Name: _____

Teacher: _____

STRATHFIELD GIRLS HIGH SCHOOL

2004

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

All questions may be attempted.
All questions are of equal value.
Start each question on a new page.
All necessary working should be shown in every question.
Full marks may not be awarded for careless or badly arranged work.
Approved calculators may be used.
A standard integral sheet is provided at the end of this paper.

Total marks - 120

Reading time - 5 minutes
Working time - 3 hours

Exam Requirements

1 examination paper
1 Standard Integrals sheet (detach from back of exam)
20 sheets of writing paper

Marks

Question 1 (12 marks) Start a new page

(a) Evaluate $\left(\frac{\sqrt{17}}{6\times10^{-2}}\right)^3$, correct to 3 significant figures **2**

(b) Simplify $\frac{x+2}{2}+\frac{2}{x-2}$. **2**

(c) Solve $x^2+x-12=0$. **2**

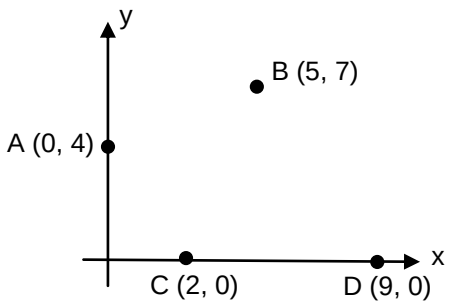
(d) Differentiate $\sqrt{3}e^{4\pi}$ with respect to x **1**

(e) Solve $|7-2x|\geq 5$ and graph your solution on a number line. **3**

(f) Helen and Paris paid \$264 for dinner at the Illium Restaurant. This included a 10% tip. How much was the tip? **2**

Marks

Question 2 (12 marks) Start a new page



The diagram shows the points A(0,4), B(5,7), C(2,0) and D(9,0). Copy this diagram onto your page.

(i) Show that the equation of AB is $3x - 5y + 20 = 0$. **2**

(ii) Show that the length of AB is $\sqrt{34}$. **1**

(iii) Find the coordinates of the midpoint of AB **1**

(iv) Calculate the perpendicular distance of C to AB **2**

(v) Calculate the exact area of triangle ABC **2**

(vi) Show that the size of $\angle BCD$ is $66^\circ 48'$ (to the nearest minute). **2**

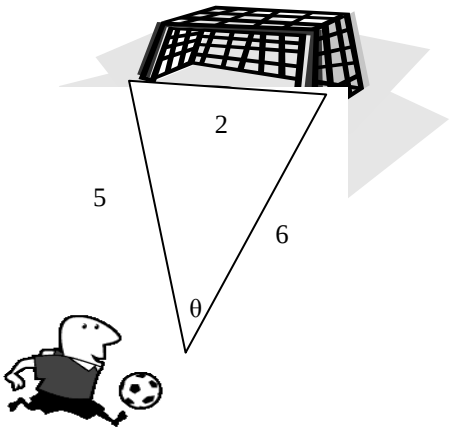
(vii) Hence, or otherwise, calculate the size of $\angle ACB$ (to the nearest degree) **2**

Marks

Question 3 (12 marks) Start a new page

(a) Show that $\frac{4x+5}{2x+1} = 2 + \frac{3}{2x+1}$ and hence find $\int \frac{4x+5}{2x+1}\,dx$ **3**

(b) In a game of playground soccer the goal is 2 metres wide. Tony is 5 metres from the closest goalpost and 6 metres from the other post, as shown in the diagram below. Calculate the size of the angle θ, to the nearest minute. **2**



(c) Differentiate with respect to x:

(i) $\ln(5-2x^3)$ **1**

(ii) x^2e^{2x} **2**

(d) For the parabola $(x-1)^2 = -8(y-2)$,

(i) Find the focal length, *a* **1**

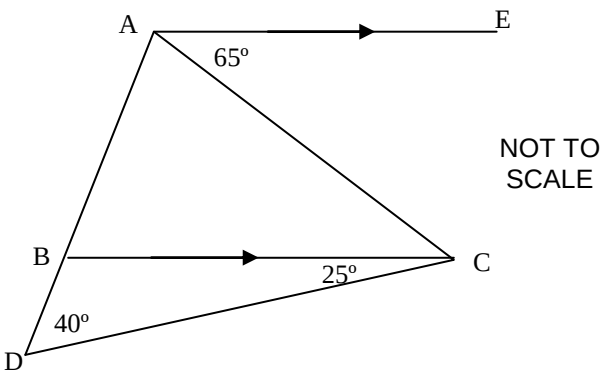
(ii) Sketch it showing clearly the coordinates of the focus and vertex. **2**

(iii) State the equation of the directrix. **1**

Marks

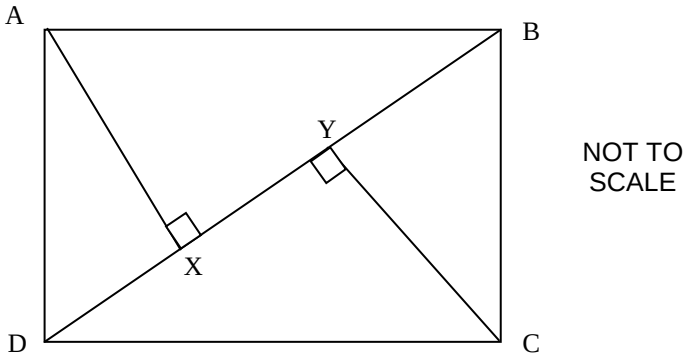
Question 4 (12 marks) Start a new page

(a)



In the diagram above, AE is parallel to BC, <BDC = 40^o, <BCD = 25^o and <EAC = 65^o. **2**
Copy this diagram onto your worksheet.
Prove that ∆ABC is isosceles.

(b)



In the diagram, ABCD is a rectangle. X and Y are points on the diagonal BD such that AX and CY are perpendicular to DB.
Copy the diagram onto your worksheet.

(i) Explain why $\angle ADX = \angle CBY$ **1**

(ii) Prove that $\Delta ADX \equiv \Delta CBY$ **3**

(iii) Hence show that AX = CY **1**

(iv) What type of quadrilateral is AXCY? Justify your answer. **2**

….Question 4 continues next page

Question 4 continued

(c) The roots of the equation $2x^2 - 5x + 12 = 0$ are α and β.

Find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha \beta$ **1**

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ **1**

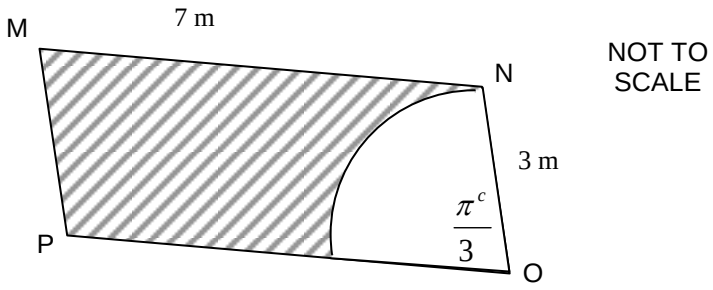
Marks

Question 5 (12 marks) Start a new page

(a) State the domain and range of the function $y = \dfrac{1}{x+2}$ **2**

(b) Find the value of $\log_6 30$ correct to 2 decimal places. **1**

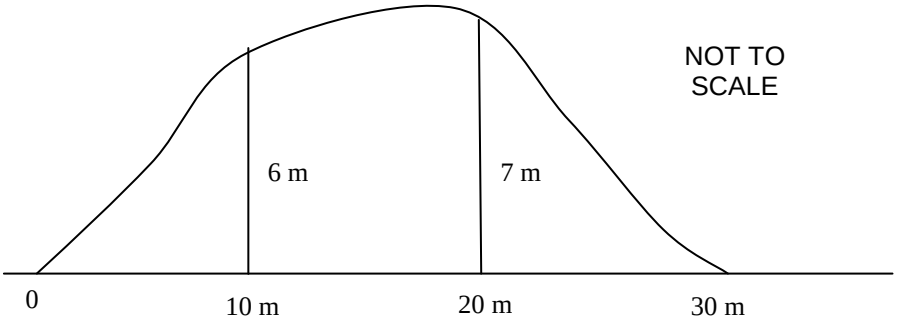
(c)



(i) Find the area of the shaded region of this figure, where the arc is part of a circle centre O and *MNOP* is a parallelogram. **3**

(ii) The shaded region is to be a garden bed. Calculate the perimeter. **2**

(d) The diagram shows a cross-section of a hill to be excavated to make way for a highway 12 metres wide. The height of the hill is shown in relation to the proposed road at 10 metre intervals as shown.



(i) Use the trapezoidal rule to approximate the area of the cross-section. **3**

(ii) Assuming the cross-section to be excavated is constant, find the volume of soil to be removed to the nearest cubic metre. **1**

Marks

Question 6 (12 marks) Start a new page

(a) The first three terms of a sequence are $2, x, 8$

(i) Find x if the sequence is geometric **2**

(ii) Find x if the sequence is arithmetic **1**

(iii) Hence find for the arithmetic sequence

1. the 100^{th} term and **1**

2. the sum of the first 100 terms **1**

(b) For the function $y = x^3 - 3x^2 - 9x + 20$

(i) Sketch the curve.
Clearly indicate the stationary point(s), point(s) of inflection and the y-intercept. **5**

(ii) Find the equation of the tangent to the curve at the point of inflection. (Give your answer in general form) **2**

Marks

Question 7 (12 marks) Start a new page

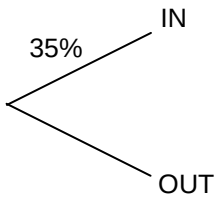
(a) A curve $y = x^2$ and $y = 3x - 2$ intersect at two points.

(i) Show that the points are $(1,1)$ and $(2,4)$ **3**

(ii) Find the area enclosed by the curve and the line. **3**

(b) A tennis player gets a second serve only if the first serve does not go in. Lleyton's first serve has a probability of 35% of going in, and his second serve has a probability of 95% of going in.

(i) Copy and complete the tree diagram below showing the probability on each branch.



1

(ii) What is the probability that Lleyton double faults? (A double fault occurs when a tennis player's first serve is out and then the second is also out.) **1**

(iii) What is the probability that he gets one serve in? **1**

(c) Solve $6e^x = 10319$, giving your answer correct to 3 decimal places **2**

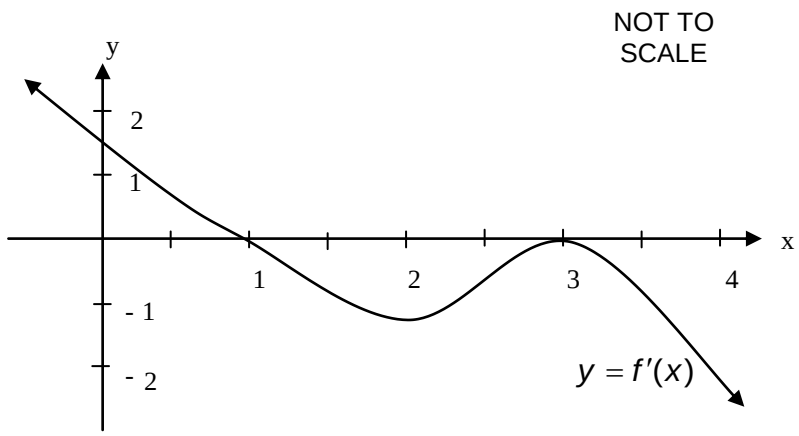
(d) Find the limiting sum of the series $12, 3, \frac{3}{4}, ...$ **1**

Marks

Question 8 (12 marks) Start a new page

(a) Solve $12 + 5x - 2x^2 < 0$ **3**

(b) **3**



Above is a sketch of the gradient function $y = f'(x)$
On your answer page, sketch the function of $y = f(x)$

(c) A pair of dice, numbered 1 to 6, are thrown.

(i) Use a table, or other method, to show all possible outcomes and find the probability that the sum of the two numbers is greater than 9. **1**

(ii) If one of the numbers is a 5, find the probability that the sum of the two numbers is greater than 9. **1**

(d)

(i) Copy and complete the table of values below for the function $y = \log_e(x + 2)$. Record your values to 2 decimal places. **2**

$$y = \log_e(x + 2)$$

x	-1	-0.5	0	0.5	1
y					

(ii) Using the five function values in the table and Simpson's Rule, find an approximate value for $\int_{-1}^{1} \log_e(x + 2)dx$, correct to 1 decimal places. **2**

Marks

Question 9 (12 marks) Start a new page

(a) Jennifer joined a superannuation fund earning 5% pa. She contributed \$1000 at the beginning of each year, to mature at the end of 16 years. At the beginning of the 7^{th} year she increased her contribution to \$1500 per year.

(i) What was the total amount in her fund at the end of the 6^{th} year? **3**

(ii) Show that the total amount in her fund at the end of the 16 years was approximately \$31 444. **3**

(b) At the end of the 16 years Jennifer loaned \$31 000 to her daughter. Under the terms of the loan the interest rate was 8% pa compounded quarterly and the loan was to be repaid quarterly in 12 equal instalments of \$$R$ over 3 years.

Let \$$A_n$ be the amount owing after the *nth* repayment.

(i) Show that after the first repayment **1**

$$A_1 = 31\,620 - R$$

(ii) By writing expressions for A_2, A_3 and A_4 show that **2**

$$A_n = 31\,000 \times 1{\cdot}02^n - R(1 + 1{\cdot}02 + 1{\cdot}02^2 + \ldots + 1{\cdot}02^{n-1})$$

(iii) Find the value of each repayment \$$R$. **3**

Marks

Question 10 (12 marks) Start a new page

(a) For the quadratic **3**

$$\left(p^2+q^2\right)x^2+2q(p+q)x+\left(q^2+r^2\right)=0\,,$$

find in terms of p, q and r the condition for which it will have real roots. Simplify your answer.

(b) An inter-city bus is to travel 800 kilometres at a constant speed of v km/h. When travelling at v km/h, the bus consumes fuel at the rate of

$$\left(6+\frac{v^2}{40}\right) \text{ litres per hour.}$$

The bus company pays 50 cents/litre for LPG fuel and pays each of the two drivers $35 per hour for the duration of the trip.

(i) Let the total cost, for the fuel and drivers, for the duration of the trip be C dollars. Show that **4**

$$C=10v+\frac{58400}{v}$$

(ii) For timetabling reasons the bus must take no longer than 11 hours to complete the trip and may not exceed the speed limit of 100km/h. **5**
At what speed should the bus travel to minimise the cost, C?

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$