

STRATHFIELD GIRLS HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE

2002

MATHEMATICS

*Time allowed – Three hours
(Plus 5 minutes reading time)*

DIRECTIONS TO CANDIDATES

- Attempt **ALL** questions.
- **ALL** questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied.
- Board-approved calculators may be used.
- Each question attempted is to be started on a new page clearly marked Question 1, Question 2, etc.
- You may ask for extra paper if you need it.
- Diagrams are **NOT TO SCALE**.

Students are advised that this is a Trial Examination only and cannot in any way guarantee the content of the HSC Examination

QUESTION 1

Marks

- a) Express $0.\dot{3}8\dot{1}$ as a fraction in simplest form 3
- (b) Solve $|3x - 2| \geq 1$ and graph the solutions on a number line 3
- (c) Evaluate $\frac{2^3}{\sqrt{8+3 \times 2}}$ correct to 3 decimal places 2
- (d) Find the exact value of $\sin^2 60^\circ + \tan^2 30^\circ + \cos^2 60^\circ$ 2
- (e) Find a primitive function for $\frac{2}{x} + \cos 2x$ 2

QUESTION 2 (Start a new page)

Marks

(a) Differentiate with respect to x

5

i) $\log_e x^3$

ii) $x \cos x$

iii) $\frac{e^x}{1-x}$

(b) 

$ABCD$ is a rhombus which has $AB = 65\text{mm}$ and $BX = 60\text{mm}$ as shown

4

i) Explain why $\angle BXA = 90^\circ$

ii) Find the length of AX

iii) State the length of BD giving reasons.

NOT TO SCALE

(d) In an experiment the following values of $f(x)$ were recorded:

3

x	1	1.5	2
$f(x)$	1	1.5	2.7

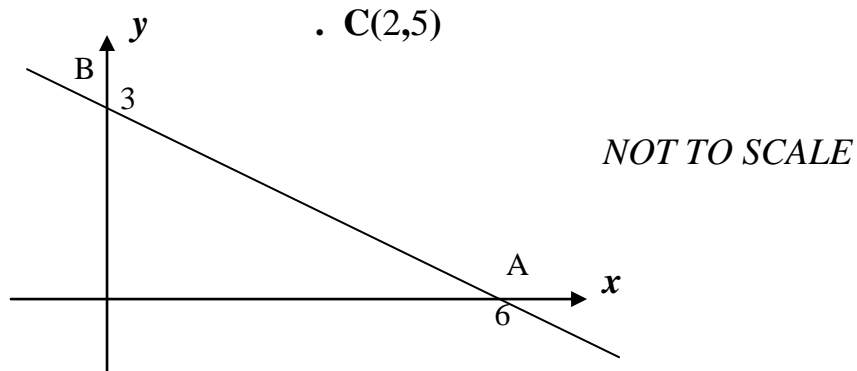
Use the trapezoidal rule to find an

approximation for $\int_1^2 f(x) dx$ giving your answer to

2 significant figures.

QUESTION 3 (start a new page)

Marks



- (a)
- i) Find the gradient of the line **AB**. 1
 - ii) Show that the equation of the line **AB** is $x + 2y - 6 = 0$ 1
 - iii) Find the length of **AB**. 1
 - iv) Find the perpendicular distance of **C** from **AB**. 2
 - v) Hence find the area of the triangle **ABC**. 1
 - vi) There is a point **D** such that **ABCD** is a parallelogram.
Give the coordinates of the point **D**. 2
- (b) The area under the curve $y = e^{2x}$ from $x=0$ to $x=2$ is rotated about the x-axis. 4
Find its volume to 1 decimal place.

QUESTION 4 (Start a new page)

Marks

(a) Evaluate $\int_0^{\frac{\pi}{3}} \sec^2 x \, dx$ 2

(b) The second term of a geometric series is 324 and the fifth term is **12** 3

i) Find the common ratio, r

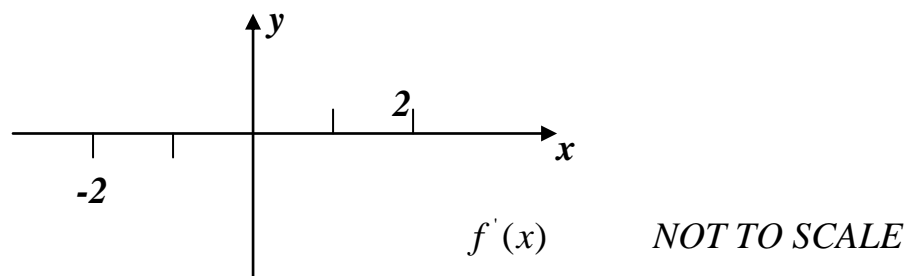
ii) Find the first term, and hence, the sum to infinity

(c) In an arithmetic sequence the tenth term is 58 and the sum of the first **10** terms is **355**. 2

i) Find the first term

ii) Find the common difference

(d) The graph shows the gradient function $y = f'(x)$ 3



Copy this graph onto your answer page and below it sketch $y = f(x)$ showing all essential features.

(e) The number, N , of koalas in a certain area was studied over a period of time, t . At the beginning of the study there were 450 koalas. 2

i) During this period, $\frac{dN}{dt} < 0$.

What does this say about the number of koalas over this time?

ii) During this period $\frac{d^2N}{dt^2} > 0$.

Sketch a graph showing the number of koalas against time.

QUESTION 5 (Start a new page)

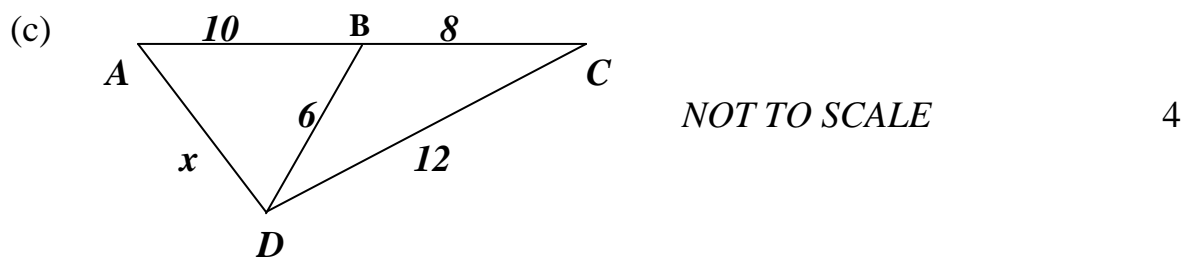
Marks

- (a) If α and β are the roots of the equation $3x^2 - 7x - 6 = 0$, find the value of 3

i) $\alpha + \beta$ and $\alpha\beta$

ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

- (b) Find all the values for θ if $2\sin\theta = 1$ for $0 \leq \theta \leq 2\pi$. 3



- i) Prove that $\triangle DCA$ is similar to $\triangle BCD$.

- ii) Hence find the value of x .

- (d) If $\cos\theta = \frac{7}{25}$ and $\sin\theta$ is negative find $\tan\theta$ as a fraction 2

QUESTION 6 (Start a new page)

Marks

- (a) Six books labeled **A, B, C, D, E** and **F** are placed at random on a shelf. What is the probability that book **A** or book **E** is placed third from the left? 1
- (b) A drawer contains 5 red socks, 3 white socks and 1 black sock. Ms Felicietti takes out 2 socks and puts them on in the dark 7
- i) Construct a probability tree showing this information
 - ii) Calculate the probability that the first sock taken out is red
 - iii) Calculate the probability that Ms Felicietti's socks are the same colour
 - iv) Calculate the probability that Ms Felicietti is wearing odd socks.
- (c) i) Sketch the curve $y = \cos(x + \frac{\pi}{2})$ in the domain $-2\pi \leq x \leq 2\pi$ showing the main features of the graph. 4
- ii) Use your sketch to determine the number of solutions to the equation $\cos(x + \frac{\pi}{2}) = 3x$.

QUESTION 7 (Start a new page)

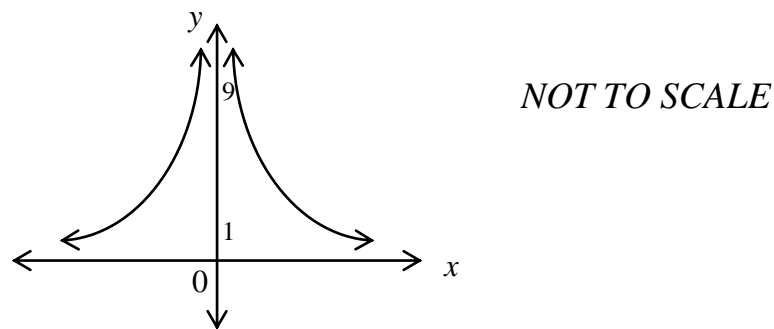
Marks

- (a) For the curve $y = x^3 - 4x$ find 9
- i) where the curve cuts the x-axis
 - ii) any stationary points and determine their nature
 - iii) any points of inflexion
 - iv) hence, sketch the curve for the domain $-3 \leq x \leq 3$
 - v) for what range of x values is the curve concave up and decreasing.
- (b) For what values of k does the quadratic equation, $kx^2 - 8x + k = 0$ have real roots? 3

QUESTION 8 (Start a new page)

Mark

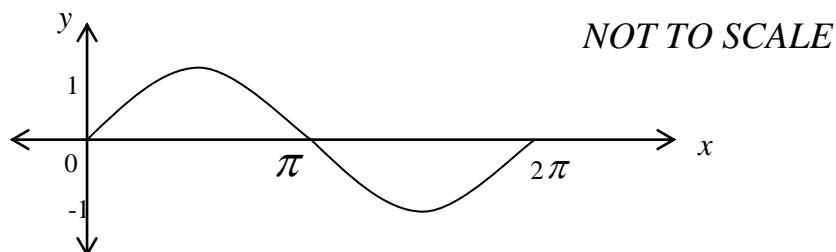
- (a) The region in the first quadrant enclosed by the curve $y = \frac{1}{x^2}$ the y-axis and the line $y = 1$ and $y = 9$ is shown below. Find the area of the region. 3



- (b) Consider the sketch of $y = \sin x$ and explain why

$$\int_0^{2\pi} \sin x \, dx = 0$$

1



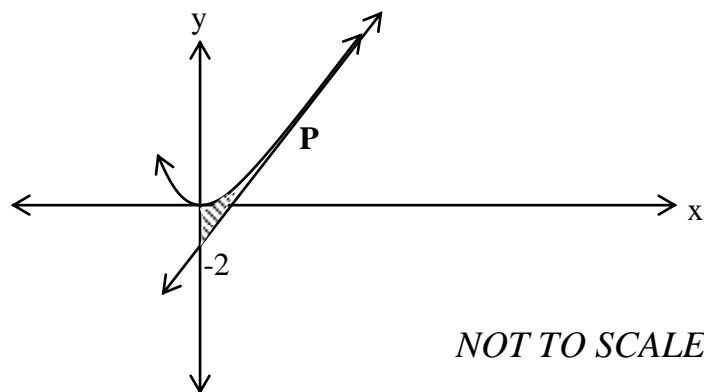
QUESTION 8 (Continued)

Mark

(c) Solve $\log_2 x + \log_2 x^3 = -8$ 2

(d) **\$2,880** is paid into a superannuation fund at the beginning of each year. Interest is paid at the end of each year at the rate of **8%p.a.** compounded annually. Find the amount in the fund at the end of 15 years. 3

(e)



For the shaded area shown 3

i) Show that $(1,1)$ is one point of intersection P of $y = x^2$ and $y = 3x - 2$.

ii) Hence, calculate the shaded area.

QUESTION 9 (Start a new page)

Mark

(a) For the parabola $x^2 = 12y$ find 4

- i) the focus
- ii) the directrix
- iii) the equation of the tangent to the parabola at $x = 3$

(b) If $\log_x 3 = 1.5$ and $\log_x 2 = 1.2$ evaluate 3

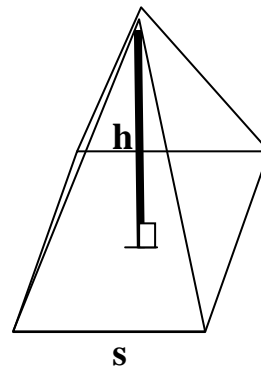
- i) $\log_x 12$
- ii) $\log_x \sqrt[3]{3}$

(c) The diagram shows a right square pyramid. The height (h) plus the side length (s) equals 30cm . 5

- i) Show that the volume of the pyramid is given by

$$V = \frac{1}{3}s^2(30 - s)$$

NOT TO SCALE



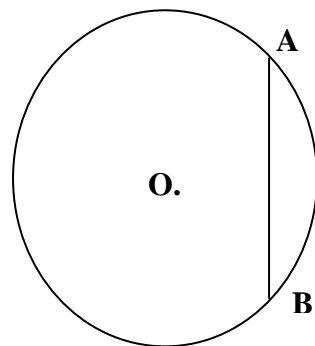
- ii) Find the value of s which maximizes the volume of the pyramid.
- iii) Calculate this maximum volume.

QUESTION 10 (Start a new page)

Mark

(a) Simplify $\frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\sin(\pi - \theta)}$ 2

- (b) A garden bed is in the shape of a circle with a segment removed as shown. The circle has centre O and radius 5 metres. The length of the straight edge AB is $5\sqrt{3}$ metres. Find the exact area of the garden bed. 5



NOT TO SCALE

- (c) A farmer buys a new tractor for \$100 000. The interest rate is 12% p.a. compounded monthly. If the farmer wishes to repay the loan in half yearly instalments, calculate the amount of each instalment if he is to repay the loan in 10 years. 5

END OF EXAM