QUESTION 1 (12 marks) Marks

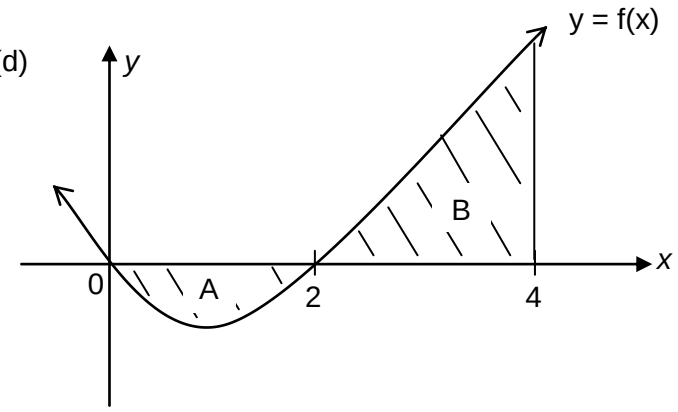
(a) Simplify $(2\sqrt{7}-3)(2\sqrt{7}+3)$ 2

(b) Find $\lim_{x \to 0} \frac{x}{x^2+x}$ 2

(c) Solve for x,

$$3-x \leq \frac{x-1}{2}$$ 2

(d)



NOT TO SCALE

The diagram shows the curve $y=f(x)$ and the areas bounded by the curve and the x-axis.

Area A = 1.3 units² and Area B = 1.8 units²

Write down the value of:

(i) $\int_0^2 f(x)dx$ 1

(ii) $\int_0^4 f(x)dx$ 1

(e) Solve $3^{2x}+2(3^x)-15=0$ 4

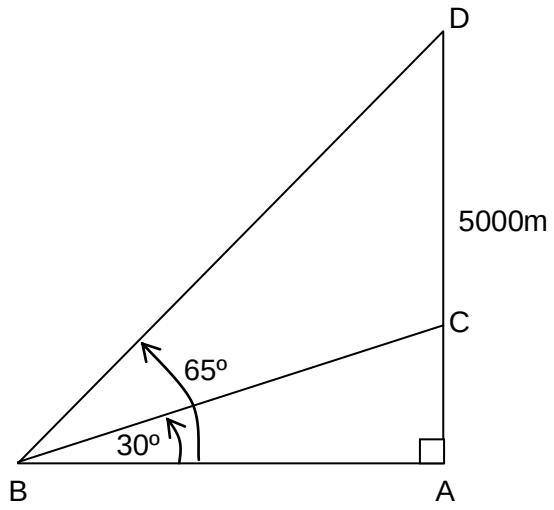
QUESTION 2 (12 marks) Marks

(a) Prove $\dfrac{2\cos^2\theta}{1-\sin\theta} = 2+2\sin\theta$ 3

(b) Solve $ln(2x+15) = 2lnx$ 4

(c) A rocket launched vertically from A is observed from B.

Soon after the launch when at position C its angle of elevation is 30°. After it climbs 5000 metres from this position to D its angle of elevation is 65º



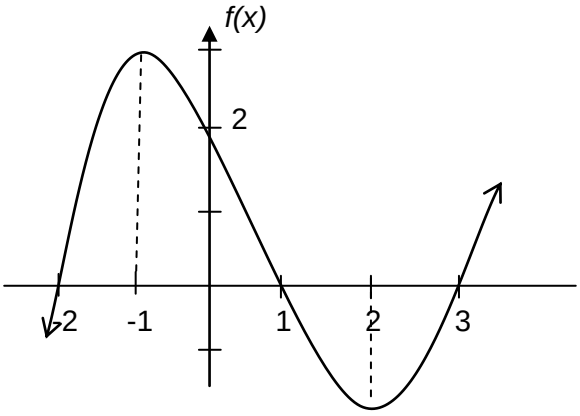
(i) Find BC to the nearest metre. 3

(ii) Find how far the observer is from the launching pad to the nearest metre. 2

QUESTION 3 (12 marks) Marks

(a) Given that the minimum value of the quadratic function $f(x) = x^2 + 2x + k$ is 6, find k. 3

(b) Draw the derivative graph of the given graph below:



2

(c) Is the function $f(x) = \dfrac{x}{x^2 - x^4}$ odd, even or neither? 3

Show full working for your answer.

(d) The line *k* passes through the points (3,0) and (7,2).
What angle does it make with the positive direction of the *x* - axis? 2

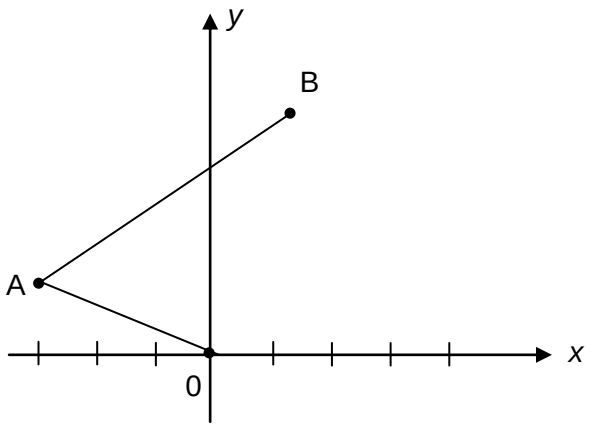
(e) If $\cos x > 0$ and $\sin x = \dfrac{-8}{17}$ find the exact value of $\tan x$. 2

QUESTION 4 (12 marks) Marks

(a) For what value(s) of *x* will this series have a limiting sum? 2

$$1+(x+1)^3+(x+1)^6+(x+1)^9+.....$$

(b)



NOT TO SCALE

A is the point (-3,1) and B is (2,5).
Draw the diagram on your answer page.

(i) Show that the equation of AB is $4x-5y+17=0$ 2

(ii) Show that the length of AB is $\sqrt{41}$ 1

(iii) Calculate the perpendicular distance from 0 to AB 2

(iv) Show that if C is the point (5,4), then A0CB is a parallelogram 3

(v) Find the area of parallelogram A0CB 2

QUESTION 5 (12 marks) Marks

(a) The equation of a parabola is $(x-3)^2 = 12(y+4)$ 4

Find (i) the coordinates of the focus

(ii) the equation of the directrix

(b) Differentiate $y = \dfrac{e^x}{x^2}$ with respect to x. 3

(c) A square chess board contains alternating black and white squares with eight squares along each side. Dylan places **one** grain of rice on the first square, **two** grains of rice on the second square, **four** grains on the third, **eight** on the fourth, and so on until every square is covered.

(i) Find the number of grains she places on the last square. 3

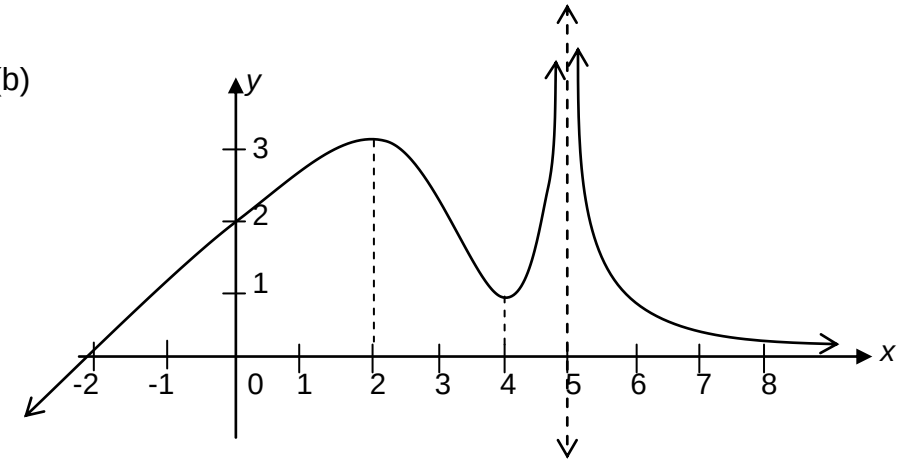
(ii) How many grains does she need altogether? 2

[Leave your answers to (i) and (ii) in index form]

QUESTION 6 (12 marks) Marks

(a) Differentiate $y = 3x^2(x-2)^4$ giving your answer in fully factorised form. 3

(b)



The diagram above shows the graph of a certain function $y = f(x)$.
It is known that the graph has stationary points at $x = 2$ and $x = 4$.

(i) By studying the graph, complete the statement below:

$f(0) =$ ☐ 1

(ii) By studying the graph, complete the statement below by inserting =, >, <

(α) $f'(1)$ ☐ 0 1

(β) $f'(2)$ ☐ 0 1

(c) Sketch a curve satisfying the following conditions for $a \le x \le b$:

$f'(x) > 0$ *and* $f''(x) < 0$ 2

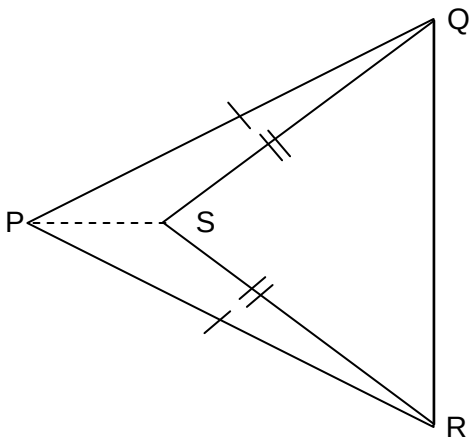
(d) Use the Trapezoidal Rule with 5 function values to find the area represented by:

$\int_1^3 \log_e x \, dx$ 4

Answer to 2 decimal places

QUESTION 7 (12 marks) Marks

(a)



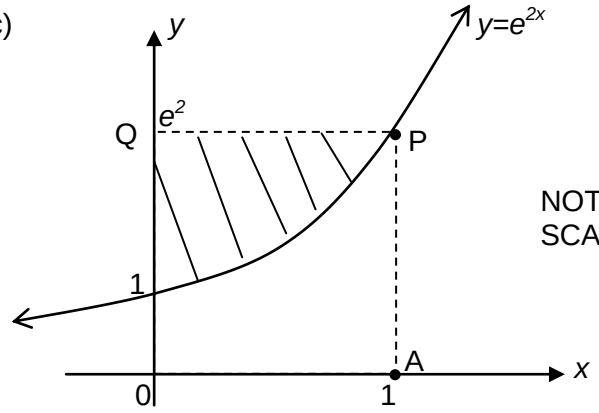
In the diagram PQ = PR and SQ = SR

Copy the diagram onto your answer sheet. 4

Prove giving reasons, that SP bisects $\angle$QPR

(b) Find $\int \frac{dx}{2x+3}$ 2

(c)



NOT TO SCALE

Q is $(0,e^2)$
A is $(1, 0)$

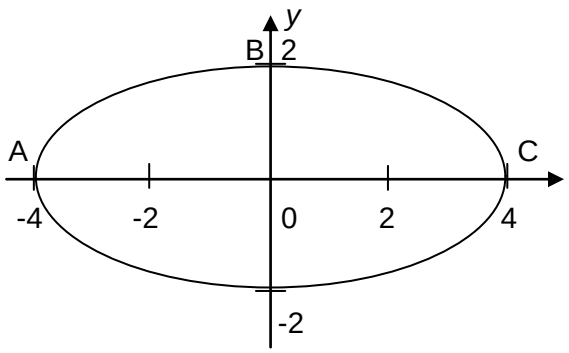
(i) Find the coordinates of P 1

(ii) Find the area of rectangle 0APQ 1

(iii) Find the shaded area (give your answer in terms of e) 4

QUESTION 8 (12 marks) Marks

(a)



5

This is the sketch of an ellipse whose equation is $x^2 + 4y^2 = 16$.
The arc ABC is rotated about the *x*-axis.
Find the volume enclosed by the surface that is traced out (give answer in exact form).

(b) Consider the function $f(x) = x^3 - 12x + 20$ and its domain $-3 \le x \le 3$ 7

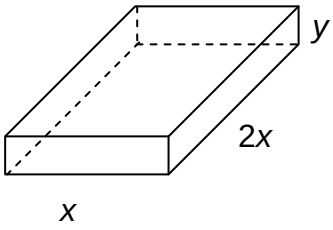
(i) Find the stationary point(s) and determine their nature.

(ii) Use the results to clearly sketch the curve showing where it cuts the y- axis.(Points of inflexion not necessary).

QUESTION 9 (12 marks) Marks

(a) The sum S_n of the first n terms of a certain arithmetic series is $2n + 3n^2$. Find an expression for the nth term T_n of the series. 3

(b) Boxes in the shape of rectangular prisms are to be constructed from special materials. The width of the base (x metres) is to be half the length of the base and each box is to hold a volume of 4 cubic metres. Material that is used to build the base and top costs \$15 per m². A cheaper material at \$10 per m² is used for the four sides.



(i) Show the height of the box is given by $y = \frac{2}{x^2}$ 1

(ii) Show that the total cost (\$C) of building each box is given by

$$C = 60x^2 + \frac{120}{x^2}$$ 3

(iii) What is the width of the base of the cheapest box that can be constructed. 5

QUESTION 10 (12 marks) Marks

(a) On Lauren's birth her grandmother opened an account for her and deposited $10 000. Thereafter on each birthday, she deposited $1000, making her last deposit on Lauren's 18th birthday. The money remained in the account until Lauren turned 21. On that day, her grandmother withdrew the money and presented it to Lauren.

If the account earned 6% p.a. compound interest, find:

(i) The value of the first $10 000 when the account closed 1

(ii) Find the total amount of money that Lauren's grandmother presented to her on her 21st birthday. 4

(b) α and β are the roots of the equation $x^2 + 4x + 2 = 0$

(i) Find the value of $\alpha^2 + \beta^2$ given that

$\alpha + \beta = -4$ and $\alpha\beta = 2$ 2

(ii) Hence or otherwise write down the equation whose roots are

$\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ 5