

Question One: (12 marks)**Marks**

(a) Find the value of $\sqrt{\frac{12.35 - 8.66}{6.5}}$, correct to 2 decimal places.

2

(b) A function, $f(x)$, is defined as:

$$f(x) = \begin{cases} 2x + 1, & \text{for } x \leq 1 \\ -4, & \text{for } x > 1 \end{cases}$$

Evaluate $f(1) + f(4)$.

2

(c) For what value of 'k' will $kx^2 + 2kx - 3 = 0$ have equal roots?

3

(d) Write the inequality which represents the locus of all points which lie less than 6 units from the point (1,-2).

1

(e) Write $\frac{7}{\sqrt{10} - 2}$ in the form $a + b\sqrt{10}$.

2

(f) Solve $|x - 2| = 7$.

2

**End of Question One.
Start Question Two on a new page!**

Start a new page!

Question Two: (12 marks)

Marks

(a) If $\log_a 2 = 0.41$ and $\log_a 3 = 0.55$ find $\log_a 54$.

3

(b) Find the primitive function of $3x^2 - \frac{1}{x^4}$.

2

(c) Differentiate with respect to 'x':

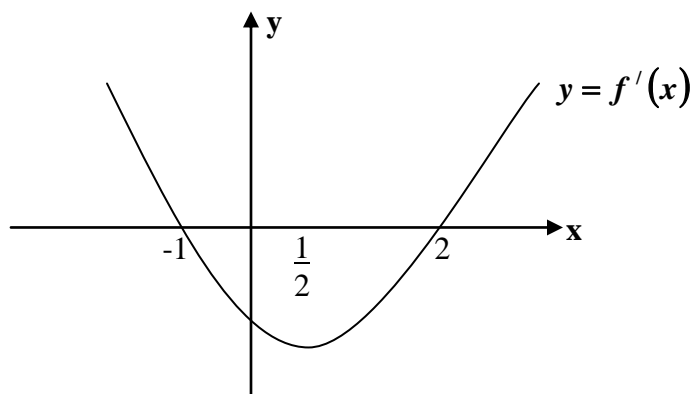
$$f(x) = \sqrt{1-x^2}$$

2

(d) State the exact value of $\cot 300^\circ$.

2

(e) The graph of $y = f'(x)$ is shown.



Draw a neat sketch of $y=f(x)$.

3

**End of Question Two.
Start Question Three on a new page!**

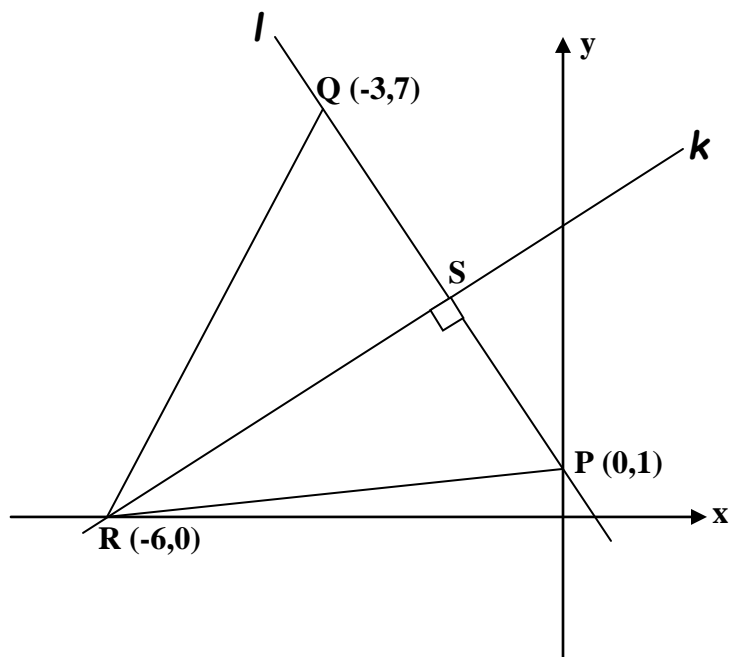
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Question Three: (12 marks)

Marks

In the diagram below, P, Q and R are the points (0,1), (-3,7) and (-6,0), respectively.

Use this information to answer the following questions, (a) to (e).



(a) Calculate the

(i) gradient of PQ 1

(ii) exact length of PQ 1

(b) Line 'l' passes through P and Q. Find the equation of line 'l'. 2

(c) Line 'k' is perpendicular to line 'l' and passes through R.
Show that the equation of line 'k' is $x - 2y + 6 = 0$. 2

Question Three is continued on the next page!

Question Three: (continued)

	Marks
(d) Find the coordinates of S, the point of intersection of lines ' l ' and ' k '.	2
(e) (i) Calculate the shortest distance from R to line ' l '.	2
(ii) Hence find the area of the triangle PQR.	2

**End of Question Three.
Start Question Four on a new page!**

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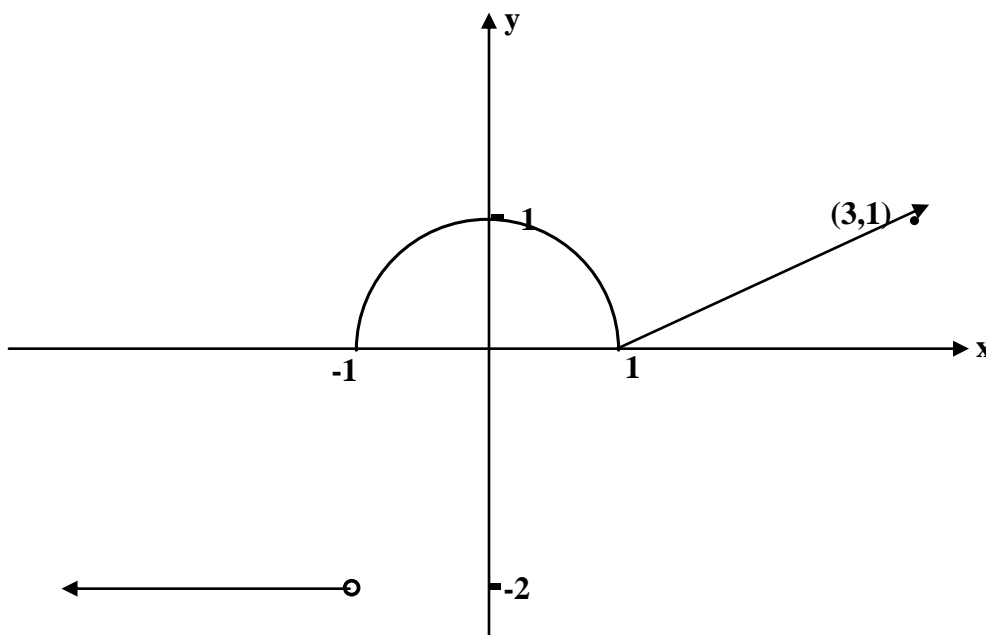
Question Four: (12 marks)

Marks

- (a) Find the equation of the normal to the function $y = x^3 - 4x - 1$ at the point where the curve crosses the y-axis.

3

- (b) The diagram above represents the function $y = f(x)$.



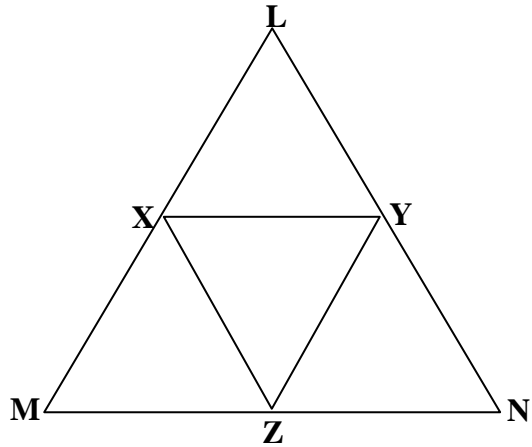
For each partial domain, write the equations which define $f(x)$.

5

Question Four continued on next page

Question Four: (continued)

- (c) In the diagram below, points X, Y and Z are the midpoints of the sides of triangle LMN and $LM = LN$.



- (i) Copy the diagram onto your writing paper and label this information.
- (ii) Prove $XM = YN$. **1**
- (iii) Prove $XZ = YZ$. **3**

End of Question Four.
Start Question Five on a new page!

Start a new page!

Question Five: (12 marks)

Marks

(a) A parabola is defined by the equation $(x - 3)^2 = -8(y - 3)$.

Find

(i) the coordinates of the vertex **1**

(ii) the coordinates of the focus **1**

and (iii) the equation of the directrix **1**

(b) Find the area of the circle whose equation is given by
 $x^2 + 10x + y^2 - 2y + 1 = 0$. **2**

(c) Find the gradient function of $y = x \ln(1 - 3x)$ **2**

(d) (i) Use one application of Simpson's Rule to estimate the area between the curve $y = e^{-2x}$ and the x-axis, between $x = -1$ and $x = 1$. Give your answer in terms of 'e'. **3**

(ii) Find the exact area, in terms of 'e', between the curve $y = e^{-2x}$ and the x-axis, from $x = -1$ to $x = 1$. **2**

End of Question Five.
Start Question Six on a new page!

Start a new page!

Question Six: (12 marks)

Marks

(a) (i) State the condition for a function to be odd. **1**

(ii) If a function $g(x)$ is odd and $g(-9)=5$, find $g(9)$. **1**

(b) Solve $\log_4 x = 3$. **1**

(c) Find the equation of the curve whose gradient function is given by

$$\frac{dy}{dx} = \frac{1}{2x+1} \quad \text{and when } x = 0, y = 2. \quad \mathbf{4}$$

(d) A series is defined by $\sum_{n=3}^{200} (5 - 2n)$.

(i) Write the first three terms of the series. **1**

(ii) Hence evaluate $\sum_{n=3}^{200} (5 - 2n)$ using the appropriate formula.

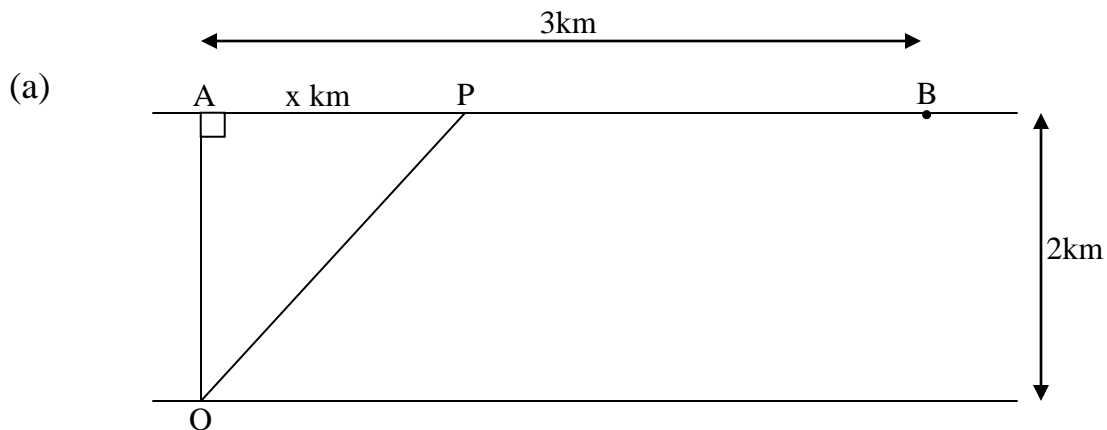
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End of Question Six.
Start Question Seven on a new page!

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Question Seven: (12 marks)

Marks



The diagram above shows a straight section of river with width 2km. Ariel is at the point O on the south bank and wishes to reach point B, on the opposite bank. Point A is directly opposite O and B is 3km down river from A.

Ariel can row at 5km per hour and jog at 13km per hour. She intends to row in a straight line from O to P and then jog directly from P to B. Let the distance from A to P be x km.

Show that

- (i) the time T , in hours, for Ariel to reach B is given by

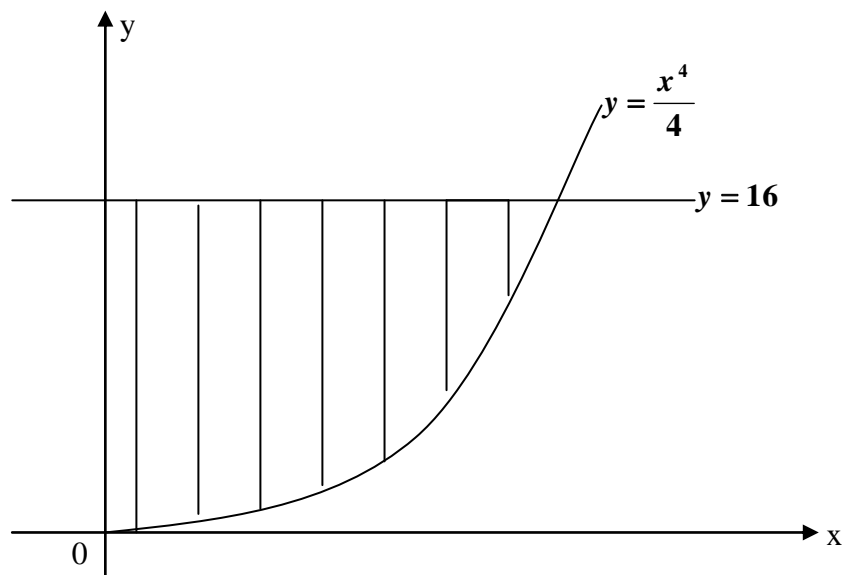
$$T = \frac{\sqrt{x^2 + 4}}{5} + \frac{3 - x}{13}. \quad 2$$

- (ii) if Ariel wishes to minimize the time taken to complete her journey from O to B, she should row to a point P which is five-sixths of a kilometre from A. 3

Question Seven continued on next page!

Question Seven: (continued)**Marks**

(b)



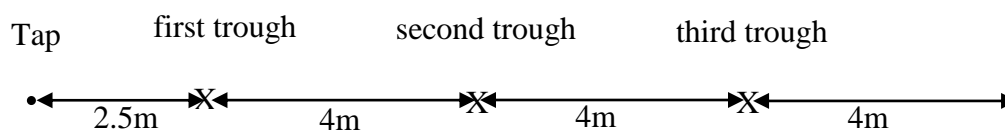
The shaded region is bounded by the curve $y = \frac{x^4}{4}$, the y-axis and the line $y = 16$.

Calculate the volume of the solid of revolution, formed by rotating this region about the y-axis.

3

(c) A stable hand fills troughs with water. She carries a bucket of water from the tap to each trough and then returns to the tap. When the last trough is filled, she returns to the tap to hang up the bucket.

The tap and the 'n' troughs are in a straight line. The tap is 2.5 metres from the first trough and the distance between each pair of consecutive troughs is 4 metres, as shown in the diagram.



Question Seven continued on next page!

Question Seven: (continued)**Marks**

(i) How far does the stable hand walk to fill the n th trough?

2

(ii) If the stable hand walks 1314 metres to fill all n troughs, how many water troughs are there?

2

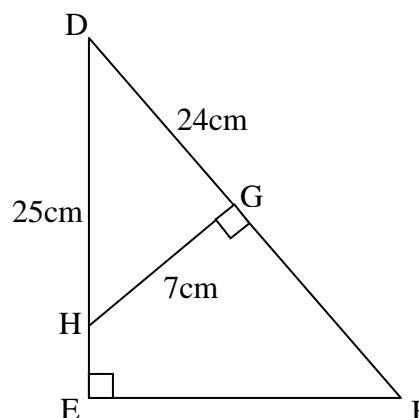
**End of Question Seven.
Start Question Eight on a new page!**

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Question Eight: (12 marks)

Marks

- (a) In the diagram, $\angle DEF$ and $\angle DGH$ are right angles, $DH = 25\text{cm}$, $DG = 24\text{cm}$, $GH = 7\text{cm}$ and G is the midpoint of DF .



- (i) Prove that triangles DGH and DEF are similar. **2**
- (ii) Find the length of EF . **3**
- (b) For all the values of 'x' in the domain $-2 \leq x \leq 4$, a function $f(x)$ satisfies the conditions $f'(x) < 0$ and $f''(x) > 0$.
- Sketch a possible graph of $y = f(x)$ in this domain. **3**
- (c) The sum of the first three consecutive terms of an arithmetic sequence is 24 and the sum of the squares of the same three terms is 200.
- (i) Find an expression for 'a' in terms of 'd'. **4**
- (ii) Hence, or otherwise, find the three terms.

End of Question Eight.
Start Question Nine on a new page!

Start a new page!

Question Nine: (12 marks)

Marks

(a) Kate is buying a new car and has signed a contract to borrow \$18 000 from the Loans R Us finance company. She has agreed to repay the loan over the next four years, incurring an interest rate of 6.6% per annum, with equal monthly instalments.

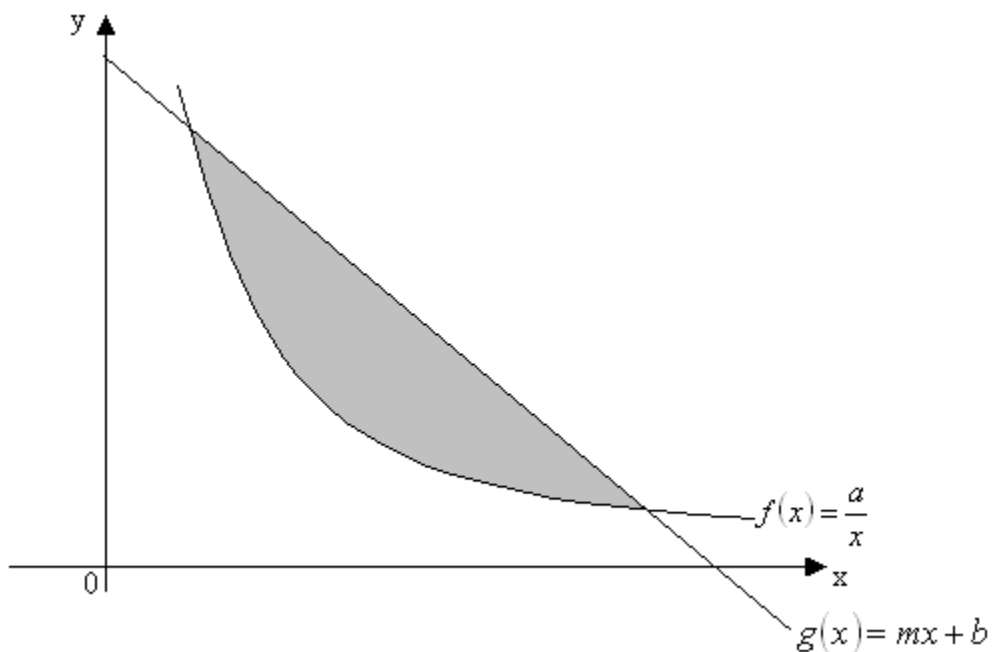
Kate has met the criteria to be considered an ‘outstanding customer’ and, consequently, the first month has been declared an interest-free period.

- (i) If the amount of each monthly instalment is \$M, find:
1. the amount Kate will owe after she makes the first instalment; **1**
 2. the amount owing after the fourth instalment. **1**
- (ii) Find the value of M. **3**
- (iii) What will be the total cost of the car? **1**

Question Nine continued on next page!

Question Nine: (continued)**Marks**

- (b) The diagram shows two functions, $f(x) = \frac{a}{x}$ and $g(x) = mx + b$.
The points of intersection are (2,4) and (8,1).



- (i) Find the values of 'a', 'm' and 'b' and hence write the equations of $f(x)$ and $g(x)$.

3

- (ii) Show that the shaded area bounded by the curves $f(x)$ and $g(x)$ is $A = (15 - 16 \ln 2)$ square units.

3

**End of Question Nine.
Start Question Ten on a new page!**

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Question Ten: (12 marks)

Marks

- (a) If α and β are the roots of the quadratic equation $5x^2 - 3x - 1 = 0$, find the value of the expression

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2}.$$

3

- (b) Consider the function $y = \frac{\ln x}{x}$, in the domain $0 < x < 5$.

- (i) Find the coordinates of the stationary point, in terms of 'e', and determine its nature.

4

- (ii) State the condition/s for points of inflexion and then find the coordinates of the point of the point of inflexion for this function.

2

- (iii) Sketch the graph of $y = \frac{\ln x}{x}$, in the stated domain.

3

**End of Question Ten.
End of examination.**