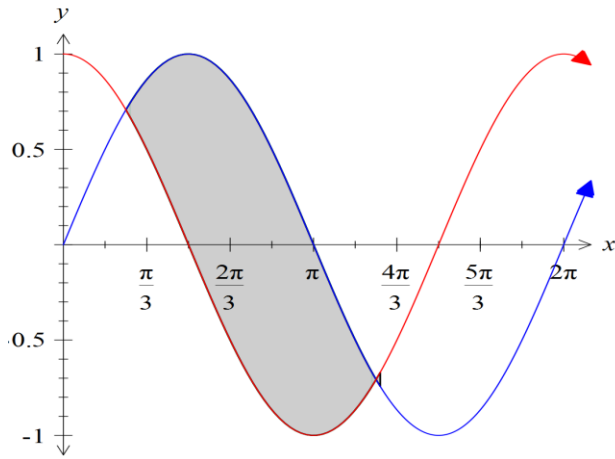
7. Consider the series $\sqrt{5} + \sqrt{45} + \sqrt{125} + \dots + z = 225\sqrt{5}$.
How many terms are there in this series?

- (A) 15
 - (B) 16
 - (C) 225
 - (D) 226
-

8. Which of the following is equal to $\sin \theta$?

- (A) $\tan(90^\circ - \theta)$
 - (B) $\cos(\theta - 90^\circ)$
 - (C) $\sin(180^\circ - \theta)$
 - (D) $\sin(360^\circ - \theta)$
-

9.



Which of the following describes the area given in the graph above?

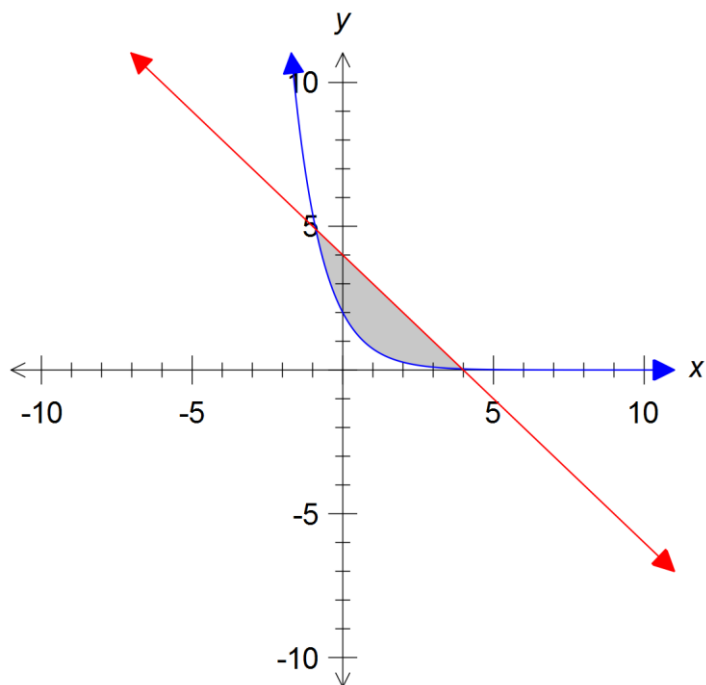
(A)
$$\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} \sin x - \cos x \, dx$$

(B)
$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin x - \cos x \, dx$$

(C)
$$\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} \cos x - \sin x \, dx$$

(D)
$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos x - \sin x \, dx$$

10.



Which of the following describes the region given in the graph above?

- (A) $y \geq e^{-2x}, x + y \geq 4$
 - (B) $y \geq 2e^{-x}, x + y \leq 4$
 - (C) $y \geq e^{-2x}, x + y \leq 4$
 - (D) $y \geq 2e^{-x}, x + y \geq 4$
-

Section 2

Question 11 (Start a new Booklet)

Marks

- (a) Factorise completely $4x^3 - 32$. 2
- (b) Solve $|3x + 6| = 12$ 2
- (c) Solve $10^x = 178$, correct to 4 decimal places 2
- (d) Draw the graph of $x^2 + 4x - 21 + y^2 = 0$ 3
- (e) $A(-2,4)$ and $B(6, -2)$ are points on the number line.
- (i) Calculate the gradient of the line AB . 1
- (ii) Hence show that the equation of the line AB is $3x + 4y = 10$ 1
- (iii) Find the distance between the x and y intercepts of the line AB . 2
- (iv) On the same graph show the region described by 2
- $3x + 4y > 10, x \geq 0, y \geq 0$

Question 12 (Start a new Booklet)

Marks

(a) Differentiate the following:

(i) $3x e^{2x^2}$ 2

(ii) $(3 + \sin(x^2))^4$ 2

(b) (i) Evaluate $\int_1^e \frac{5}{x} dx$ 2

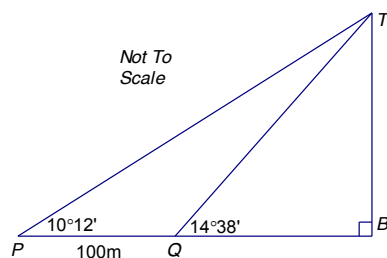
(ii) Evaluate $\int_0^3 2\sqrt{x} + x^3 dx$ 2

(c) An AP has a first term of 2 and a last term of 126. If there are 32 terms in the series, find the sum of the series. 2

(d) 3

The angle of elevation of the top of tree BT when viewed from point P is $10^\circ 12'$.

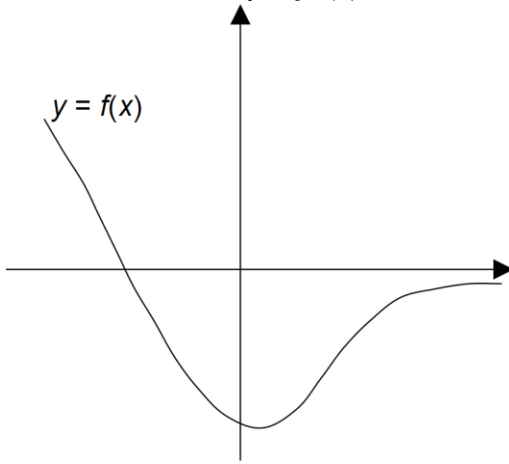
After walking 100m directly towards the tree one arrives at Q where the angle of elevation is $14^\circ 38'$.



Find the height of the tree to the nearest centimetre.

- (e) Copy the following graph into your answer booklet and on the same graph draw the function $y = f'(x)$

2



Question 13 (Start a new Booklet)

Marks

(a) Solve $\cos 2\theta = \frac{1}{\sqrt{2}}$ in the domain $0 \leq \theta \leq 2\pi$

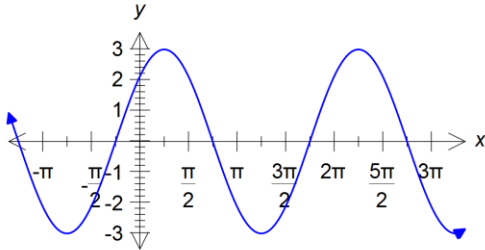
3

(b)

2

The graph given is in the form $y = A \sin(x + \alpha)$.

Find the values of A and α .



(c) Given the parabola $x^2 + (m-2)x + 4 = 0$, find the values of m for which the parabola has no real roots.

3

(d) If α and β are the roots of the quadratic equation $x^2 + 4x - 8 = 0$, calculate:

(i) $\alpha + \beta$

1

(ii) $\alpha\beta$

1

(iii) $\alpha^2\beta + \alpha\beta^2$

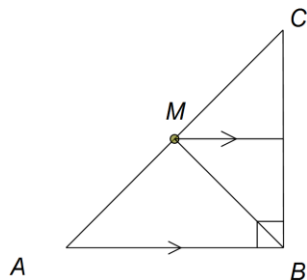
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(iv) $\alpha^2 + \beta^2$

1

(e) In the triangle ABC , M is the midpoint of AC . Prove that M is equidistant from all three vertices of the right angle triangle.

3



Question 14 (Start a new Booklet)

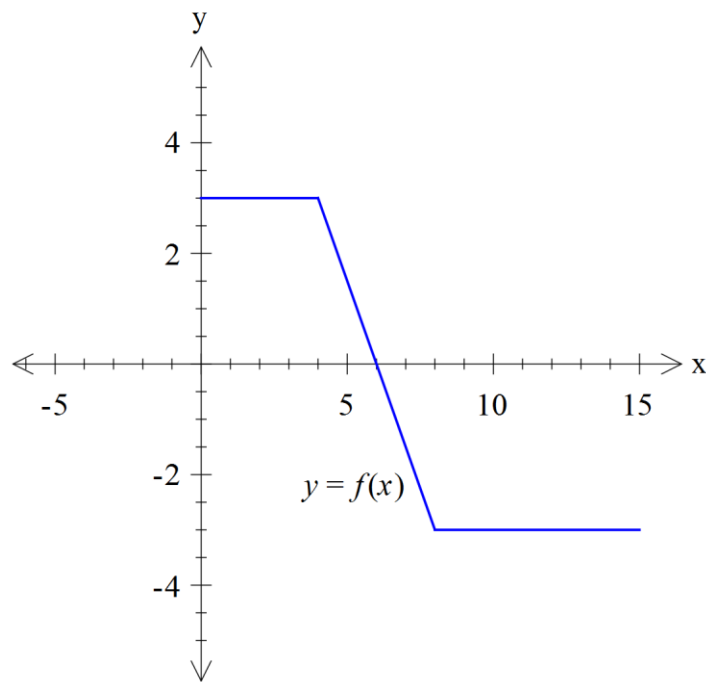
Marks

- (a) Find the equation of the tangent to the curve $y = 2x e^x$ at the point $(1, e)$. 2
- (b) Consider the parabola $y = x^2 + 12$
- (i) Find the coordinates of the vertex and focus of the parabola. 2
- (ii) The area between the parabola and the line $y = 16$ is rotated about the y -axis. Calculate the volume of the solid formed by this rotation leaving your answer in terms of π . 3
- (c) Calculate the approximate area (to two decimal places) between the curve $y = \ln 2x$, the x -axis and the line $x = 2$, using the Trapezoidal Rule with four function values. 4

Question 14 continues on page 12

Question 14 continued

(d)



- (i) Calculate $\int_0^4 f(x) dx$ 1
- (ii) Explain why $\int_4^8 f(x) dx = 0$ 1
- (iii) What is the value of a if $\int_1^a f(x) dx = -6$ 2

Question 15 (Start a new Booklet)

Marks

- (a) Radioactive material is decaying according to the function $R = R_0 e^{-kt}$. There is initially 1 kg of the material and after 20 years there is 0.95 kg of the material remaining.
- (i) Calculate the value of R_0 and k in exact form 2
- (ii) Determine the half-life of the material 2
- (b) A particle is traveling with the acceleration in terms of time given by the expression $\ddot{x} = 4e^{-2t}$. The particle is initially at rest.
- (i) Explain why the particle moves in a positive direction for $t > 0$ 1
- (ii) Find an expression for the velocity of the particle. 2
- (iii) Find the value of the velocity as the acceleration approaches zero. 2
- (c) A couple is wishing to buy a home for \$650 000. They take out a loan at 12% p.a. interest compounded monthly. The term of the loan is 25 years, with repayments paid monthly.
- (i) Show that after the second repayment has been made, the amount outstanding is given by the expression. 1
- $$A_2 = 650\,000(1.01)^2 - M(1.01) - M$$
- where M is the amount of the monthly repayment.
- (ii) Calculate the value of M . 2
- (iii) Instead of paying the amount in (ii) for the loan repayment, the couple pays \$250 more on their loan so that they will pay the amount in less time. By paying this extra money per month, how many months does the couple save on their home loan? 3

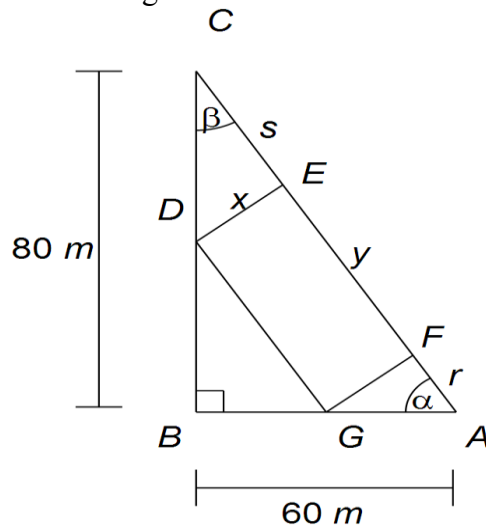
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Question 16 (Start a new Booklet)

Marks

- (a) Consider the curve $y = \frac{2}{3}x^3 - \frac{5}{2}x^2 - 3x + 2$.
- (i) Find the stationary points of the curve and determine their nature. 3
- (ii) Show that there is an inflexion point at $x = 1\frac{1}{4}$ 1
- (iii) Sketch a graph of the function for the domain $-2 \leq x \leq 6$. 2

- (b) A lot of land has the form of a right triangle, with perpendicular sides 60 and 80 metres long.



- (i) Show that $r = \frac{3}{4}x$ and $s = \frac{4}{3}x$ 2
- (ii) Show that $y = 100 - \frac{25}{12}x$ 1
- (iii) Find the length and width of the largest rectangular building that can be erected, facing the hypotenuse of the triangle. 3
- (c) The centres of two circles are 7 cm apart, with one circle having a radius of 5 cm and the other a radius of 3 cm. Find the area of their intersection. 3

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$